Oligopoly Theory

This might be revision in parts, but (if so) it is good stuff to be reminded of...

Oligopoly Theory

We will cover the following topics:

- Cournot
- Cournot with Sequential Moves
- Bertrand
- Cournot versus Bertrand

After these basic static models we will examine:

- Dynamic oligopoly and Self-enforcing Collusion

Cournot wrote in 1838 - well before John Nash!

He proposed an oligopoly-analysis method that is (under many conditions) in fact the Nash equilibrium w.r.t. quantities.

Firms choose. . . production levels.

Version 1: Two-seller game

Cost: $TC_i(q_i) = c_i q_i$

Demand: $p(Q) = a - bQ$ where $Q = q_1 + q_2$

Define a game:

- Players: firms.
- Action/Strategy set: production levels/quantities.
- Payoff function: profits, defined:

$$\pi_1(q_1, q_2) = p(q_1 + q_2)q_1 - TC_1(q_1)$$

$$\pi_2(q_2, q_1) = p(q_2 + q_1)q_2 - TC_2(q_2)$$

- Now we need an equilibrium concept.

$\{p^c, q_1^c, q_2^c\}$ is a Cournot equilibrium ("Nash-in-quantities" equilibrium) if:

1. a) given $q_2 = q_2^c$, q_1^c solves $\max_{q_1} \pi_1(q_1, q_2^c)$
   b) given $q_1 = q_1^c$, q_2^c solves $\max_{q_2} \pi_2(q_2, q_1^c)$
2. $p^c = a - b(q_1^c + q_2^c)$, for $p^c, q_1^c, q_2^c \geq 0$

"No firm could increase its profit by changing its output level, given what other firms produced."

F.O.C. (for firm 1):

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = a - 2bq_1 - bq_2 - c_1 = 0$$

As always, set marginal revenue $(a - 2bq_1 - bq_2)$ equal to marginal cost (c_1).

How do we know this is a maximum and not a minimum?

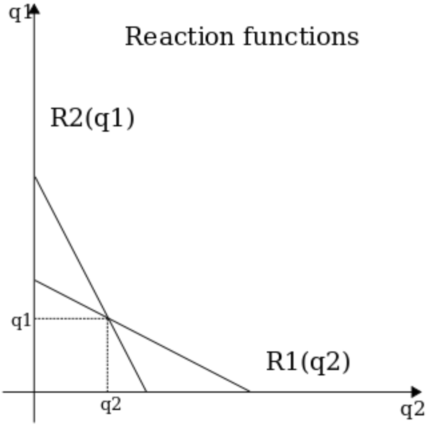
$$\frac{\partial^2 \pi_1(q_1, q_2)}{\partial (q_1)^2} = -2b < 0 \quad \forall q_1, q_2$$

Solve the F.O.C. for q_1 ...

Note: q_1 is a function of q_2. It is the *best-response function*, a.k.a. *reaction function* of firm 1. Call this $R_1(q_2)$.

$$q_1 = R_1(q_2) = \frac{a - c_1}{2b} - \frac{1}{2}q_2$$

Oligopoly Theory
Cournot



Note both best-response functions are downward sloping. For each firm: if the rival's output increases, I lower my output level. (i.e., an increase in a rival's output shifts the residual demand facing my firm inward, leading me to produce less.) This feature of "strategic substitutes".

Now we can compute Cournot equilibrium output levels.

$$q_1^c = \frac{a - 2c_1 + c_2}{3b}$$

$$q_2^c = \frac{a - 2c_2 + c_1}{3b}$$

Equilibrium quantity supplied on the market is

$$Q^c = q_1^c + q_2^c = \frac{2a - c_1 - c_2}{3b}$$

We can also find equilibrium price.

$$p^c = a - bQ^c = \frac{a + c_1 + c_2}{3}$$

What are the pay-off functions in equilibrium?

$$\pi_i^c = (p^c - c_i)(q_i^c) = b(q_i^c)^2$$

Version 2: Extending this to N firms.
It's harder to see the reaction functions, but the story is exactly the same.
Now each firm maximizes profits according to:

$$\pi_i(q_1, q_2, ...q_N) = p(Q)\, q_i - TC_i(q_i)$$

We would derive the best response function for all N firms. For firm 1,

$$q_1 = R_1(q_2, ...q_N) = \frac{a - c_1}{2b} - \frac{1}{2}\left(\sum_{j=2}^{N} q_j\right)$$

We need N of these equations. However, if we assume that firms' marginal costs are the same ($TC_i(q_i) = cq_i \ \forall i$), it's a lot easier. Each firm has the same reaction function, which is

$$q_i = R_i(q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}\left(\sum_{j \neq i} q_j\right)$$

Look at the 2-firm example. When costs (c_1 and c_2) are the same, output levels of the two firms are the same. So, we guess in this case that all the output levels $(q_1, ..., q_N)$ are going to be the same too. If so, denote $q_i = q \ \forall i$.

WATCH OUT! We cannot substitute in $q_i = q$ before we've derived the best response function. This is a common mistake. Why is it a mistake?

Assuming that $q_i = q$ before we take first order conditions implies that firm i has control over all firms' output decisions (the q_{-i}'s). Here, we substitute $q_i = q$ into the already-derived best-response functions. We do this to make the process of solving N equations for N unknowns easier. And, hopefully because we believe the assumption that firms' cost functions are truly the same.

Back to the reaction function.

$$q_i = R_i(q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}\left(\sum_{j \neq i} q_j\right)$$

$$q = \frac{a-c}{2b} - \frac{1}{2}(N-1)q$$

Thus,

$$q^c = \frac{(a-c)}{(N+1)b}$$

$$Q^c = \frac{N(a-c)}{(N+1)b}$$

Now it is straightforward to solve for p^c (the market price) and π_i^c (profits for each firm).

Sanity checks. . .
Do we get the monopoly result for $N = 1$?
Do we get the duopoly result for $N = 2$?
What is the Cournot solution for $N = \infty$?

Take the limit as $N \longrightarrow \infty$ for q^c and Q^c and p^c. They are:

$$\begin{aligned}\lim_{N\to\infty} q^c &= 0 \\ \lim_{N\to\infty} Q^c &= \frac{a-c}{b} \\ \lim_{N\to\infty} p^c &= c\end{aligned}$$

Under the assumption of Cournot competition, market supply approaches the competitive supply as $N \longrightarrow \infty$.

Note that market supply depends on the slope and intercept of demand, and the (common) marginal cost. Individual firms' output levels approach zero as $N \longrightarrow \infty$.

Convince yourself that you can find the equilibrium quantity supplied and price in the market under the assumption of perfect competition.
Simple way: let number of firms = 1, but assume that

$$\frac{\partial p}{\partial Q} = 0$$

and take F.O.C. to find the Q that maximizes profits for the firm under assumptions of perfect competition. This is what we did in the first week.

We have obtained producer surplus under the assumption that N firms in a market set outputs levels simultaneously according to a "Nash-in-quantities" equilibrium. What about consumer surplus?

We can compute consumer surplus as the area under the demand curve up to the output level that is supplied on the market.

$$CS^c(N) = \frac{N^2(a-c)^2}{2b(N+1)^2}$$

Does consumer surplus go up or down as the number of firms increases in this model?

As for social welfare, we want to compare the sum of firm profits and consumer surplus as the number of firms increases. (Firms are owned by consumers in the end...)

We can compute this as a function of the number of firms in the market.

The bottom line: firm profits go down as more firms enter, but consumer surplus goes up, and goes up by more.

$$W^c(N) = CS^c(N) + N\pi^c(N) = \left(\frac{(a-c)^2}{2b}\right)\left(\frac{N^2+2N}{N^2+2N+1}\right)$$

The important point:

$$\frac{\partial W^c(N)}{\partial N} > 0.$$

Version 3: Different Marginal Costs

We have been assuming that all firms are identical, which implies symmetric equilibrium outcomes.

Take a look at the results when firms have different levels of marginal cost.

Let marginal cost of firm i be c_i. The F.O.C. is only a slight modification from before.

$$\frac{\partial \pi_i}{\partial q_i} = a - 2bq_i^* - b\sum_{i \neq j} q_j^* - c_i = 0$$

Rewrite the F.O.C. as

$$a - bQ^* - bq_i^* = c_i$$

Substitute in for price:

$$p - c_i = bq_i^* \frac{Q^*}{Q^*}$$

Rearrange...

$$\frac{p - c_i}{p} = \frac{bq_i^*}{Q^*} \frac{Q^*}{p}$$
$$\frac{p - c_i}{p} = \frac{-\partial p}{\partial Q} \frac{q_i^*}{Q^*} \frac{Q^*}{p}$$

The term $\frac{q_i^*}{Q^*}$ the market share of firm i. Denote this simply as s_i.

We now have the inverse elasticity rule of Cournot oligopoly:

$$\frac{p - c_i}{p} = \frac{s_i}{-\varepsilon_d}$$

Cournot with Sequential Moves:

We could also think about this in a game where firm 1 moves first, firm 2 moves second, etc. We call this a *leader-follower* market structure, or a *Stackelberg* game.

Keep everything as in the 2-firm cournot model, but assume firm 1 sets quantity (q_1) first, then firm 2 sees this qunatity and then chooses its quantity, q_2.

- Work backwards: Suppose firm 1 sets output level to q_1. What would firm 2 do?
- $R_2(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1$
- Firm 1 can figure this out. What will Firm 1 do in response?
- $\max_{q_1} [p(q_1 + R_2(q_1))q_1 - cq_1]$

- Firm 1 can figure this out. What will Firm 1 do in response?
- $\max_{q_1} [p(q_1 + R_2(q_1))q_1 - cq_1]$
- What's different between this profit function and the Cournot profit function?

- Turns out leader output level is: $q_1^S = \frac{a-c}{2b} = \frac{3}{2}q^c$.
- Follower output level is: $q_2^S = \frac{a-c}{4b} = \frac{3}{4}q^c$.
- Equilibrium price is lower than Cournot, output is larger than Cournot – hence more consumer surplus.
- First-mover advantage: "leader" makes more profit than "follower". "Leader" is better off than in Cournot.

When do you think price setting makes more sense than setting quantity?

In general, economists may believe that different assumptions hold for different settings. Then we have to argue about which one is more consistent with the data.

Bertrand reviewed Cournot's work 45 years later.

Go through a two-firm example again. Now firms set prices. We need two assumptions.

1. Consumers always purchase from the cheapest seller (recall definition of homogeneous goods).
2. If two sellers charge the same price, consumers are split 50/50.

The second assumption is that demand takes the form:

$$q_i = \begin{cases} 0 & \text{if } p_i > p_j \quad \text{or } p_i > a \\ \frac{a-p}{2b} & \text{if } p_i = p_j = p < a \\ \frac{a-p}{b} & \text{if } p_i < \min(a, p_j) \end{cases}$$

Equilibrium:
$\{p_1^b, p_2^b, q_1^b, q_2^b\}$ is a Bertrand equilibrium ("Nash-in-prices" equilibrium) if:

1. a) given $p_2 = p_2^b$, p_1^b solves $\max_{p_1} \pi_1(p_1, p_2^b) = (p_1 - c_1)q_1$
   b) given $p_1 = p_1^b$, p_2^b solves $\max_{p_2} \pi_2(p_1^b, p_2) = (p_2 - c_2)q_2$
2. q_1 and q_2 are determined as above.

That is:

"No firm could increase its profit by changing its price, given the price set by the other firm."

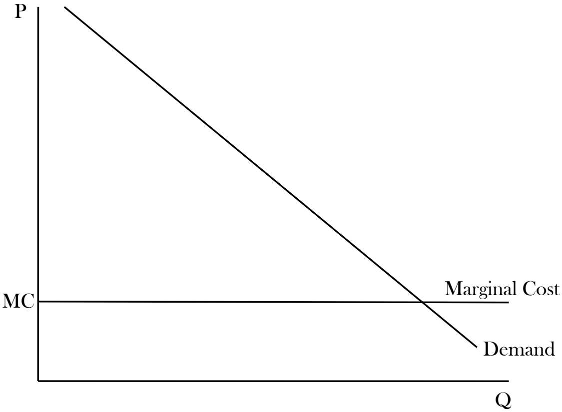
1. If costs are the same, a Bertrand equilibrium is price $=$ marginal cost, with quantity supplied on the market equal to the perfectly competitive outcome (equally split between the two firms).
2. If costs differ, (say firm 1 has cost $= c_1$ where $c_1 < c_2$), then the firm with the lower cost charges $p_1 = c_2 - \varepsilon$, firm 2 sells zero quantity, and firm 1 sells quantity given by $q_1^b = \frac{(a - c_2 + \varepsilon)}{b}$.

Intuition:

If costs are the same, undercutting reduces price to marginal cost. If costs differ, undercutting reduces price to "just below" the cost of the high-cost firm.

Oligopoly Theory

Bertrand



Oligopoly Theory

Bertrand w capacity constraints

(keep the assumption that costs are the same):

Previous results (with no capacity constraints):

1. price competition reduces prices to unit costs
2. firms earn zero profits.

Do we believe these results? Maybe, maybe not.

We might worry about the fact that the number of firms makes no difference. Undercutting reduces price to marginal cost with only 2 firms.

(Consider the policy implications of this for the anti-trust authorities.)

Edgeworth had the idea that in the short run, firms may be constrained by capacity that limits their production levels.

Bertrand with capacity constraints:

- Consider the following example. There are two hair salons, both charging an initial price of $35.
- Under the basic Bertrand model (no capacity constraints), Salon-2 could take all customers if it set its price at $34.
- Suppose initial business is 12 haircuts per day per firm, and that the maximum capacity for either firm is 15 haircuts in a day.

Now the basic Bertrand result of marginal cost pricing won't arise.

Assume the marginal cost of a haircut is simply c. The regular Bertrand solution is of course

$$\begin{aligned} p_1 &= p_2 = c \\ \pi_1 &= \pi_2 = 0 \end{aligned}$$

And the total market output is the same as the competitive level of output

$$q_1 + q_2 = Q^{Competn} = a - bp = a - bc$$

But now add in the capacity constraint (k_i) with the following features:

$$k_i < Q^{Competn}$$

$$k_i > \frac{1}{2}(Q^{Competn})$$

Oligopoly Theory

Bertrand w capacity constraints

If $p_2 > p_1 = c$, firm 2 loses some customers, but not all of them, because firm 1 cannot serve the entire residual demand.
$\Longrightarrow \pi_2 > 0$ for $p_2 = p_1 + \varepsilon$

In other words, there exists a unilateral profitable deviation from $p_1 = p_2 = c$.

$p_1 = p_2 = c$ is not a Nash equilibrium of the Bertrand game with capacity constraints (of the type described above).

In order the determine the Nash equilibrium of the price-setting game with capacity constraints, we can consider a two-stage game:

1. Firms invest in (choose) capacities; then
2. Firms choose prices

Example:

- Let $c = 0$, $p = 1 - Q$, for $Q = q1 + q2$.
- Capacity of each firm: k_1, k_2
- Capacity has been acquired at unit cost $c_0 \in [\frac{3}{4}, 1]$

- Claim 1: Firms won't invest in capacities above $\frac{1}{3}$
- Proof 1: Monopoly profit is $\frac{1}{4}$. Net profit is negative for $k_i > \frac{1}{3}$.

- Let $c = 0$, $p = 1 - Q$, for $Q = q1 + q2$.
- Capacity of each firm: k_1, k_2
- Capacity has been acquired at unit cost $c_0 \in [\frac{3}{4}, 1]$

- Claim 2: If $k_i \leq \frac{1}{3}$, both firms charging $p^* = 1 - (k_1 + k_2)$ is an equilibrium, and this equilibrium is unique

- Proof: Suppose firm 2 is charging p^* and has capacity k_2. What should firm 1 do?
- Can firm 1 lower price? No, because at p^*, both firms are at capacity, so there's no use in lowering price.
- Can firm 1 raise price? Assume she charges $p_1 \geq p^*$. Let q_1 be the quantity that firm 1 sells at this high price. Then profit is $(1 - q_1 - k_2)q_1$. Derivative is $1 - 2q_1 - k_2$. This is greater than 0 when $q_1 = k_1$ since $k_i \leq \frac{1}{3}$, i.e. lowering quantity, or equivalently, raising price is not profitable.

- Claim 3: Given Claim 2, it is optimal for firms to set capacities equal to Cournot levels in the first stage.
- Proof 3, step 1: In second stage, $p^* = 1 - (k_1 + k_2)$.
- Proof 3, step 2: Hence, firm 1's profit function is: $(1 - k_1 - k_2)k_1 - c_0 k_1$, with derivative (or best-response function), $1 - 2k_1 - k_2 - c_0$. Hence, $k_1 = k_2 = \frac{1-c_0}{3}$.
- CRUCIAL ASSUMPTION: setting capacities above $\frac{1}{3}$ is not profitable. This was due to the high investment costs.

Oligopoly Theory

Bertrand w capacity constraints

- We made this rather problematic assumption since it turns out that if firms build capacity above the Cournot level, then there is no pure strategy equilibrium in the second stage. Mixed strategies are pretty tough to derive; you can look at Kreps and Scheinkman (1983) for a full derivation.
- Main message of the above model, however, is that capacity constraints can soften Bertrand competition.
- Sometimes regulation/government can help firms "coordinate" on capacity limits. Example: zoning laws in communities prevent large stores from being set up.

Comparison of Equilibrium in Models Covered So Far:

Monopoly (M), Cournot (C), Stackelberg

$$CS^M < CS^C < CS^S < CS^{PC}$$

$$PS^M > PS^C > PS^S > PS^{PC} = 0$$

$$DWL^M > DWL^C > DWL^S > DWL^{PC} = 0$$