# "A Computational Framework for Analyzing Dynamic Procurement Auctions: The Market Impact of Information Sharing"

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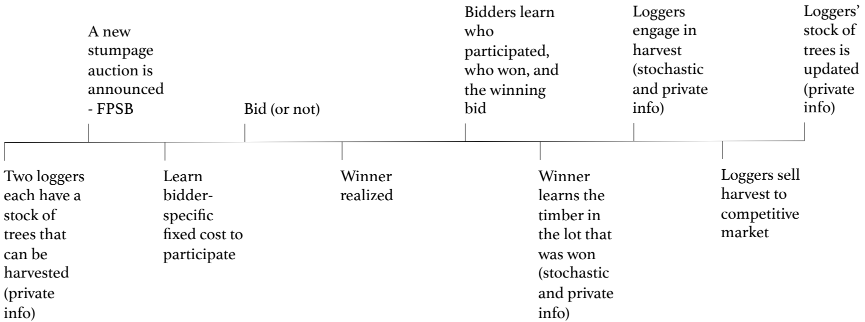
# Motivation

- Research Question:
  - What is the impact on competition of information sharing among bidders in a series of interlinked auctions?
    - How might dynamic consideration shape thinking?
  - How to formulate a modeling environment that can address questions like these, that also allows for substantive dynamics
- Approach:
  - Computational model - new theory and analysis.
- Why is this interesting?
  - Information sharing is somewhat neglected area of competition policy, recently relevant in several merger and conduct settings
  - Common applied mechanism design concern
  - Computational oligopoly models have tended to struggle with asymmetric information

# The setting: Infinitely repeated game, stage game looks like:



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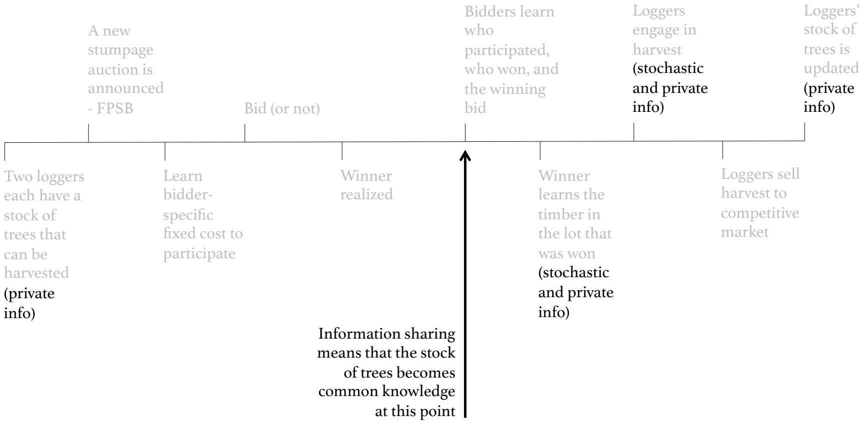


# Context: Antitrust treatment of information sharing

- US: "The sharing of information relating to price, cost, output, customers, or strategic planning is more likely to be of competitive concern than the sharing of less competitively sensitive information."
  - FTC/DOJ Collaboration Guidelines
  - Rule of reason approach in conduct cases. Cases are pretty rare in modern era. Issues have arisen in mergers: falls under broad rubric of coordinated effects.
- EU: Sharing of information relating to future price is a "restriction of competition by object". This may include non-price but strategically relevant information (See *Dole Foods*).
  - Conduct cases are more common in modern era.

# Takeaways

- Information sharing increases the precision with which a firm knows its rivals' states
  - For some states, this intensifies competition (e.g. when both firms have low inventory)
  - Firms increase participation to avoid these states
  - Thus, participation increases, and quantity increases.
  - But prices drop, as more time in spent in states where competition is less intense.

- The "price low bad, price high good" intuition for assessing competition between bidders seems poorly suited to this environment.

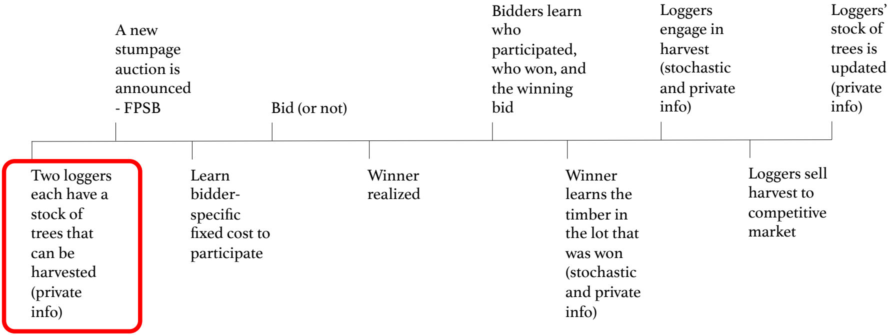
# Related literature

- Dynamic oligopoly games
  - Doraszelski and Pakes (2007)
  - Saini (2013), Jeziorski and Kasnokutskaya (2016), Jofre-Bonet and Pesendorfer (2003)
  - Fershtman and Pakes (2012)
- Auctions
  - Maskin and Riley (2000), Athey, Levin and Seira (2011)
- Bid rigging
  - Baldwin, Marshall and Richard (1997), Marshall and Marx (2013)
- Information sharing
  - Gal-Or (1985,1986), Shapiro (1986), Kuhn and Vives (1995)

# Map of talk

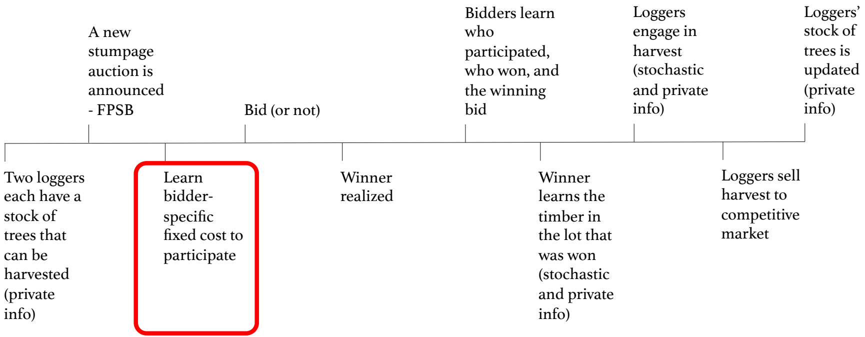
- **Model set-up**
- Equilibrium
- Adding information sharing
- Computation and parametrization
- Results
- Conclusion

# Model set-up




- Inventory of timber is $\omega_{it}$, will usually drop the $t$ subscript

# Model set-up




- Inventory of timber is $\omega_{it}$
- Fixed cost is $F_{it} \sim U[F_l, F_h]$, i.i.d. across bidders (private info)

# Model set-up

Two loggers each have a stock of trees that can be harvested (private info)

A new stumpage auction is announced - FPSB

Learn bidder-specific fixed cost to participate

Bid (or not)

Winner realized

Bidders learn who participated, who won, and the winning bid

Winner learns the timber in the lot that was won (stochastic and private info)

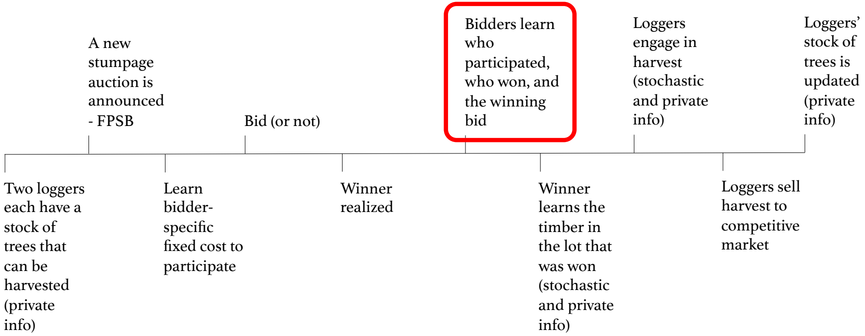
Loggers engage in harvest (stochastic and private info)

Loggers sell harvest to competitive market

Loggers' stock of trees is updated (private info)

- Inventory of timber is $\omega_{it}$
- Fixed cost is $F_{it}$
- Bid $\in \{\varnothing, \underline{b}, ..., 3\underline{b}, ...\overline{b}\}$

# Model set-up




- Inventory of timber is $\omega_{it}$; Fixed cost is $F_{it}$; Bid $\in \{\varnothing, \underline{b}, ..., 3\underline{b}, ...\overline{b}\}$
- $I_{i,t} = (J_{i,t}, F_{i,t})$, $J_{i,t} = (\omega_{i,t}, \xi_t)$, $\xi_t^n \equiv [i_t^w, b_t^*, p_t]$ or, if information exchange, $\xi_t^n \equiv [i_t^w, \omega_t]$

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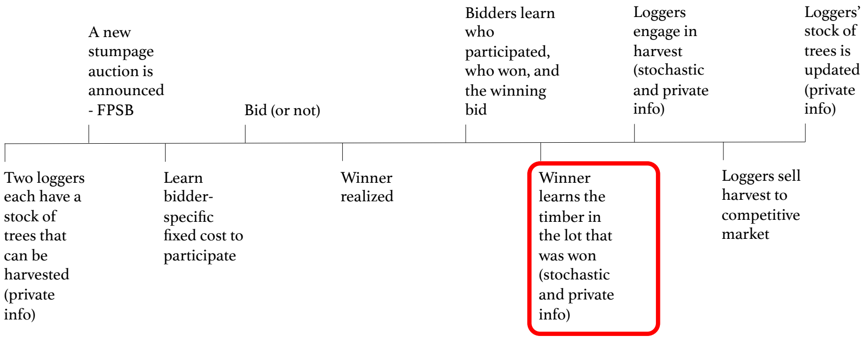


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- Timber in lot is given by $\theta + \eta_t$ where $\theta$ is the average amount and $\eta_t$ is an i.i.d discrete random variable.

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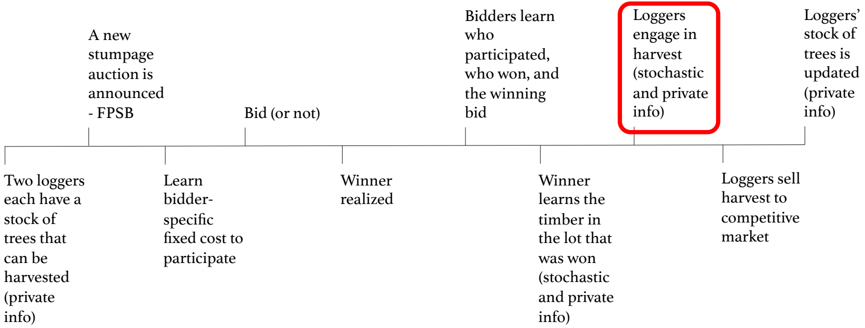


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- Timber in lot is given by $\theta + \eta_t$
- Harvest is given by $e + \epsilon_{i,t}$ where $\epsilon_{i,t}$ is a discrete random variable

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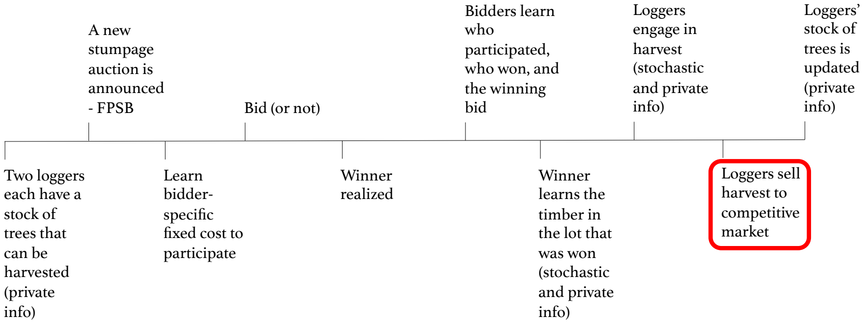


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- Timber in lot is given by $\theta + \eta_t$
- Harvest is given by $e + \epsilon_{i,t}$
- Each unit of harvested $\omega$ sells for a price of 1

# Model setup, other issues

- State space consists of pay-off relevant and informationally relevant variables - not full history
  - Pay-off relevant: current profits depend on it and it is not a control
  - Informationally relevant: even if no other player conditions on the variable for play, it is profit increased by conditioning on it (i.e. revealing about private states in some way)
- Every T periods all information revealed to everyone. Needed for current existence proofs and computational feasibility (finite state space).
- There is a discount factor, $\beta$

# Model set-up, dynamic system

$$V(J_i, F_i) = \max\left\{W(\varnothing|J_i), \max_{b\in\mathcal{B}}[W(b|J_i) - F_i]\right\} \tag{1}$$

Letting $\beta$ be the discount factor, the firm's expectation of current period revenue (which excludes $F_i$) is

$$\pi^e(b|J_i) = \sum_{\epsilon_i,\eta}\left[p^w(b|J_i)\Big(\min\{\omega_i+\theta+\eta, e+\epsilon_i\}-b\Big)+[1-p^w(b|J_i)]\min\{\omega_i, e+\epsilon_i\}\right]p(\epsilon \tag{2}$$

It follows that, for $b \in \mathcal{B}$,

$$W(b|J_i) = \pi^e(b|J_i) + \tag{3}$$

$$p^w(b|J_i)\beta \sum_{\epsilon_i,\eta,\xi' F_i'}\Big(\omega'(\omega,\eta,\epsilon_i),\xi',F_i'\Big)p(\xi'|\xi,\omega_i,b,i=i_w)p(F_i')p(\eta)p(\epsilon_i)$$

$$+(1-p^w(b|J_i))\beta \sum_{\epsilon_i,\xi',F_i'} V\Big(\omega'(\omega,\epsilon_i),\xi',F_i'\Big)p(\xi'|\xi,\omega_i,b,i\neq i_w)p(F_i')p(\epsilon_i)$$

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# Model set-up, dynamic system

$$V(J_i, F_i) = \max\left\{W(\varnothing|J_i), \max_{b\in\mathcal{B}}[W(b|J_i) - F_i]\right\} \tag{4}$$

Letting $\beta$ be the discount factor, the firm's expectation of current period revenue (which excludes $F_i$) is

$$\pi^e(b|J_i) = \sum_{\epsilon_i,\eta}\Big[p^w(b|J_i)\Big(\min\{\omega_i{+}\theta{+}\eta, e{+}\epsilon_i\}{-}b\Big){+}[1{-}p^w(b|J_i)]\min\{\omega_i, e{+}\epsilon_i\}\Big]p(\epsilon \tag{5}$$

It follows that, for $b \in \mathcal{B}$,

$$W(b|J_i) = \pi^e(b|J_i) + \tag{6}$$

$$p^w(b|J_i)\beta \sum_{\epsilon_i,\eta,\xi' F_i'}\Big(\omega'(\omega, \eta, \epsilon_i), \xi', F_i'\Big)p(\xi'|\xi, \omega_i, b, i = i_w)p(F_i')p(\eta)p(\epsilon_i)$$

$$+(1 - p^w(b|J_i))\beta \sum_{\epsilon_i,\xi',F_i'} V\Big(\omega'(\omega, \epsilon_i), \xi', F_i'\Big)p(\xi'|\xi, \omega_i, b, i \neq i_w)p(F_i')p(\epsilon_i)$$

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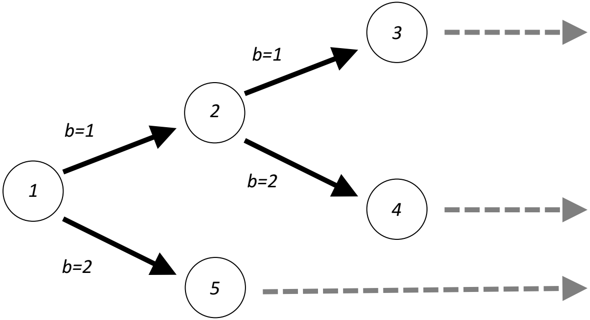
# Equlibrium, REBE



# Equlibrium, REBE

**Definition of a REBE:**
A restricted experience based equilibria consists of the following three objects.

1. A set $\mathcal{R}$ that is a subset of the state space
2. Bidding and participation strategies, $b^*(J_i, F_i)$
3. A set of numbers $\mathcal{W} \equiv \{W^*(b|J_i)_{b \in \mathcal{B} \cup \varnothing}\}$ representing the firm's perceptions of the expected discounted value of bid $b$

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For these objects to define a REBE they must satisfy the following three conditions.

**C1:** $\mathcal{R}$ **is a recurrent class.** That is, with probability one, any subgame starting from an $s_0 \in \mathcal{R}$ will generate sample paths that are within $\mathcal{R}$ forever.

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**C1:** $\mathcal{R}$ **is a recurrent class.**

**C2: Optimality of strategies.** Conditional on $\mathcal{W} \equiv \{W * (b|J_i)_{b \in \mathcal{B} \cup \varnothing}\}$, the strategies are optimal. That is

$$b^*(J_i, F_i) = \arg \max_{b \in \mathcal{B} \cup \varnothing} \left[ W^*(b|J_i) - \{b \neq \varnothing\} F_i \right].$$

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**C1:** $\mathcal{R}$ **is a recurrent class.**

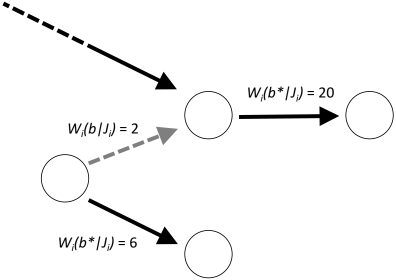
**C2: Optimality of strategies.**

**C3: Consistency of values on** $\mathcal{R}$**.** Consistency requires that the perception of discounted values, generated by every possible choice at every $J_i$ that is a component of an $s \in \mathcal{R}$ equals the expected discounted value of returns generated by that choice from that $J_i$; where expectations are taken using the empirical distribution of outcomes from that $J_i$.

# Equilibrium, relationship to other equilibrium notions, issues

"In an self-confirming equilibrium, each players strategy is a best response to his beliefs about the play of his opponents, and each player's beliefs are correct along the equilibrium path of play"

- Substantive difference between REBE and SCE is that REBE requires beliefs about non-equilibrium path play that keeps you in the recurrent class to be consistent.



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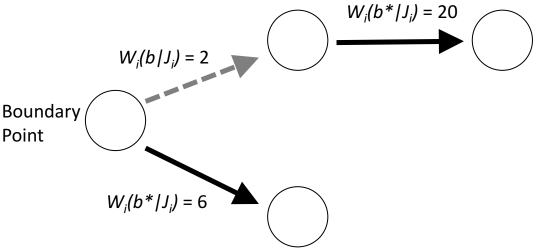
- Return from non-optimal play at boundary points (i.e. doing something that takes us outside recurrent class) need not be consistent.
- This is a source of multiplicity and potentially problematic equilibrium selection if computation poorly initiated

# Equilibrium, adding boundary consistency

Boundary Consistency: the perceived value of off-equilibrium-path play from a boundary point $\geq$ the expected discounted value of profits from that point when all agents use their equilibrium policies



# Equilibrium, adding boundary consistency

**C4:Boundary Consistency.**
Let $\pi_i(b^*, s, F) \equiv \pi(b_i^*(J_i, F_i), b_{-i}^*(F_{-i}, J_{-i}), F_i, J_i)$ and $\pi_i(b, b_{-i}^*, s, F) \equiv \pi(b, b_{-i}^*(F_{-i}, J_{-i}), F_i, J_i)$. Then our condition is $\forall (b, J_i)$ component of $(b, s) \in B$ and for every $F_i$,

$$W(b^*|I_i) - \{b^*(J_i, F_i) \neq \varnothing\}F_i \geq$$

$$\sum_{J_{-i},F_{-i}} \Big[\pi_i(b, b_{-i}^*, s, F_i)) + \sum_{\gamma=1}^{\infty} \beta^\gamma \sum_{s_\gamma, F_\gamma} \pi_i(b^*, F_\gamma, s_\gamma) p(s_\gamma|s_{\gamma-1}, b^*, F_\gamma) p(F_\gamma)\Big] p(F_{-i})\mu^E$$

where $p(s_\gamma|s_{\gamma-1}, b^*, F_\gamma)$ is the probability of reaching state $s_\gamma$ at time $\gamma$ given that at time $\gamma - 1$ the state is $s_{\gamma-1}$, participation fees are $F_\gamma$ and the players play the equilibrium strategies $b^*$.

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# Adding information sharing: IE with commitment

- Baseline (B) is above model with T=4
- Information Exchange (IE) treatment is above model with T=1 (learn $\omega$'s every period)

# Model set-up

- Two loggers each have a stock of trees that can be harvested (private info)
- A new stumpage auction is announced - FPSB
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# Adding information sharing: Voluntary IE (no commitment)

- Baseline (B) is above model with T=4
- Information Exchange (IE) treatment is above model with T=1 (learn $\omega$'s every period)
- Voluntary IE (VIE) introduces a choice to reveal information for next 4 periods. Choice must be unanimous. Choice made at same time as bid. Bid is a $b$ and a 'yes/no'
  - Simplest way to put in a endogenous switch between B and IE

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# Computation: testing for equilibrium

- Computation is done via a reinforcement learning algorithm with v. high starting values
- Testing for convergence to REBE is done by comparing the $\mathcal{W} \equiv \{W^*(b|J_i)_{b \in \mathcal{B} \cup \varnothing}\}$ in memory, to the estimated analog from simulating a long path holding strategies constant. Details in the paper.
- Testing for boundary consistency
  - Simulate to find boundary points (see where each possible action at a point in the recurrent class go to)
  - Take boundary points, for each action compare $\mathcal{W} \equiv \{W^*(b|J_i)_{b \in \mathcal{B} \cup \varnothing}\}$ to the estimated continuation value generated by simulating many iterations that each travel outside the recurrent class for a long time (stop if return).
  - Details in the paper

# Parameterization

| | | $B$ | $IE$ | $VIE$ |
|---|---|---|---|---|
| Parameters that vary: | | | | |
| Distribution of fixed cost of participation | $F_i$ | U[0,1] | U[0,1] | U[0,1] |
| Mean timber in a lot | $\theta$ | 3.5 | 3.5 | 3.5 |
| Periods between $\omega$ revelation | $T$ | 4 | 1 | {1,4} |
| Discount factor | $\beta$ | 0.9 | 0.9 | 0.9 |
| Other parameter values: | | | | |
| Mean harvest capacity | $e$ | | 2 | |
| Disturbance around $\theta$ | $\eta$ | | {-0.5,0.5} | |
| Probability on $\eta$ realizations | | | {0.5,0.5} | |
| Disturbance around $e$ | $\epsilon$ | | {-1,0,1} | |
| Probability on $\epsilon$ realizations | | | {0.33,0.33,0.33} | |
| Bidding grid | | | {0.5,1,1.5,2} | |
| Number of firms/bidders | | | 2 | |
| Retail price of a unit of timber | | | 1 | |

# Computational burden

Size of recurrent class:

| $B$ | $IE$ | $VIE$ |
|---|---|---|
| 325,843 | 2,089 | 328,688 |

Number of all states visited during computation:

| $B$ | $IE$ | $VIE$ |
|---|---|---|
| 7,495,307 | 2,724 | 7,908,122 |

Computation times per 5 million iterations (in hours):

| $B$ | $IE$ | $VIE$ |
|---|---|---|
| 1:38 | 1:06 | 1:56 |

Computation times for testing for a REBE (5 million iterations, in hours):

| $B$ | $IE$ | $VIE$ |
|---|---|---|
| 1:43 | 1:09 | 2:00 |

Computation times for testing for boundary consistency (100,000 iterations, in hours):

| $B$ | $IE$ | $VIE$ |
|---|---|---|
| 3:03 | 0:16 | 75:41 |

Total time = about a week for B and VIE, and half a day for IE. We used matlab on a 3.7GHz/16Gb RAM windows desktop.

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# Results: Judging competition, price vs participation

| | $B$ | $IE$ | $VIE$ | $SP$ |
|---|---|---|---|---|
| Avg. bid | 1.09 | 0.94 | 1.04 | - |
| Avg. winning bid (revenue for the auctioneer) | 1.11 | 0.98 | 1.07 | - |
| Avg. winning bid conditional on $\geq 1$ firm participating | 1.16 | 0.98 | 1.12 | - |
| Avg. winning bid conditional on 1 firm participating | 1.06 | 0.67 | 0.99 | - |
| Avg. winning bid conditional on 2 firms participating | 1.23 | 1.16 | 1.20 | - |
| Avg. # of participants | 1.52 | 1.63 | 1.52 | 1 |
| Avg. # of participants, conditional on $\geq 1$ firm participating | 1.59 | 1.63 | 1.59 | 1 |
| Avg. participation rate | 0.76 | 0.81 | 0.76 | 0.50 |
| % of periods with no participation | 4.39 | 0.15 | 3.85 | 0.004 |
| Avg. total revenue | 3.35 | 3.49 | 3.37 | 3.50 |
| Avg. profit | 0.81 | 0.87 | 0.84 | - |
| % of periods in which a firm with the lowest omega wins | 66.37 | 60.80 | 65.32 | 85.96 |
| conditional on $\geq 1$ firm participating Average total social surplus | 2.73 | 2.72 | 2.74 | 3.10 |

Notes: Here, and in tables 4, 5, 6, and 7, the per-period profit is defined as $\pi(\omega_i) - \mathbb{I}_{\{i=win\}} b_i - \{b_i \neq \varnothing\} F_i = \min\left\{\omega_i + \mathbb{I}_{\{i=win\}}(\theta+\eta), e+\epsilon_i\right\} - \mathbb{I}_{\{i=win\}} b_i - \{b_i \neq \varnothing\} F_i$. Total revenue is defined as $\sum_i \pi(\omega_i) = \sum_i \min\left\{\omega_i + \mathbb{I}_{\{i=win\}}(\theta+\eta), e+\epsilon_i\right\}$. Total social surplus is defined as $\sum_i \{\pi(\omega_i) - \{b_i \neq \varnothing\} F_i\}$. Averages are taken over periods. The statistics are computed based on a 5 million iteration simulation of each model.

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| % of periods in which a firm with the lowest omega wins | 66.37 | 60.80 | 65.32 | 85.96 |
| conditional on $\geq 1$ firm participating Average total social surplus | 2.73 | 2.72 | 2.74 | 3.10 |

Notes: Here, and in tables 4, 5, 6, and 7, the per-period profit is defined as $\pi(\omega_i) - \mathbb{I}_{\{i=win\}}b_i - \{b_i \neq \varnothing\}F_i = \min\left\{\omega_i + \mathbb{I}_{\{i=win\}}(\theta+\eta), e+\epsilon_i\right\} - \mathbb{I}_{\{i=win\}}b_i - \{b_i \neq \varnothing\}F_i$. Total revenue is defined as $\sum_i \pi(\omega_i) = \sum_i \min\left\{\omega_i + \mathbb{I}_{\{i=win\}}(\theta+\eta), e+\epsilon_i\right\}$. Total social surplus is defined as $\sum_i \{\pi(\omega_i) - \{b_i \neq \varnothing\}F_i\}$. Averages are taken over periods. The statistics are computed based on a 5 million iteration simulation of each model.

# Results

- IE generates more participation and lower prices - hard to reconcile w static intuition

# Results: IE reduces profits conditional on the state

| $(\omega_i, \omega_{-i})$ | Prob. Dist. (%) $B$ | $IE$ | $SP$ | Profit $B$ | $IE$ |
|---|---|---|---|---|---|
| $(\leq 4, \leq 4)$ | 65.51 | 32.59 | 90.12 | 0.68 | 0.52 |
| $(\leq 4, 5-7)$ | 12.61 | 19.09 | 4.52 | 0.57 | 0.58 |
| $(\leq 4, \geq 8)$ | 4.05 | 10.55 | 0.28 | 0.60 | 0.59 |
| $(5-7, \leq 4)$ | 12.61 | 19.09 | 4.52 | 1.51 | 1.26 |
| $(5-7, 5-7)$ | 0.88 | 5.72 | 0.22 | 1.49 | 1.46 |
| $(5-7, \geq 8)$ | 0.14 | 1.12 | 0.02 | 1.49 | 1.13 |
| $(\geq 8, \leq 4)$ | 4.05 | 10.55 | 0.28 | 1.62 | 1.58 |
| $(\geq 8, 5-7)$ | 0.14 | 1.12 | 0.02 | 1.66 | 1.87 |
| $(\geq 8, \geq 8)$ | 0.01 | 0.17 | 0.00 | 1.72 | 1.56 |

# Results: Changing the transitions through the state space

| $(\omega_i, \omega_{-i})$ | Prob. Dist. (%) $B$ | $IE$ | $SP$ | Profit $B$ | $IE$ |
|---|---|---|---|---|---|
| $(\leq 4, \leq 4)$ | 65.51 | 32.59 | 90.12 | 0.68 | 0.52 |
| $(\leq 4, 5-7)$ | 12.61 | 19.09 | 4.52 | 0.57 | 0.58 |
| $(\leq 4, \geq 8)$ | 4.05 | 10.55 | 0.28 | 0.60 | 0.59 |
| $(5-7, \leq 4)$ | 12.61 | 19.09 | 4.52 | 1.51 | 1.26 |
| $(5-7, 5-7)$ | 0.88 | 5.72 | 0.22 | 1.49 | 1.46 |
| $(5-7, \geq 8)$ | 0.14 | 1.12 | 0.02 | 1.49 | 1.13 |
| $(\geq 8, \leq 4)$ | 4.05 | 10.55 | 0.28 | 1.62 | 1.58 |
| $(\geq 8, 5-7)$ | 0.14 | 1.12 | 0.02 | 1.66 | 1.87 |
| $(\geq 8, \geq 8)$ | 0.01 | 0.17 | 0.00 | 1.72 | 1.56 |

# Results

- IE generates more participation and lower prices - hard to reconcile w static intuition
- Transitions are changing, likely in response to increased competition on specific states
  - Since the control is the bid, to understand this, need to look at bids
  - How does the information structure generate bids that keep bidders in higher inventory states?

# Results: Shaded cells are bid-state pairs where Prob(IE)>Prob(B)

| $(\omega_i, \omega_{-i})$ | Bids | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $B$ | | | | | $IE$ | | | | |
| | $\varnothing$ | 0.5 | 1 | 1.5 | 2 | $\varnothing$ | 0.5 | 1 | 1.5 | 2 |
| $(\leq 4, \leq 4)$ | 0.22 | 0.13 | 0.27 | 0.31 | 0.07 | 0.07 | 0.13 | 0.28 | 0.47 | 0.06 |
| $(\leq 4, 5-7)$ | 0.11 | 0.32 | 0.45 | 0.11 | 0.02 | 0.02 | 0.53 | 0.37 | 0.08 | 0.00 |
| $(\leq 4, \geq 8)$ | 0.08 | 0.58 | 0.29 | 0.04 | 0.02 | 0.00 | 0.88 | 0.12 | 0.00 | 0.00 |
| $(5-7, \leq 4)$ | 0.43 | 0.18 | 0.34 | 0.04 | 0.01 | 0.33 | 0.10 | 0.52 | 0.05 | 0.00 |
| $(5-7, 5-7)$ | 0.37 | 0.50 | 0.09 | 0.02 | 0.01 | 0.40 | 0.59 | 0.01 | 0.00 | 0.00 |
| $(5-7, \geq 8)$ | 0.39 | 0.53 | 0.06 | 0.01 | 0.01 | 0.11 | 0.89 | 0.00 | 0.00 | 0.00 |
| $(\geq 8, \leq 4)$ | 0.51 | 0.25 | 0.22 | 0.02 | 0.00 | 0.60 | 0.14 | 0.26 | 0.00 | 0.00 |
| $(\geq 8, 5-7)$ | 0.53 | 0.39 | 0.06 | 0.01 | 0.00 | 0.84 | 0.16 | 0.00 | 0.00 | 0.00 |
| $(\geq 8, \geq 8)$ | 0.61 | 0.36 | 0.03 | 0.00 | 0.00 | 0.47 | 0.53 | 0.00 | 0.00 | 0.00 |

# Results: Shaded cells are bid-state pairs where Prob(IE)>Prob(B)

Low inventory states

| $(\omega_i, \omega_{-i})$ | Prob. Dist. (%) | | Bids | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $B$ | $IE$ | $B$ | | | | | $IE$ | | | | |
| | | | $\varnothing$ | 0.5 | 1 | 1.5 | 2 | $\varnothing$ | 0.5 | 1 | 1.5 | 2 |
| $(0,0)$ | 3.17 | 0.50 | 0.12 | 0.07 | 0.12 | 0.41 | 0.28 | 0.01 | 0.00 | 0.09 | 0.12 | 0.78 |
| $(0,1)$ | 3.70 | 0.88 | 0.12 | 0.08 | 0.13 | 0.46 | 0.20 | 0.04 | 0.00 | 0.09 | 0.44 | 0.43 |
| $(0,2)$ | 4.91 | 1.48 | 0.11 | 0.09 | 0.17 | 0.49 | 0.15 | 0.05 | 0.08 | 0.05 | 0.60 | 0.23 |
| $(1,0)$ | 3.70 | 0.88 | 0.18 | 0.06 | 0.13 | 0.49 | 0.15 | 0.01 | 0.04 | 0.00 | 0.29 | 0.66 |
| $(1,1)$ | 2.36 | 0.80 | 0.18 | 0.12 | 0.23 | 0.40 | 0.07 | 0.03 | 0.09 | 0.00 | 0.74 | 0.15 |
| $(2,0)$ | 4.91 | 1.48 | 0.28 | 0.07 | 0.19 | 0.41 | 0.05 | 0.05 | 0.10 | 0.00 | 0.86 | 0.00 |

# Results: Shaded cells are bid-state pairs where Prob(IE)>Prob(B)

Asymmetric inventory states

| $(\omega_i, \omega_{-i})$ | Prob. Dist. (%) $B$ | $IE$ | Bids $B$ $\varnothing$ | 0.5 | 1 | 1.5 | 2 | $IE$ $\varnothing$ | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (0, 7) | 1.49 | 2.36 | 0.05 | 0.23 | 0.61 | 0.09 | 0.03 | 0.01 | 0.33 | 0.62 | 0.03 | 0.00 |
| (1, 7) | 0.40 | 0.83 | 0.08 | 0.50 | 0.38 | 0.03 | 0.01 | 0.00 | 0.79 | 0.21 | 0.00 | 0.00 |
| (2, 7) | 0.35 | 0.89 | 0.14 | 0.64 | 0.18 | 0.02 | 0.01 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 |
| (4, 7) | 0.13 | 0.69 | 0.26 | 0.61 | 0.10 | 0.02 | 0.02 | 0.04 | 0.96 | 0.00 | 0.00 | 0.00 |
| (7, 0) | 1.49 | 2.36 | 0.46 | 0.10 | 0.41 | 0.03 | 0.01 | 0.26 | 0.00 | 0.74 | 0.00 | 0.00 |
| (7, 1) | 0.40 | 0.83 | 0.48 | 0.23 | 0.26 | 0.02 | 0.00 | 0.40 | 0.03 | 0.57 | 0.00 | 0.00 |
| (7, 2) | 0.35 | 0.89 | 0.48 | 0.29 | 0.21 | 0.02 | 0.00 | 0.50 | 0.11 | 0.39 | 0.00 | 0.00 |
| (7, 4) | 0.13 | 0.69 | 0.46 | 0.43 | 0.09 | 0.02 | 0.01 | 0.76 | 0.24 | 0.00 | 0.00 | 0.00 |
| (7, 7) | 0.02 | 0.26 | 0.45 | 0.47 | 0.06 | 0.01 | 0.00 | 0.47 | 0.53 | 0.00 | 0.00 | 0.00 |

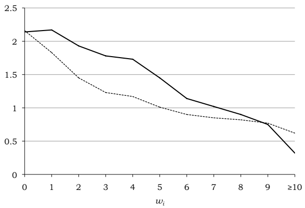
# Results

- IE generates more participation and lower prices - hard to reconcile w static intuition
- Transitions are changing, likely in response to increased competition on specific states
  - Since the control is the bid, to understand this, need to look at bids
  - How does the information structure generate bids that keep bidders in higher inventory states?
- Precision of information about states in IE allows for more targeted bidding strategies
  - Vigorous competition in low inventory states
  - Use fixed costs to reduce auction to lottery in symmetric high inventory states
  - Asymmetric bidding in the asymmetric states ("tough love" by the high type)

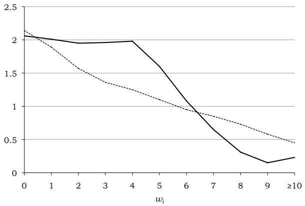
# Results: How much of a role is dynamics taking?

- Dynamics is about the impact of the continuation value
- Compute $D$ as the difference between the continuation value of optimal strategy at a state and the continuation value from doing what is statically optimal at that state.

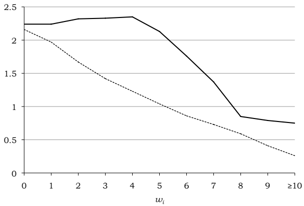
# Results: Impact of dynamic incentives




(a) $E_{J_i,F_i}\left[D(b^*, J_i, F_i)|\,\omega_i, \omega_{-i} \leq 4\right]$



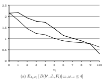

(b) $E_{J_i,F_i}\left[D(b^*, J_i, F_i)|\,\omega_i, \omega_{-i} \in \{5, 6, 7\}\right]$

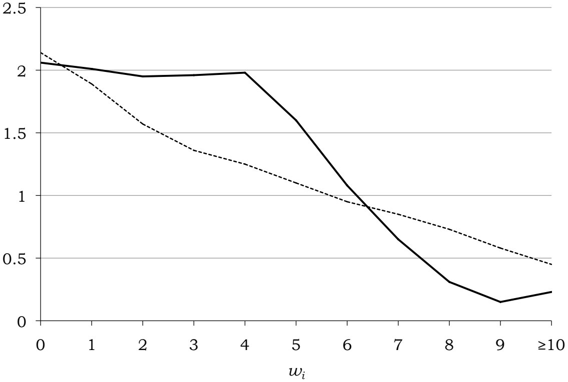



(c) $E_{J_i,F_i}\left[D(b^*, J_i, F_i)|\,\omega_i, \omega_{-i} \geq 8\right]$

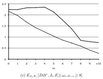
Solid is IE, dash is B

# Results: Impact of dynamic incentives






(b) $E_{J_i,F_i}\left[D(b^*, J_i, F_i) | \omega_i, \omega_{-i} \in \{5,6,7\}\right]$



Solid is IE, dash is B

# Conclusion

- IE generates more participation and lower prices - hard to reconcile w static intuition
- Transitions are changing, in response to increased competition on specific states
- Precision of information about states in IE allows for more targeted bidding strategies
- Commitment crucial to IE having any impact
- Welfare intuitions from static intuition fail

- Developed a computational framework that allows these issues to be explored in a auction setting (non-capital accumulation game)
- Extended REBE to check for boundary consistency