Product differentiation

Product Differentiation

We now finally drop the assumption that firms offer homogeneous products.
Differentiated products are "similar", but not identical (they are close, but not perfect, substitutes).
There are two major models of differentiation:

1. Horizontal differentiation: if all products were the same price, consumers disagree on which product is most preferred Eg. films, cars, clothes, books, cereals, icecream flavors, Starbucks (by geographic location), ...
2. Vertical differentiation: if all products were the same price, all consumers agree on the preference ranking of products, but differ in their willingness to pay for the top ranked versus lesser ranked products Eg. computer memory/processors, airline tickets, different quantities, car packages, ...

Product Differentiation

We can think of a product as being a bundle of different attributes. For example, a car is bundle of a certain amount of horsepower, a color, weight, size, etc.

From this point of view (actually called a hedonic approach), products have multiple attributes to them, and some attributes are vertical attributes (eg. horsepower) while other attributes are horizontal attributes (eg. color).

So in general we think products are differentiated in both vertical and horizontal dimensions. For these products, the overall differentiation will be horizontal. It takes only one dimension of horizontal differentiation for products for the products to be horizontally differentiated overall.

Product Differentiation

Analytical framework

Price Competition Among Horizontally Differentiated Firms: AKA Bertrand with Differentiated Products

The main idea comes from Hotelling (1929), referred to as “Hotelling’s Linear City”

Product Differentiation

Analytical framework

1. Imagine a beach that is one mile long
2. Suppose that 1,000 people are uniformly distributed along the beach
3. Thus, in each quarter mile there are 250 people, for example
4. There are two ice-cream sellers, one at each end of the beach
5. People like ice-cream, but an individual may have to walk as far as half a mile to get to the nearest seller
6. Say the cost of walking one mile (because of the effort) is t
7. Ignore the fact that the individual would have to walk back to their spot on the beach after they buy an ice-cream (we can include it or not, makes no difference)

Product Differentiation

Analytical framework

If a person walks half a mile to buy an ice-cream, and then pays price p for the ice-cream when she gets there, the total cost to the individual is $\frac{1}{2}t + p$

The point is, the total price a person pays to consume a product is equal to the price of the product plus some amount reflecting how close the product is to their ideal product.

The closer a person is to the ice-cream seller, the closer the product is to their ideal point, the lower will be the overall price the person pays to consume the ice-cream.

A person on the beach located at any distance x from the ice-cream seller at the left-hand-side, will be at a distance of $(1 - x)$ from the seller on her right-hand-side.

Product Differentiation

Analytical framework

Say the price at the left-seller is p_l and the price at the right-seller is p_r. For the person sitting at x, the effective prices are:

$$p_l + xt$$

$$p_r + (1 - x)t$$

We generally presume there exists the option of not buying an ice-cream at all, in which case the person pays nothing but also gains nothing. Suppose all people regardless of where they are located on the beach, have willingness-to-pay of v for an ice-cream.

What is the consumer's problem? maximize utility:

$$\max(v - (p_l + xt), v - (p_r + (1 - x)t), 0)$$

You don't always go to the closest guy: it depends on relative prices p_l, p_r.

Suppose everyone buys an ice-cream. Then there is a marginal consumer sitting at x^* for whom

$$v - (p_l + x^* t) = v - (p_r + (1 - x^*)t)$$

$$p_l + x^* t = p_r + (1 - x^*)t$$

solve for x^*:

$$x^* = \frac{1}{2t}(p_r - p_l + t)$$

All people to the left of x^* will strictly prefer going to the left seller.

All people to the right of x^* will strictly prefer going to the right seller.

Since we assumed there were 1000 people uniformly distributed along the beach, this means that

$$q_l = 1000(\frac{1}{2t}(p_r - p_l + t)) = 500 - \frac{500}{t}p_l + \frac{500}{t}p_r$$

$$q_r = 1000(1 - \frac{1}{2t}(p_r - p_l + t)) = 500 + \frac{500}{t}p_l - \frac{500}{t}p_r$$

These are demand functions for horizontally differentiated products.

Product Differentiation

Analytical framework

Points to note:

1. if $p_l > p_r$, then $q_l \neq 0$ (unlike homogeneous goods)
2. demand is downward sloping in firms'own-price, and upward sloping in their competitors' price
3. instead of a beach + walking cost, we could think about the location of my "tastes" and various products in "product space"
4. i.e., although people may differ in their preferences, they may still be willing to consume a product that is not closest to their ideal, as long as the price is low enough
5. we can also solve this model when the firms are located at points other than the ends. We can also have more than one dimension.

Product Differentiation

Analytical framework

In general, the demand functions will have a form like:

$$q_1 = a_1 - b_1 p_1 + \theta p_2$$

$$q_2 = a_2 - b_2 p_2 + \theta p_1$$

where a_1, b_1, b_2, θ are all positive numbers.

The parameter θ, the partial derivative of demand w.r.t. the other firm's price will always have the same value in each demand function. This is an assumption that there are symmetric effects around the marginal consumers.

We calculate own-price demand elasticities just like usual.

But now the cross-price demand elasticities are interesting. We can look at the percentage change in demand for firm 1 due to a one percent increase in price by firm 2:

$$\varepsilon_{1,2} = \frac{\partial q_1}{\partial p_2} \frac{p_2}{q_1}$$

Since products are imperfect substitutes in this framework, the cross-price elasticities will be positive. A one percent increase in price by firm 2 will lead to an x percent increase in demand for firm 1. Own-price elasticities are negative.

Go back to the ice-cream example.

Firm 1 located at $x = 0$.

Firm 2 located at $x = \frac{1}{5}$

Firm 3 located at $x = 1$.

(Note firms 1 and 2 are much closer to each other than firms 2 and 3.)

To illustrate the point here, let's compute the demand functions.

1. Assuming firm 2 has positive demand, no consumer is choosing between firms 1 and 3.
2. What is the location of the consumer who's indifferent between firms 1 and 2? (As before...)

$$x^*_{1,2} = \frac{1}{10} + \frac{1}{2t}(p_2 - p_1)$$

3. What is the location of the consumer who's indifferent between firms 2 and 3?

$$x^*_{2,3} = \frac{3}{5} + \frac{1}{2t}(p_3 - p_2)$$

4. Everyone to the left of $x^*_{1,2}$ goes to firm 1. So his demand is (as before...):

$$q_1 = 100 - \frac{500}{t}p_1 + \frac{500}{t}p_2$$

5. Everyone between $x^*_{1,2}$ and $x^*_{2,3}$ goes to firm 2. So his demand is (as before...):

$$q_2 = 500 + \frac{500}{t}p_1 - \frac{1000}{t}p_2 + \frac{500}{t}p_3$$

6. All consumers located to the right of $x^*_{2,3}$ go to firm 3...

$$q_3 = 400 + \frac{500}{t}p_2 - \frac{500}{t}p_3$$

Product Differentiation

Analytical framework

Demand for each firm is negative in its own price and positive in the price of competitors. Demand for firm 1 does not depend on price at firm 3. Firm 2 is the only firm whose demand depends on everyone's prices.

Suppose that every firm sets a price of \$5 and suppose that travel cost is \$1 per mile (for example). Then we find:

$$q_1 = 100, \; q_2 = 500, \; q_3 = 400$$

And we can compute a table of own and cross-price elasticities at these prices.

The bottom line: because firm 1 is much closer to firm 2 (compared to how close firm 3 is to firm 2), demand at firm 1 is much more sensitive to p_2 than is the case for firm 3. And, the own price elasticity for firm 1 is the highest of all firms.

This is the general result that firms with products in "crowded" regions of product space will face flatter residual demand curves, which means they have less market power, which will give rise to lower markups and profits in equilibrium (relative to firms in uncrowded parts of product space).

Differentiated Products Bertrand Equilibrium

We will assume the firms compete by setting prices. Let's assume product characteristics are somehow already determined. Suppose there are two firms, with marginal costs of c_1 and c_2, and fixed cost f. Suppose these firms face the following demands (which may be derived from an underlying model of locations in product-characteristic space):

$$q_1 = a_1 - b_1 p_1 + \theta p_2$$

$$q_2 = a_2 - b_2 p_2 + \theta p_1$$

The profit function for firm 1 is

$$\pi_1 = p_1 q_1 - c(q_1)$$

$$\pi_1 = p_1 (a_1 - b_1 p_1 + \theta p_2) - c_1 q_1 - f$$

With homogeneous goods, we could not differentiate the profit function because of the discontinuities in the demand curve, but now the demand functions are smooth. We don't get the extreme "tipping" effect because the goods are differentiated.
F.O.C.:

$$\frac{\partial \pi_1}{\partial p_1} = 0 = a_1 - 2b_1 p_1 + \theta p_2 + b_1 c_1$$

Best-response functions (as before) are:

$$p_1^*(p_2) = \frac{1}{2b_1}(a_1 + \theta p_2 + b_1 c_1)$$

Note: The best-response function is upward sloping in the price of the competitor. Note also that $p_2^*(p_1)$ is calculated similarly:

$$p_2^*(p_1) = \frac{1}{2b_2}(a_2 + \theta p_1 + b_2 c_2)$$

Product Differentiation

Bertrand

Let's work out the algebra for the case where $a_1 = a_2$; $b_1 = b_2$; and $c_1 = c_2 = 0$. (I.e., Assume firms are symmetric.)

$$p_i = \frac{1}{2b}(a + \theta p_j)$$

What is the content of these assumptions?

The assumption that $c_1 = c_2 (= 0)$ implies that firms' marginal cost curves are the same (and equal zero here w.l.o.g.). (And for that matter, neither firm faces fixed costs here either.) The assumptions that $a_1 = a_2$ and $b_1 = b_2$ mean that the firms are symmetrically located in product space.

In many cases, we won't believe these assumptions, so we will have to solve N equations for N unknowns.

The solution for Nash-in-prices equilibrium strategies (under these simplifying assumptions) is:

$$p_i = \frac{a}{2b - \theta} \ \forall i$$

Product Differentiation

Bertrand

Proof of this result: we had

$$p_1^*(p_2) = \frac{1}{2b_1}(a_1 + \theta p_2 + b_1 c_1)$$

$$p_2^*(p_1) = \frac{1}{2b_2}(a_2 + \theta p_1 + b_2 c_2)$$

- $a_1 = a_2 = a$, $b_1 = b_2 = b$, $c_1 = c_2 = 0$

$$\implies p_1 = \frac{1}{2b}(a + \theta p_2)$$

$$p_2 = \frac{1}{2b}(a + \theta p_1)$$

$$\implies p_1 = \frac{1}{2b}(a + \theta . \frac{1}{2b}(a + \theta p_1))$$

Product Differentiation

Bertrand

Solve for p_1:

$$4b^2 p_1 = 2ba + \theta a + \theta^2 p_1$$

$$(4b^2 - \theta^2) p_1 = (2b + \theta) a$$

$$\implies p_1 = \frac{a}{2b - \theta} = p_2$$

To calculate quantities and profits just plug p_1^*into demand equation.

$$p_1^* = \frac{a}{2b - \theta}$$

$$q_1^* = \frac{ab}{2b - \theta}$$

$$\pi_1^* = \frac{a^2 b}{(2b - \theta)^2}$$

Empirical Examples: McFadden and BLP