Dynamic Inputs and Resource (Mis)Allocation*

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Abstract

We investigate the role of dynamic production inputs and their associated adjustment costs in shaping the dispersion of static measures of capital misallocation within industries (and countries). Across 9 datasets, spanning 40 countries, we find that industries exhibiting greater time-series volatility of productivity have greater cross-sectional dispersion of the marginal revenue product of capital. We use a standard investment model with adjustment costs to show that variation in the volatility of productivity across these industries and economies can explain a large share (80-90%) of the cross-industry (and cross-country) variation in the dispersion of the marginal revenue product of capital.

Keywords: Misallocation; Adjustment Costs; Dynamic Inputs; Dispersion.

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1 Introduction

It is well documented that firms differ in productivity within even narrowly defined industries.\footnote{We define our measure of productivity, TFPR, below and discuss its measurement in detail. For recent work, see Syverson (2011), Bartelsman and Doms (2000), Bartelsman, Haltiwanger, and Scarpetta (2013) and references therein.} Moreover, across countries, the extent of this dispersion varies considerably, particularly when comparing countries at different stages of economic development. Dispersion is also observed in the marginal revenue products of inputs, particularly capital. Viewed through a standard static model of production and demand, variation in marginal products across firms suggests the existence of frictions that prevent the efficient allocation of resources in an industry, or an economy at large.

Mirroring this observation, quantitative studies find that reallocation of capital to more productive uses has important implications for aggregate productivity and welfare, within industries, countries, and over time (see, as examples, Olley and Pakes (1996), Hsieh and Klenow (2009), Bartelsman, Haltiwanger, and Scarpetta (2013), and Collard-Wexler and De Loecker (2013)). Spurred by this set of facts, a number of recent papers have tried to identify specific mechanisms to explain why productivity differences are not eliminated by market-based resource reallocation.\footnote{See Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Collard-Wexler (2009), Midrigan and Xu (2013), Moll (2012), Bollard, Klenow, and Sharma (2012), and Peters (2012) for recent work.}

This paper investigates the role of dynamically chosen inputs, such as capital, in shaping the dispersion of the marginal product of inputs. Specifically, we consider a variant of a standard dynamic investment model in which firms: a) face costs when adjusting one factor of production (capital); b) can acquire all inputs in a frictionless spot market and; c) get a firm-specific productivity shock (measured using revenue total factor productivity, or TFPR) in each period generated by an AR(1) process.\footnote{Throughout the paper, in discussing our own work, we consider productivity to be TFPR, and use the terms interchangeably.}

Thus, a capital stock determined in some previous period may no longer appear to be optimal after a productivity shock hits. As a result, dispersion in the marginal revenue product of capital arises naturally.\footnote{We focus on the marginal revenue product of capital as this seems the input that is most prone to} A literal implication is that resource allocation,
while appearing inefficient in a static setting, may well be efficient in a dynamic sense.\(^5\)

We then evaluate the empirical value of this model, employing two types of data: The first is country-specific data, which we will refer to as Tier 1 Data, on establishment/firm production in manufacturing in each of the U.S., Chile, France, India, Mexico, Romania, Slovenia, and Spain (all of which have been widely used in the development and productivity literatures). The second data, which we call Tier 2, are from the World Band Enterprise Survey (WBES), which allow us to exploit production data on firms in 33 countries.\(^6\) Each type of data has different strengths: the country-specific data sets have many more observations and tighter data collection protocols, while the WBES data allow us to access a broader set of countries.

The basic reduced-form pattern implied by the model—that as the volatility of TFPR increases, so does the dispersion of marginal product of capital—is strongly supported by data. It is supported both across industries within a country (using the Tier 1 data), as well as across countries (using the Tier 2 data).

After documenting this basic reduced-form pattern, we take a more structural approach to see how well the model captures cross-industry variation in dispersion. For this exercise, we first estimate capital adjustment costs. These adjustment-cost estimates, along with an (industry-country specific) AR(1) shock process, are used to generate model predictions (that is, we hold all other parameters constant). We then confront the model predictions with the data.

We make three specific contributions in this paper: First, we show that the model of dynamic inputs can quantitatively replicate dispersion of the marginal revenue product of capital that is found in the data. This indicates that the model of dynamically chosen inputs provides a natural benchmark for the dispersion of marginal revenue products in an undistorted economy. Indeed the literature on misallocation acknowledges that adjustment costs. This is consistent with data, as we discuss in Section 4.3: we observe more dispersion in the marginal product of capital relative to that of other inputs.

\(^5\) The validity of this literal interpretation rests on the relationship between the TFPR process and policy. Whether this process is amenable to policy adjustment is a question we turn to in the conclusion.

\(^6\) The WBES data covers firms in the manufacturing, construction, services, and transport, storage, and communications sectors.
dispersion of marginal revenue products alone is not evidence of misallocation, and that adjustment costs may play an important role.

Second, we find meaningful differences in the size of TFPR shocks across industries within countries, as well as across countries, of the same relative magnitude as differences in the cross-sectional dispersion the marginal revenue product of capital. Moreover, industries (countries) with the greatest volatility of TFPR also have the greatest dispersion of the marginal revenue product of capital. These reduced form results are robust to a wide range of measurement and model specification concerns, such as alternative specifications of the TFPR process and alternative measures of volatility; and these results hold both across industries within a country, and across countries.

Third, we show that a structural implementation of this model can capture, both qualitatively and quantitatively, much of the cross-industry (country) variation in the dispersion of marginal revenue products of capital. The model performs strongly: when confronted with industry-country data on dispersion in the marginal revenue product of capital it generates a measure of fit equivalent to an uncentered $R^2$ of around 0.8 – 0.9, depending on the specification. Our results indicate that, perhaps surprisingly, the exact level of adjustment costs does not change this measure of fit greatly: whether we rely on the U.S. estimated adjustment costs or a country-specific one, the measure of fit is about the same. The absence of adjustment costs, holding all other parts of the model fixed, leads to a drop in our measure of fit; which suggests that adjustment cost and volatility play an important role in shaping differences in the dispersion of marginal revenue product of capital (across industries and countries), and as a consequence are crucial to understand income differences across countries.

Taken as a whole, these results highlight the importance of dynamic inputs in explaining, both in levels and differences, the dispersion of the marginal product of capital. Furthermore, our analysis suggests that producer-level volatility may be an important factor in explaining aggregate welfare. The productivity process we employ is a reduced-form for a range of time-varying shocks to production, including (but not
limited to): demand shocks; natural disasters; infrastructure shocks; variation in the incidence of corruption or nepotism; changes in markups due to demand shocks or market-structure changes; and changes to informational barriers. This paper suggests a channel through which these micro effects can have aggregate implications.\footnote{For micro-based studies that consider the effect of each see: (Collard-Wexler, 2013)(on demand shocks); (De Mel, McKenzie, and Woodruff, 2012) (on natural disasters), (Fisher-Vanden, Mansur, and Wang, 2012) (on infrastructure); (Fisman and Svensson, 2007) (on corruption); (De Loecker et al., 2012) (on markups); and (Bloom et al., 2013) (on information barriers).}

The remainder of the paper is organized as follows: In Section 2, we present our dynamic model of investment. Section 3 presents the data and discusses the measurement of productivity across several countries. We turn to reduced-form empirical evidence, and subjects it to a variety of robustness checks in Section 4. Section 5 confronts the predictions of the dynamic investment model with the data using a structural approach. In Section 6 we consider cross-country variation using the WBES data. Finally, we conclude with a discussion of our findings in Section 7.

2 Theoretical Framework

In this section, we posit a simple model that allows us to consider how the time-series process of TFPR should affect the cross-sectional dispersion of the (static) marginal revenue product of capital, and other variables. Central to the model is the role of capital adjustment costs, and a one-period time-to-build, in making optimal capital investment decisions. These adjustment frictions create links between the time-series process generating firm-level TFPR shocks and firm-level heterogeneity in the adjustment of capital stocks.

2.1 Modeling preliminaries

We begin by providing an explicit model of TFPR, in the context of a profit-maximizing firm (since we assume that establishments operate as autonomous units, firms and establishments, for our purposes, are synonymous). A firm $i$, in time $t$, produces

\[ Y_{i,t} = K_{i,t}^a \cdot F(L_{i,t}, Z_{i,t}) - wL_{i,t} - rK_{i,t} \]

\[ \pi_{i,t} = Y_{i,t} - wL_{i,t} - rK_{i,t} \]

\[ \max_{K_{i,t}, L_{i,t}} \pi_{i,t} \]

subject to

\[ K_{i,t+1} = K_{i,t} + \Delta K_{i,t} \]

where $Y_{i,t}$ is the output, $K_{i,t}$ is the capital stock, $L_{i,t}$ is the labor input, $F$ is a production function, $w$ is the wage rate, $r$ is the rental rate of capital, and $\Delta K_{i,t}$ is the adjustment in capital stock.
output \( Q_{it} \) using the following (industry-specific) constant-returns technology:

\[
Q_{it} = A_{it} K_{it}^{\alpha_K} L_{it}^{\alpha_L} M_{it}^{\alpha_M},
\]

where \( K_{it} \) is the capital input, \( L_{it} \) is the labor input, \( M_{it} \) is materials, and we assume constant returns to scale in production so \( \alpha_M + \alpha_L + \alpha_K = 1 \). The demand curve for the firm’s product has a constant elasticity:

\[
Q_{it} = B_{it} P_{it}^{-\epsilon}.
\]

Combining these two equations, we obtain an expression for the sales-generating production function:

\[
S_{it} = \Omega_{it} K_{it}^{\beta_K} L_{it}^{\beta_L} M_{it}^{\beta_M},
\]

where \( \Omega_{it} = A_{it}^{1-\frac{1}{\epsilon}} B_{it}^{\frac{1}{\epsilon}} b \), and \( \beta_X = \alpha_X (1 - \frac{1}{\epsilon}) \) for \( X \in \{K, L, M\} \). For the purposes of this paper, productivity or TFPR is defined as \( \omega_{it} \equiv \ln(\Omega_{it}) \).

The production function and sales-generating function are industry-specific; and throughout the paper, the coefficients \( \beta \) and \( \alpha \) are kept country- and industry-specific unless noted otherwise. For ease of measurement, we set \( \epsilon \) to be constant for all firms, industries and countries.

A fact that we will use repeatedly is that, in a static model with no frictions, profit maximization implies that the marginal revenue product (MRP) of an input should be equal to its unit input cost. For capital, this static marginal revenue product is given by:

\[
\frac{\partial S_{it}}{\partial K_{it}} = \beta_K \frac{\Omega_{it} K_{it}^{\beta_K} L_{it}^{\beta_L} M_{it}^{\beta_M}}{K_{it}}.
\]

We will frequently refer to the marginal revenue product of capital (MRPK), which we measure in logs:

\[
MRPK_{it} = \log(\beta_K) + \log(S_{it}) - \log(K_{it}) = \log(\beta_K) + s_{it} - k_{it}.
\]

\(^8\)Throughout the paper, lower case denotes logs, such that \( x = \ln(X) \).
The marginal revenue products of labor and materials are defined similarly.\footnote{Due to the Cobb-Douglas specification the marginal and average products are equivalent in our setup. Hence, in the data we measure the average product and, using the model, interpret it as marginal.}

Our notion of productivity is a revenue-based productivity measure, or TFPR (as introduced by Foster, Haltiwanger, and Syverson (2008)). As is common in this literature, we do not separately observe prices and quantities at the producer level, and, therefore, we can only directly recover a measure of profitability or sales per input precisely.

This implies that all our statements about productivity refer to TFPR, and, therefore, deviations across producers in our measure of productivity, or its covariance with firm size, could reflect many types of distortion, such as adjustment costs, markups or policy distortions, as Hsieh and Klenow (2009) discuss in detail.

\section{2.2 A dynamic investment model}

We now articulate a dynamic investment model that allows us to examine the link between TFPR volatility and dispersion in both the static marginal revenue product of capital and other variables of interest. Our model follows, and builds on, a standard model of investment used in the work of Bloom (2009), Cooper and Haltiwanger (2006), Dixit and Pindyck (1994), and Caballero and Pindyck (1996).\footnote{The model used in this paper is a partial equilibrium model, that can be rationalized from a general equilibrium perspective only if there are no aggregate shocks and a continuum of firms. Bloom et al (2012) discuss the implications of putting this type of model into a general equilibrium framework with aggregate shocks, versus using a partial equilibrium model.}

Taking the structure in Section 2.1 as given, we begin by assuming that firms can hire labor in each period for a wage $p_L$ and acquire materials in each period at a price $p_M$. Both of these inputs have no additional adjustment costs. Thus, conditional on $\Omega_{it}$ and $K_{it}$, we can substitute in the statically optimal amount of labor and materials. This leads to a ‘period-profit’ (ignoring capital costs for the moment) of:

$$\pi(\Omega_{it}, K_{it}) = \lambda \Omega_{it}^{-\frac{1}{\beta}} K_{it}^{\frac{\beta K}{3 \beta + 1}} K_{it}^{\frac{\beta K}{3 \beta + 1}},$$

(6)
where $\lambda = (\beta K + \epsilon^{-1}) \left( \frac{\beta L}{\beta L} \right)^{\frac{\beta L}{\beta L}} \left( \frac{\beta M}{\beta M} \right)^{\frac{\beta M}{\beta M}}$.

Capital depreciates at rate $\delta$ so $K_{it+1} = (1 - \delta)K_{it} + I_{it}$ where $I_{it}$ denotes investment.

These investment decisions are affected by a one-period time to build and a cost of investment $C(I_{it}, K_{it}, \Omega_{it})$.

We employ an adjustment cost function composed of: 1) a fixed disruption cost of investing and 2) a convex adjustment cost expressed as a function of the percent investment rate. Formally:

$$C(I_{it}, K_{it}, \Omega_{it}) = I_{it} + C^F_K \{ I_{it} \neq 0 \} \pi(\Omega_{it}, K_{it}) + C^Q_K K_{it} \left( \frac{I_{it}}{K_{it}} \right)^2.$$  \(7\)

Next, let $\omega_{it}$ follow an AR(1) process given by:

$$\omega_{it} = \mu + \rho \omega_{it-1} + \sigma \nu_{it},$$  \(8\)

where $\nu_{it} \sim N(0,1)$ is an i.i.d. standard normal random variable. This implicitly defines the transition function of $\Omega$: $\phi(\Omega_{it+1} | \Omega_{it}, \cdot)$.

A firm’s value function $V$ is given by the Bellman equation:

$$V(\Omega_{it}, K_{it}) = \max_{I_{it}} \pi(\Omega_{it}, K_{it}) - C(I_{it}, K_{it}, \Omega_{it})$$

$$+ \beta \int_{\Omega_{it+1}} V(\Omega_{it+1}, \delta K_{it} + I_{it}) \phi(\Omega_{it+1} | \Omega_{it}) \, d\Omega_{it+1},$$  \(9\)

and, thus, a firm’s policy function $I^*(\Omega_{it}, K_{it})$ is just the investment level that maximizes the firm’s continuation value less the cost of investment.

Note that since there is neither entry nor exit in this model, there is no truncation of the TFPR distribution.\(^{11}\) Thus, given the AR(1) structure above, the cross-sectional

\(^{11}\)The absence of entry and exit is a consequence of the decreasing returns to scale in the revenue equation (yielded by constant returns to scale in the production function and an elastic demand curve) and the absence of fixed costs, which make it profitable for any firm to operate at a small enough scale. See Midrigan and Xu (2013) for a discussion of the role of entry and exit in a similar model.
standard deviation of TFPR is given by the ergodic distribution of \( \Omega_{it} \). Hence,

\[
\text{Std}_t(\omega_{it}) = \frac{\sigma}{\sqrt{1 - \rho^2}}. \quad (10)
\]

### 2.3 Moments of interest

In examining the data we will focus on a set of moments that can be generated by the model, three of which warrant explicit definitions. For ease of reference, we provide these definitions here.

The first moment is the dispersion in the (static) marginal revenue product of capital (MRPK, as defined in equation 5). Dispersion in MRPK is defined as: \( \text{Std}_{st}(MRPK_{it}) \), where the \( st \) subscript indicates that the standard deviation is taken within industry-country \( s \) in year \( t \). This will be our most common specification, although at times we will use different configurations (indicated in the subscript).

We next define the computed volatility in the static marginal revenue product of capital over time as:

\[
\text{Std}_{st}(\Delta MRPK) = \text{Std}_{st}[MRPK_{it} - MRPK_{it-1}]. \quad (11)
\]

Third, the volatility in firms’ capital over time is defined as:

\[
\text{Std}_{st}(\Delta k) = \text{Std}_{st}[k_{it} - k_{it-1}]. \quad (12)
\]

It is important to note that the magnitudes of these three moments are unchanged if we adopt an alternate specification of the model in which each firm’s TFPR process has a firm-specific fixed effect; that is, if \( \mu \) is firm-specific. This result is established formally in Theorem 1 in Appendix A. At the heart of the result is the property that a different constant term in the AR(1) results in a level shift in the process, and this generates level shifts in the inputs (\( K, L, \) and \( M \)). These level shifts then get cancelled out when taking differences at the firm level.
2.4 Comparative statics

We analyze the model using computation. Like Bloom (2009), we use a model in which investment decisions are made each month (a period in the model). Modeling decisions at a monthly level is an attractive approach, as the model incorporates the likely time aggregation embedded in annual data.\(^{12}\) The results we report are in terms of what one would see in annual data — that is, we aggregate up from monthly decision-making to the year.

Figure 1 examines the way Std\(_{st}(MPRK)\), the dispersion in the static marginal revenue product of capital, changes as \(\sigma\), the volatility of TFPR, changes.\(^{13}\) To generate this figure we use parameters estimated using U.S. Census data as described in Section 5. Parameters and details of computation can be found in Appendix C. In the figure there are three lines that correspond to the model with both a one-period time-to-build and adjustment costs, but with different persistence parameters in the AR(1) process. From top to bottom, these lines correspond to \(\rho\) equal to 0.94, 0.85, and 0.65 respectively.\(^{14}\) Note that, for any specification and any level of \(\rho\), as \(\sigma\) increases so does dispersion in the static marginal revenue product of capital.

To further understand the pattern in Figure 1, note that this dispersion reflects the optimal investment choices of firms facing different TFPR shocks over time and, hence, different state variables. To make the effect of this clear, note that if all firms had the same capital stock, this graph would contain a series of upward sloping, straight, lines out of the origin. Yet (focusing on the solid black lines) the relationship between Std\(_{st}(MPRK)\) and \(\sigma\) is not linear and has a slope change in the region of \(\sigma = 0.5\) for \(\rho = 0.94\) and in a (wider) region around \(\sigma = 0.6\) for \(\rho = 0.85\). There is no readily

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\(^{12}\)This interpretation requires transforming the AR(1) process—which is quoted to reflect, and empirically estimated off, annual data—into its monthly equivalent. After noting that the sum of normal random variables with the same mean is distributed normally, this reduces to a straightforward algebraic exercise.

\(^{13}\)Where \(\rho = 0\), we can show that the dispersion of \(MPRK\) is given by \(\text{Std}(s - k) = \frac{1}{\rho K} \sigma\). Thus more volatility of productivity leads to higher dispersion \(MPRK\). This result is straightforward to show, since with \(\rho = 0\), productivity is no longer a state, and firms choose the same steady-state level of capital. Where \(\rho \neq 0\), we use the computational simulations in the paper to establish the possibility that volatility increases misallocation.

\(^{14}\)These three values of \(\rho\) represent the 90th percentile, median, and 10th percentile in the U.S. Census data respectively.
discernible slope change in this range of $\sigma$ for $\rho = 0.65$.

To see why this is happening, note that initially, as volatility increases, firms will engage in more investment and disinvestment. Since greater volatility leads to larger changes in TFPR, it is natural that firms respond by altering their capital stock more frequently. However, past a certain point, firms begin to reduce their response to TFPR shocks. This begins as $\sigma$ approaches 0.5 for $\rho = 0.94$ and 0.6 for $\rho = 0.85$, while for $\rho = 0.65$, the same pattern exists but is much more gradual.

At these high levels of volatility, current TFPR is a weaker signal of the future marginal value of capital. Hence, firms respond less to shocks today because those current shocks are more likely to be swamped by future shocks. In the limit, where the TFPR process is an i.i.d. draw, current TFPR provides no information about future profitability. Firms would choose an optimal level of capital and stick to it forever, resulting in no variance in investment across firms. Thus, the slope changes evident in the relationships in Figure 1 reflect a flattening out of capital-adjustments to volatility.

3 Data and Measurement

3.1 Data

We employ multiple datasets in our analysis. Table 1 describes the Tier 1 data. It consists of country-specific producer-level data from eight countries: the United States, Chile, France, India, Mexico, Romania, Slovenia, and Spain. Each of these data sets have been used extensively in the literature; most commonly in the analysis of productivity.\footnote{See, for instance, Tybout and Westbrook (1995), Roberts (1996), Pavcnik (2002), De Loecker and Konings (2006); De Loecker (2007), Goldberg et al. (2009), Bloom, Draca, and Van Reenen (2011), and Konings and Vandenbussche (2005).}

The data sets differ in the time period covered, and in how producers are sampled. Table 1 summarizes the main features of the various datasets. Below we briefly discuss the various Tier 1 datasets and defer more details to Appendix B.
**United States** The data for the United States comes from the U.S. Census Bureau’s Research Data Center Program. We use data on manufacturing plants from the Census of Manufacturers (henceforth, CMF), and the Annual Survey of Manufacturers (henceforth, ASM) from 1972 to 1997. The CMF sends a questionnaire to all manufacturing plants in the United States with more than 5 employees every five years, while the ASM is a four-year rotating panel with replacement, sent to approximately a third of manufacturing plants, with large plants being over-represented in the sampling scheme. The final dataset contains 735,342 plants over a 26-year period.

An industry is defined as a four digit SIC code. Labor is measured using the total number of employees at the plant. Materials are measured using total cost of parts and raw materials.

Capital is constructed in two ways. For the majority of plants, including all plants in the CMF, capital is measured using a question on total assets – be they machines or buildings – at the plant. For the remaining observations, capital is constructed using the perpetual inventory method, using industry-specific depreciation rates and investment deflators from the Bureau of Economic Analysis and the National Bureau of Economic Research. Capital, materials and sales are deflated using the NBER-CES industry-level deflators into 1997 dollars.

**Chile** Annual plant-level data on all manufacturing plants with at least ten workers were provided by Chile’s Instituto Nacional de Estadística (INE). These data, which cover the period 1979–1986, include production, employment, investment, intermediate input, and balance-sheet variables. Industries are classified according to their four digit ISIC industry code. The data contain 37,600 plant-year observations. The smallest number of plants observed in any year is 4,205 in 1983.

**France, Romania, and Spain** Annual firm-level data on manufacturing firms for France, Romania, and Spain are obtained from Bureau Van Dijck’s (BvD) Amadeus dataset and cover firms reporting to the local tax authorities and/or data collection agencies for the period between 1999 and 2007. We selected three relatively large European countries at different stages of economic development. The coverage for all
three countries is substantial in that we cover approximately 90 percent of economic activity in each of the three manufacturing sectors. For example for France, in 2000, we record total sales of 739 billion Euros, whereas the OECD reports total sales to be 768 billion Euros. This implies coverage of 96 percent of total economic activity in manufacturing. For Spain we find, using the same coverage calculation, coverage of 88 percent. The collection protocol of BvD is consistent across countries. We focus on the manufacturing sector to facilitate the measurement of TFPR. Industries are classified according to the two digit NACE Rev 1.1. code for all three countries. Our data covers firms that are primarily active in sectors NACE Rev 1.1. 15 to 36. This leaves us with 391,422, 174,435, and 457,934 firm-year observations for France, Romania, and Spain, respectively. The data include standard production data, including sales, employment, investment, intermediate input and other balance-sheet variables.

**India** Annual firm-level data on manufacturing firms were provided by Prowess, and are collected by the Centre for Monitoring the Indian Economy (CMIE). Prowess is a panel that tracks firms over time for the period 1989–2003. The data contain mainly medium and large Indian firms. Industries are classified according to the 4 digit PNIC (the Indian industrial classification code). These data include sales, employment, investment, capital, intermediate input, and various balance-sheet variables. The final data set comprises 30,709 firm-year observations.

**Mexico** Annual plant-level data on manufacturing plants are recorded by Mexico’s Annual Industrial Survey and are provided by Mexico’s Secretary of Commerce and Industrial Development (SEC-OFI). These data, which cover the period 1984–1990, include production, employment, investment, intermediate input, and balance-sheet variables. The sample of plants represents approximately eight percent of total output, where the excluded plants are the smallest ones. Industries are classified according to the Mexican Industrial Classification (a four digit industrial classification system). The final data contain 21,180 plant-year observations. The minimum number of observed

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16 The Indian data do not report the wage bill separately from the number of workers. We do, however, take care to appropriately deflate the wage bill.
firms in a year is 2,958 in 1989.

**Slovenia** The data are taken from the Slovenian Central Statistical Office and are the full company accounts of firms operating in the manufacturing sector between 1994 and 2000. We have information on 7,915 firms: an unbalanced panel with information on production, employment, investment, intermediate input, and balance-sheet variables. Industries are classified according to the two digit NACE Rev 1.1. code.

### 3.2 Measurement

To guide the measurement of TFPR, we build on the model in Section 2.1 and, in particular, rely on the sales-generating production function in equation (3). In order to recover a measure of TFPR, \( \omega_{it} \), we need to compute the value of \( \beta_L, \beta_M \) and \( \beta_K \) by industry-country. Profit maximization implies that for each input facing no adjustment costs, the revenue production function coefficient equals the share of the input’s expenditure in sales, or formally:

\[
\beta_X = \frac{P_{it}^X X_{it}}{S_{it}} \quad \text{for } X \in \{L, M\}.
\]  

(13)

As mentioned before, we allow \( \beta_X \) to vary at the industry level within a country, thereby allowing the production function to vary across industries and countries. In practice, in order to obtain a robust measure of these shares, we rely on the median of the expenditure share for labor and intermediate inputs, in a given industry-country \( (sc) \), or

\[
\beta^{sc}_{X} = \text{median} \left( \left\{ \frac{P_{it}^X X_{it}}{S_{it}} \right\} \right) \quad \text{for } X \in \{L, M\}, i \in sc.
\]

(14)

To recover the coefficient on capital, \( \beta_K \), we use our assumption of constant returns to scale in production—i.e., \( \sum_x \alpha_x = 1 \), such that:

\[
\beta_K = \frac{\epsilon - 1}{\epsilon} - \beta_L - \beta_K.
\]

(15)

In order to compute \( \beta_K \) we need to assign a value to the elasticity parameter, \( \epsilon \). We
follow Bloom (2009) and set it equal to four.

Finally, to compute TFPR, we simply plug in the coefficients obtained above into equation (16), below, and compute for each individual firm in a given industry-country pair:

\[ \omega_{it} = s_{it} - \beta_K k_{it} - \beta_L l_{it} - \beta_M m_{it}. \] (16)

For a small fraction of the industry-country pairs for which the sum of the labor and material coefficients exceeds 0.75, and thus would imply a negative capital coefficient, we proceed by using the relevant country’s average coefficient. For the one country, Slovenia, for which the average material coefficient is above 0.75, we rely on OLS production function coefficients, effectively using the average output elasticities.\(^\text{17}\) Importantly, this approach in inferring \(\beta_K\) allows capital to have adjustment costs, since it does not rely on a static first-order condition for capital.\(^\text{18}\)

To measure the sales generating production function coefficients, and subsequently TFPR \((\omega_{it})\), we require a measure of firm-level sales \((S_{it})\), employment \((L_{it})\), material use \((M_{it})\) and the capital stock \((K_{it})\). We follow the approach to measurement described in Bartelsman, Haltiwanger, and Scarpetta (2013) which uses data sources comparable to those we use. When measuring the the value of the capital stock, we either construct the capital series from the investment data, or we directly observe the book value of a producer’s tangible fixed assets. We deflate all output and input data with the relevant country-industry specific producer price deflators.

We provide summary statistics describing our datasets in Table 2. In the left panel we report the median number of workers, and median sales and TFPR growth. The right panel lists the various standard deviations that are of direct interest for our analysis: dispersion in MRPK, dispersion in capital and TFPR, and a simple measure of volatility given by \(\text{Std}(\omega_{it} - \omega_{it-1})\).

As expected, the median size varies substantially across the various datasets due to

\(^{17}\)Alternatively, we could estimate the output elasticity directly from production data. We follow the standard in this literature and rely on cost shares to compute TFPR and thereby avoid the issues surrounding identification of output elasticities (in our case, across many industries and countries).

\(^{18}\)See De Loecker and Warzenski (2012) Section II.A for more discussion.
different data collection protocols.\textsuperscript{19} Productivity growth varies across countries, and it is no surprise that Slovenia and India are the fastest growing economies. The dispersion in MRPK ranges from 0.98 in the U.S. to 1.56 in Slovenia. The next section examines the relationship between dispersion in MRPK and volatility, a central implication of our model, in more detail.

\section{Reduced Form Evidence}

\subsection{Main Results}

We begin our analysis by plotting, in Figure 2, the relationship between the dispersion in MRPK and volatility of TFPR for the U.S. Census data, with each dot on the graph representing a specific four digit SIC code. We start with the U.S. Census data, since this is the richest data source we have access to and the dataset in which issues of measurement and sampling frame are plausibly the least important. We find a striking positive relationship between volatility of TFPR and MRPK dispersion.

To see if the relationship between MRPK dispersion and volatility of TFPR hold up more generally, Table 3 presents, for each of our Tier 1 data sets, regressions of the dispersion in $MRPK$, on TFPR volatility, controlling for industry fixed effects. The focus of Table 3 is the set of country-specific regressions, where the unit of observation is the industry-year. The last two regressions pool the data, such that the unit of observation is the industry-country-year.

For each of the countries, there is a positive, and significant, coefficient. Notably, in the U.S. Census, we see a coefficient of 0.76 (using plant-level data) and 0.68 (using firm-level data), both of which are significant at the 1\% level. Since we observe no economically significant difference between plant- and firm-level data using the U.S. census, from this point on, we use plant-level data in computing U.S. numbers.

These country-specific regressions are consistent with the model prediction that

\textsuperscript{19}In Table OA.5 in the Online Appendix we verify the robustness of our results to using a common size threshold.
dispersion in the static marginal revenue product of capital, at the country-industry level, should be positively correlated with the volatility of TFPR shocks.

Pooling across countries, we see the same pattern. The reported coefficients are 0.55, when the data is pooled in an unweighted way, and 0.74, when the weighting matrix accounts for the number of industry-year observations in a country. Figure 4a plots the dispersion in static marginal revenue product of capital against volatility. While the number of countries is very limited, it suggests a positive relationship between both variables.

An important element in these regressions is the measurement of volatility. In Table 3 we measure volatility by $\text{Std}_{st}(\omega_{it} - \omega_{it-1})$. This allows the shock process to vary over time, but is not an exact replication of the AR(1) process posited in the model. In what follows we assess the sensitivity of these baseline results to alternative specifications of the TFPR shock process.\(^{20}\)

Table 4 takes alternate measures of the volatility of the TFPR process and runs country-specific regressions mirroring those presented in Table 3. The three measures used are: $\text{Std}_{s}[\omega_{it} - \omega_{it-1}]$; an AR(1) measure which is the $\sigma_s$ term in the following specification: $\omega_{it} = \mu_s + \rho_s \omega_{it-1} + \sigma_s \nu_{it}$; and, finally, an AR(1) specification in which we replace $\mu_s$ with a producer-level fixed effects. In Table 4 we refer to this last specification as ‘AR(1)FE’. The AR(1) specifications impose the restriction that $\sigma_s$ is constant over time. To keep our alternative measures comparable, we impose the same restriction on our ‘vol’ measure.\(^{21}\) These volatility measures are highly positively correlated. The correlation coefficient for any pair of measures, for any country, exceeds 0.72 and is often above 0.9.\(^{22}\)

In all regressions the coefficient on volatility is positive, and in all but two cases – out of 24 – the coefficient is significant at the 10% level or better. In addition, the

\(^{20}\)Note that the specification for our AR(1) process rules out aggregate-level shocks to TFPR growth. However, a regression of changes in TFPR on country-year dummies yields $R^2$’s between 0.001, for Mexico, and 0.023, for Chile, when running TFPR growth against year dummies. Thus, there appears to be only a small aggregate component to TFPR change.

\(^{21}\)As a consequence, the results in Table 3 (at the country-industry-year level) differ in magnitude from those presented in Table 4 (at the country-industry level).

\(^{22}\)Table OA.4 in the Online Appendix reports these statistics.
magnitudes of the coefficients are comparable across all specifications, although the AR(1)FE specification tends to produce coefficients that are somewhat higher than the other two specifications. This is likely due to some of the $\sigma_s$ variation being absorbed by the producer fixed-effect.

Overall, the results support the conclusion that the qualitative reduced-form patterns observed in the baseline specification (Table 3) are robust to alternative specifications of the TFPR process.

### 4.2 Additional Implications

So far we have focussed on the relationship between the dispersion in MRPK and volatility of TFPR. Although, this is the main prediction of our model, there are a number of additional implications, both at the individual producer and aggregate levels. We explore these below.

#### 4.2.1 Individual Producer Implications

An essential prediction of our model is that adjustment costs in capital, coupled with TFPR shocks, lead to differences in MRPK among producers. The model thus implies that once producers install capital, TFPR shocks should manifest themselves in variation in MRPK across producers. In the absence of adjustment costs – including a one-period time-to-build – producers could simply adjust their capital, and this would lead to the equalization of MRPK across producers. To test this mechanism, we run the following regression for each of our Tier 1 countries:

$$
\text{MRPK}_{it} = \gamma_0 + \gamma_1 \xi_{it} + \gamma_2 k_{it} + \gamma_3 \omega_{it-1} + \gamma_t + \gamma_s + \nu_{it}, \tag{17}
$$

where $\xi_{it} \equiv \omega_{it} - \omega_{it-1}$ is the “shock” in TFPR between $t$ and $t - 1$. From our one-period time-to-build assumption, this shock has not been observed when the firm makes its investment decision about capital stock $k_{it}$ at time $t - 1$. We also condition on lagged TFPR to make sure we compare two firms with the same TFPR at $t - 1$.
making the same capital decision, and we ask whether their MRPK is different if they are hit by different TFPR shocks $\xi_{it}$. Our theory predicts a positive coefficient for $\gamma_1$. The null hypothesis, given by the static model, is no meaningful dispersion in MRPK as a function of TFPR shocks between $t$ and $t-1$. Table 5 lists the estimates for $\gamma_1$ by country. In every case, in every specification, we observe a significant, positive coefficient on the capital coefficient, $\gamma_1$, as predicted.

A further prediction of our framework is that a producer’s MRPK should be mean reverting. We run a regression of MRPK at time $t$ on MPRK at time $t-1$, and obtain estimates of the AR(1) coefficient. This coefficient varies by country from 0.73, for Romania, to 0.90, for Chile. The coefficient is significant at the 1% level in all cases. Hence, across all countries we find evidence for mean reversion in the MRPK. That is, in the long run, the restriction of adjustment costs on capital fades, and a firm’s capital level reverts to the time invariant mean.\(^{23}\)

### 4.2.2 Aggregate Implications

In addition to the aggregate implication that the dispersion in MRPK is strongly related to the volatility of TFPR, our model suggests two additional aggregate implications: The following moments, at the industry-year level, are all correlated with volatility: a) the dispersion in the change in MRPK; and, b) dispersion in the change in capital.\(^{24}\) We pool across all our Tier 1 countries, and run reduced-form regressions for both these aggregate variables, measured at the industry-year-country level, on volatility. We include year and country fixed effects, and cluster standard errors by country. The regression results are shown in Table 6.

We begin with the dispersion in the change in MRPK, Std$_{st}(\Delta MRPK)$. Model

\(^{23}\)We run MRPK$_{it} = \mu + \rho MRPK_{it-1} + \nu_{it}$ by country, and include year and industry fixed effects. The standard errors are clustered at the firm/plant-level to account for serial correlation and heteroskedasticity. All estimates of $\rho$ are significant at the 1 percent level. Table OA.14 in the Online Appendix lists the estimates.

\(^{24}\)A related literature explores the responsiveness of productivity dispersion to the business cycle. Baeckmann and Moscarini (2012), Bloom et al. (2012), and Kehrig (2011) all find that productivity volatility increases in recessions. We find no economically significant impact of recessions on the dispersion of MRPK, although, like Bloom et al. (2012), we see sales volatility increase. Given that MRPK is the sales to capital ratio, this suggests that capital input adjustments offset any effect coming via changes in sales.
simulations (as described in Section 2.4) indicate that the dispersion in the change in MRPK should be positively correlated with volatility.25 In Table 6, we observe a positive and significant correlation between volatility and the dispersion in the change in MRPK both within the U.S. data, and within the pooled data across all Tier 1 countries (both excluding and including the U.S.). While the degree of correlation should vary with the persistence of the AR(1) process present in each country, the positive correlation in the pooled sample is consistent with the model prediction.

The second moment we examine, the dispersion in the change in capital $\text{Std}_\text{st}(\Delta k)$, has a strongly non-linear relationship to volatility. Figure 3 shows the relationship predicted by the model using the same simulation procedure as in Section 2, where panel (a) presents this relationship for the adjustment costs we will estimate for the U.S. in the next section, and panel (b) also includes the case without any adjustment cost, but preserving the assumption of a one-period time-to-build. The reader should note the difference in the vertical scale for these two panels.

Figure 3 reflects the mechanism described in Section 2.4. That is, the flattening of the change in capital-adjustments as volatility increases reflects to the changing trade-off between the size of shocks experienced today and the likelihood that they will be swamped by future shocks. This holds for both panels (a) and (b) in figure 3.

To examine this in a reduced form, we interact the volatility coefficient with a dummy if the volatility associated with that industry-year-country observation is higher than the median for that industry-country. The model prediction is that the coefficient on this interaction is (weakly) negative. As can be seen in Table 6, this coefficient is always negative and, in the case of the U.S. and the all-country sample, significant.

### 4.3 Adjustment Costs in Other Inputs

Our model makes the stark assumption that capital is the only input that faces adjustment costs, and our empirical approach builds on this assumption. This is clearly a simplification of the data-generating process.

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25See Figure OA.1 in the Online Appendix. The lines describing the relationship is essentially straight.
This approach is based on the implicit claim that, across the span of countries and industries examined in this paper, capital adjustment costs are first-order as compared to the adjustment costs of other inputs. A simple way to evaluate this claim is to examine the (log) dispersion in the marginal revenue products of capital, labor, and intermediate inputs ($k, l,$ and $m$). To do this we compute $\text{Std}_t(\ln(\beta X) + s_t - X_{it})$ for $X \in \{k, l, m\}$ for each country. Table 7 shows the results.

Across all countries, dispersion in the marginal revenue product of capital is greater than that for any other input. Further, the order of dispersion is in line with what one might expect: $\text{Std}(\text{MRPK}) > \text{Std}(\text{MRPL}) > \text{Std}(\text{MRPM})$. Given these results we proceed with the maintained assumption that capital adjustment costs are the most important component of the adjustment costs likely facing many firms in making input decisions.\(^{26}\)

## 5 Structural Analysis

In this section we evaluate the ability of the model to capture the magnitude of the degree of dispersion in the marginal product of capital at the industry level, across our Tier 1 country datasets.

We begin by evaluating a baseline specification of our model, in which we assume all industry-countries have the same production technology and the same adjustment costs (we use the U.S. mean production coefficients, with the adjustment costs estimated from the U.S. data). In this simple version, the only thing that varies over industries in the structural model is the $\text{AR}(1)$ process governing TFPR shocks. This specification is intended to highlight the extent to which, on its own, the TFPR shock process can capture dispersion in marginal products.

Following this baseline specification, we explore the extent to which using industry-
specific production functions and different adjustment cost specifications allow us to capture additional richness. Before getting to these results, we first set out the elements in the structural estimation of the model.

5.1 Estimation

We briefly discuss the estimation of the crucial parameters of our structural model, those that vary by industry – i.e., the TFPR process, and how we recover the estimates of the adjustment cost parameters.

The AR(1) TFPR process is specified as: 
\[ \omega_{it} = \mu_{sc} + \rho_{sc} \omega_{it-1} + \sigma_{sc} \nu_{it}. \]
Note that the coefficients are country-industry specific. Estimation follows the procedure described in Section 4 and relies on standard maximum likelihood estimation techniques to recover the parameters.

Recall that the adjustment cost specification is given by:

\[ C(I_{it}, K_{it}, \Omega_{it}) = I_{it} + C_F K_{it} \{ I_{it} \neq 0 \} \pi(\Omega_{it}, K_{it}) + C_Q K_{it} \left( \frac{I_{it}}{K_{it}} \right)^2. \] (18)

We estimate \( \theta = \{ C_F, C_Q \} \) using a minimum-distance procedure very similar to that in Cooper and Haltiwanger (2006). That is, we seek parameters that minimize the distance between the moments predicted by the model, and those that are found in the data. The moments we use are: the proportion of firms with less than a 5 percent year-on-year change in capital; the proportion of firms with more than a 20 percent year-on-year change in capital; and the standard deviation of the year-on-year change in log capital.\(^{27}\)

Denote the predicted moments from the model for an industry \( s \) in country \( c \) as \( \Psi_{cs}(\theta) \), found by solving for the firms’ optimal policies and simulating the model forward for 1000 months for 10,000 firms, and computing moments based on the last

\(^{27}\)Notice that according to the results of Theorem 1, stated in the appendix, these moments are invariant to differences in the mean \( \mu \) of the TFPR process, and thus we do not need to take a stand on the presence of a firm fixed-effect in the estimation procedure. However, we have also looked at the model’s predictions using estimates of the AR(1) process that include a producer fixed-effect for the United States, and we find comparable results (contained in Table OA.14 in the Online Appendix).
two years of the simulated data set.\(^2\) These predictions may differ across industries, depending on production function coefficients \(\beta_l, \beta_m, \text{ and } \beta_k\), as well as the TFPR process estimated in the previous subsection. We then aggregate the industry prediction to the country level by taking a weighted average of the industry-level prediction; i.e.,

\[
\Psi_c(\theta) = \frac{1}{\sum_s N_{sc}} \sum_s N_{sc} \Psi_{cs}(\theta),
\]

where \(N_{sc}\) denotes the number of producers in industry \(s\) for country \(c\). Thus, the country-level predictions are matched to country-level moments on changes in capital, where the moments from the data are denoted \(\hat{\Psi}\).

We estimate the model’s adjustment costs using minimum distance with a criterion function given by the usual quadratic form, with weighting matrix \(W\):

\[
Q(\theta) = \left(\hat{\Psi} - \Psi(\theta)\right)' W \left(\hat{\Psi} - \Psi(\theta)\right).
\tag{19}
\]

As the moments in the data are similarly scaled, we pick the identity matrix as a weighting matrix (\(W = I\)). We find the minimized value of the criterion using a grid search.

Table 8 presents estimates of the adjustment costs by country, along with the moments used to estimate the model. Three aspects of the table are noteworthy.

First, the moments on the year-to-year change in capital differ substantially between countries. For the United States, over 39% of plants do not change their capital by more 5%, while this number is 20% for Spain, and 8% for Romania.\(^3\) Likewise, the share of plants that vary their capital by more than 20% is 21% for the U.S., 28% for Chile, but 76% for Slovenia. These differences in the variation of capital translate into differences in the estimated adjustment costs by country, with the U.S. having relatively high convex and fixed adjustment costs, and Mexico having convex adjustment costs that are at least five times smaller.

\(^2\)We employ a very fine grid for capital stock (of 3 percent), since fixed costs are identified from the absence of small changes in capital. With a coarser grid for capital stock, it is difficult to identify small fixed costs. This comes at the expense of computational time, and solving the value function takes over a half-hour. The total computation time required for a single 3GHz processor to complete the estimation and simulations reported in this section is 2,286,000 minutes (1,587 days). The computational burden was significantly reduced via parallel computation on a large computing cluster at NYU. For further details regarding computation, see Appendix C.

\(^3\)Note that Spain and Romania are firm-level data.
The large differences in moments on changes in capital are striking. Beyond differences in adjustment costs, they also reflect differences in patterns of aggregate growth for each of these countries, and differences in the data collection protocols for Tier 1 data. For instance, Slovenia experienced a rapid increase in output over the time period we study (1994-2000), but this is not the case for the U.S. manufacturing sector from 1963 to 1997.\footnote{Since the standard approach to estimating adjustment costs which we use, such as found in Cooper and Haltiwanger (2006) or Bloom (2009), matches moments from the steady-state distribution, this type of model has difficulty dealing with aggregate shocks.} As well, for some datasets, changes in capital are computed from the change in the reported book value of assets, while for other datasets, these are inferred from investment and depreciation. Presumably, these differences in the reporting protocol will also lead to differences in the measurement in the change in capital.

Second, for several countries—France, Mexico, Romania—we estimate no fixed costs of adjustment (beyond the one-period time-to-build, which is in itself a form of adjustment friction). In these countries, even with no fixed cost of adjustment, the model predicts that fewer firms would change their capital by less than 5% than what we find in the data. Conversely, a zero convex cost of adjustment is strongly rejected. As the convex adjustment costs get closer to zero, the volatility of capital rises sharply. Given the data, this allows us to conclusively reject the absence of any costs of adjusting a firm’s capital stock.

Third, focusing on the U.S., we obtain the following estimates: fixed adjustment costs \((C_F^F)\), 0.09; convex adjustment cost \((C_Q^Q)\), 8.8. The fixed cost of adjustment is equivalent to 1.5 months of output, while the convex adjustment costs are such that when a firm doubles its capital in a month, this component of cost is equal to 8.8 times the value of its investment.\footnote{These parameters can be compared to those found in Bloom (2009) (Table 3, column 2) who, using a sample of (only) large publicly listed firms in Compustat, obtains fixed adjustment costs of 0.01 and convex adjustment costs of 1.00.}

To assess the fit of the model, we compute the sum of squared errors, scaled by the sum of the squared ‘dependent’ variable (data). That is, if the data are a vector \(x\) that...
is predicted by a variable \( \hat{x} \), then we compute

\[
S^2 = 1 - \frac{(x - \hat{x})'(x - \hat{x})}{x'x}
\]

as our measure of fit. This measure of fit is closely related to the uncentered \( R^2 \) measure of fit familiar from regression analysis. However, because our model’s prediction does not come from a regression, but from a parameterized model, nothing in the structure restricts \( S^2 \) to lie in \([0, 1]\), though, by definition, it must be less than or equal to one. That being said, to map our measure of fit into a context equivalent to the \( R^2 \), it is correct to interpret \( S^2 \) as the proportion of the observed data captured by the model’s prediction, with the caveat that it is possible for this number to be negative.

### 5.2 Results

As noted above, our baseline specification assumes that all industries in all countries have the same adjustment costs and the same production technology. We take both from the U.S. data: we use the mean U.S. production coefficients and U.S. adjustment costs.\(^{32}\) Our objective in evaluating this specification is to highlight the extent to which (just) differences in the AR(1) process can capture dispersion in the marginal revenue product of capital, at the industry level, across a variety of data sets (equivalently, countries).

Table 9 shows the \( S^2 \) measure of fit, comparing the model prediction of the dispersion in the MRPK to that observed in our various Tier 1 data sets. Pooling across all industry-countries, the \( S^2 \) is 0.674, while, if the U.S. is excluded, the \( S^2 \) is 0.879. This suggests that the model does a good job of capturing the observed dispersion. It also highlights the curious fact that the performance of this baseline model is worst on the U.S. data, despite being based on U.S. numbers.

The U.S. \( S^2 \) is 0.223, as compared to 0.879 for all non-U.S. countries. The reason for this is that the U.S. data employs a far finer industry definition than do our other data

\(^{32}\)Mean production coefficients are computed by taking the mean of the industry labor and materials coefficients and then using these to compute the capital coefficient.
sets. In the U.S. data firms are allocated to one of 188 industry classifications, whereas in the other data the numbers of industries varies from 8 (Chile) to 52 (Mexico).\textsuperscript{33} This means that, when we impose mean production coefficients, we do so on industry definitions which incorporate differing levels of aggregation. In the U.S., where the industries are finely defined, this means that some industries will have firms that all use production technologies that differ markedly from the standard firm in the economy. As a result, the baseline model can have a hard time capturing the investment patterns observed in these industries when it has to use the production coefficients from a ‘standard’ firm.

The impact of industry heterogeneity in the U.S. data is illustrated by comparing specification (2) to specification (1) for the U.S.\textsuperscript{34} Specification (2) adds industry specific production coefficients to the model. Once industries are allowed to vary in their production technology the U.S., $S^2$ increases from 0.223 to 0.806, reflecting the model’s increased capacity to captures investment patterns across a much wider range of industries.

As illustrated by the preceding discussion of the baseline results, adding more flexibility to the model can increase the extent to which the dispersion in the data can be captured. To this end, we depart from the baseline model and allow each country to have its own adjustment costs (from Table 8), and allow each country-industry to have its own production function coefficients (specification (2) in Table 9). Following that we investigate the sensitivity of this expanded model to changes in the adjustment costs: we impose U.S. adjustment costs, twice the U.S. adjustment costs, and zero adjustment costs (aside from the one-period time-to-build) on all countries (specifications (3), (4), and (5) respectively, in Table 9).

Prior to discussing results, we outline some measurement issues: Recall that we

\textsuperscript{33}After accounting for disclosure, and basic data integrity (i.e. missing data etc.), the numbers of industries by country (data-set) are: Chile 8, France 21, India 20, Mexico 52, Romania 21, Slovenia 18, and Spain 22. The U.S. has 188. This merely reflects that the detail of industrial activity reporting varies across datasets. For example for the French data we observe the principal activity of the firm, a 2 digit industry code, while we also observe its (potentially) multiple 4 digit industry codes. However, we do not see the output and input data broken down at this level of aggregation, which is standard in these data.

\textsuperscript{34}In Table 9, specifications (2) and (3) are equivalent for the U.S.
assume \( \beta_l + \beta_m + \beta_k = 0.75 \) (given constant returns in the production function, and a demand elasticity of -4). Given this, we handle the data as described in Section 3.2, with one exception: In the Slovenian data, the material coefficient is greater than 0.75 on average. As a result a strict application of our procedure would imply negative capital coefficients for all Slovenian manufacturing sectors, which we think is not plausible.

To avoid having to omit Slovenia, an interesting country in its own right, we use the mean U.S. coefficients to generate all Slovenian results in this section.\(^{35}\)

Across specifications (2), (3), (4), and (5) there is little qualitative difference in the capacity for the model to capture dispersion. This reflects the good fit of the baseline model. That is, there is not a great deal of scope for improvement in many cases. Perhaps most interestingly, the model’s performance in capturing dispersion in MRPK is not dramatically altered by changing the level of adjustment costs. A zero adjustment cost reduces fit in most countries somewhat, but imposing twice the U.S. adjustment cost does not have an economically meaningful impact. This suggests that the presence of some capital adjustment friction is important, but that the extent of the friction is not crucial, at least as far as dispersion in MRPK is concerned.\(^{36}\) In the absence of any adjustment cost in capital, including time to build, the model evaluated under our parameter values an \( S^2 \) of zero (under-prediction). Note that, when the model over-predicts dispersion, it is possible for this \( S^2 \) measure to become negative.

\(^{35}\)Slovenia is interesting due to the volatility introduced by the transition process it experienced during our sample time period. Using U.S. production coefficients keeps the specification consistent with the structural model, albeit in a way that restricts us to examining how Slovenian firms would behave if they had the production technology of U.S. firms.

\(^{36}\)For other moments, notably the dispersion in the change in capital, the level of the adjustment can make a significant difference. See Table OA.2 in the Online Appendix, and in particular column (5) corresponding to the case of no adjustment costs. To examine the sensitivity of the models’ predictions to the potential misspecification of the productivity process, we leverage Theorem 1, and, impose the \( \sigma \) and \( \rho \) terms from an alternative productivity process allowing for firm fixed effects, where we rely various dynamic panel data estimators. Table OA.3 in the Online Appendix shows that our results are robustness to the inclusion of producer-level fixed effects in the productivity process.
6 Cross-country analysis

The main source of variation that we have relied on thus far is cross-industry variation within a country. Although our results suggest a positive correlation between dispersion and volatility in cross country settings, drawing a stronger inference is limited by only having a sample of eight countries, each with different data collection protocols. In this section, in order to provide auxiliary evidence that speaks to such a conclusion, we exploit a larger-cross section of developing countries for which we only observe a sample of firms for, at most, three consecutive periods. To this end, we rely on the WBES data. These data trade off greater cross-country variation, at the expense of stricter data collection protocols and a much larger, within-country, sample of firms. As before, we apply our reduced form and structural analysis (as carried out in Section 4 and 5, respectively) on a large cross-section of countries. We briefly introduce the data before we present our results.

6.1 The WBES Data

The WBES data were collected by the World Bank across 41 developing countries and many different industries between 2002 and 2006. Standard output and input measures are reported in a harmonized fashion. In particular, the data report sales, intermediate inputs, various measures of capital, and employment, for a three-year period, which allows us to compute changes in TFPR and capital. Out of the 41 countries in the data, 33 have usable firm-level observations. This is primarily because, for many years and countries, the World Bank did not collect multi-year data on capital stock.

To construct data on both TFPR and the change in TFPR we need two years of information on sales, assets, intermediate inputs, and employment. 5,558 firms across our 33 countries meet this criterion. The firms in the final data are almost certainly not representative of firms in their economies; for instance, the mean number of workers is 248. Thus, for instance, the data tend to oversample larger firms. In Appendix B

\footnote{We also drop countries with fewer than 25 observations.}
we provide further details on sample construction and compare the firms in our sample with the universe of sampled firms.

6.2 Cross-Country Reduced-Form Results

We start by establishing the relationship between static misallocation and volatility across countries, using a similar method to what we used at the industry-level. Figure 4b plots the volatility of TFPR against the dispersion of MRPK for our 33 countries in the WBES data. We find the same striking positive relationship as we presented in Figure 2 in Section 4 using U.S. Census data. Figure 4a shows the same exercise for the eight Tier 1 countries.

The solid black line in Panel (b) shows the line of best fit, corresponding to a regression in which dispersion is projected onto a constant and volatility, where country observations are weighted by the number of in-country observations.\textsuperscript{38} This positive, significant, correlation between dispersion and volatility is robust to alternate specifications which use alternate weights, industry fixed effects, and controls for firm size.\textsuperscript{39} The relationship can also be replicated using country-industry observations.

6.3 Structural analysis

We now perform a structural analysis of the World Bank data, in the same spirit as that conducted in Section 5. We apply the same model, with two alterations. We estimate a AR(1) process for TFPR at the country-level; and we use the adjustment costs estimated for the U.S. reported in Table 8. We use U.S. adjustment costs since we found in the analysis of the Tier 1 country datasets that the precise level of adjustment costs appears to have little influence on the ability of the model to capture the dispersion in the MRPK. Hence, we examine the capacity of the model to capture dispersion using this simple specification.

\textsuperscript{38}See the notes accompanying the figure for the coefficients and standard errors.

\textsuperscript{39}The last two specifications use firm-level observations. A complete set of results can be found in Table OA.16 in the Online Appendix.
To obtain country-level predictions, we aggregate predictions at the industry-level, using the number of producers in an industry as weights. Moreover, since dispersion in MRPK at the country-level includes both variation in MRPK within an industry, as well as between industries, we need to account for both these sources of variation when aggregating MRPK.

The results are depicted in Figure 5. The countries in the World Bank data are shown using unfilled circles, while, for comparison, the Tier 1 countries are shown using filled circles. The horizontal axis measures the model’s prediction, while the vertical axis measures the dispersion in MRPK present in the data.

The model does quite well. The $S^2$ for the WBES countries is 0.802. This is comparable to the model performance reported in Table 9 for the industry-level data from Tier 1 countries. When we treat the Tier 1 country data in the same way as the WBES data, we get an $S^2$ of 0.906. Also, if anything, the model has a tendency to over-predict the dispersion in MRPK, suggesting that the dispersion observed in data is less than what might be expected to be generated by firms operating in the U.S., facing U.S. adjustment costs, but otherwise equivalent AR(1) and technological environments.

### 6.4 Volatility and external measures

So far, our strategy has been to estimate volatility of TFPR and see how this measure of volatility is linked to the dispersion in various economic variables. We have shown that volatility varies across industries within countries, as well as across countries. Although it is beyond the scope of this paper to develop a theory of volatility to explain why volatility varies across different economic environments, it is only natural to ask whether our measures of differences in volatility across countries are related to features of these economies.

To this end we match the World Bank Doing Business Dataset (henceforth the

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40The Tier 1 country data is aggregated in the same way as the World Bank data, and we use a country-specific AR(1) process in in the model simulation.
WBDB data) with the WBES data and check whether volatility is correlated with some of the survey questions in the WBDB data. In particular we examine whether countries with greater volatility also face more frictions in contract enforcement — measured in terms of cost and duration of contract enforcement— factors that would plausibly affect the uncertainty faced by producers. Similarly, we also collected data on the extent of natural disasters per unit landmass, and an index of political stability.\footnote{The Online Appendix provides more details on the data and the analysis: Section 0.1 describes the data and variable construction. Table OA.13 presents the regression results.}

When we regress volatility against the cost of contract enforcement measure, the duration measure and a constant, we find a significant—at the ten percent level—and positive coefficient on cost, and an $R^2$ of 7%. Time to enforce is not significant, economically or statistically. This suggest that countries that exhibit larger volatility are also characterized by higher contracting costs. A regression of volatility on a constant and natural disasters per unit landmass also yields a positively coefficient on political stability, and is also significant at the 10% level. Given the limited number of countries for which we have measures of volatility, this seems as a precise an estimate as one could reasonably expect. Interestingly, the political stability index is not significant in any regression, although the correlation does have a negative sign. When we run a regression with all of our measures of the economic environment in a country against volatility and we find, as before, that the cost of contract enforcement is associated with significantly higher volatility. Moreover, the F-stat is significant at the 10% level, indicating that the combination of cost and duration of contract enforcement, political instability, and natural disasters does explain some component of the cross-country differences we observe in volatility. In particular this simple linear cross-sectional regression leads to a $R^2$ of 14.3 percent. While speculative, these reported correlations suggest that there may be linkages between volatility and features of a country’s operating environment that are worth investigating further.
7 Conclusion

The primary contribution of this paper is to establish the link between the dynamic process governing TFPR changes over time, and cross-sectional measures of dispersion in the marginal product of capital. We have shown that a parsimonious model of the TFPR process coupled with capital adjustment costs explains both the level and variation of the dispersion in the static marginal revenue product of capital across industries within countries, and across countries. We do this by examining eight large-scale country-level data sets, including the U.S. Census, (Tier 1 data) and then extend the analysis with data from the World Bank on a further 33 developing countries (Tier 2 data). The cross-industry findings are primarily supported by the Tier 1 data, while the cross-country findings are primarily supported by the Tier 2 data.

These findings suggest that producers in industries (countries) that experience larger ‘uncertainty’ in the future operating environment (i.e., higher volatility in TFPR) make different investment decisions than those producers active in less volatile environments. This leads to different levels of capital and output and, moreover, means that the welfare gains from policies inducing reallocation of factors of production are likely to be lower than otherwise implied by static models, at least to the extent that the TFPR process is exogenous. Indeed, if one has the view that the productivity process is an exogenous, or primitive, feature of the model, then our findings suggest that, in an aggregate sense, the firms in the countries we studied are acting much as the social planner in our model would have them act (assuming that the social planner takes the capital adjustment costs as given). This suggests that there are few welfare implications for differences in cross-sectional measures of (static) capital misallocation across industries or countries. On the other hand, if government policy can affect the productivity process, then there may be significant welfare dividends to policy interventions aimed at moving toward some socially-optimal productivity process. However, characterization of what this optimal process is likely requires a more subtle modeling approach than that offered here.
This raises the important issue of the specific sources of adjustment costs and TFPR volatility, a topic on which we provide some suggestive evidence, but otherwise leave open for future research. In particular, TFPR is not just technological in nature. Our measure of TFPR volatility will capture changes in managerial and physical technology. It will also capture year-on-year variation in the intensity of corruption (and the implicit tax therein); other aspects of the application of the rule of law relevant to business (such as erratic contract enforceability); changing regulatory frictions; environmental factors (e.g., floods and other natural disasters) and the efficacy of infrastructure used to cope with them; and year-on-year variation in markups and product market competition. Many of these elements of measured productivity volatility may be effectively influenced by appropriate policy aimed at providing a stable business environment.
References


Tables and Figures
Table 1: Tier 1 Data sources

<table>
<thead>
<tr>
<th>Country</th>
<th>Plant</th>
<th>Firm</th>
<th>Provider – Survey Type</th>
<th>Size Threshold</th>
<th>Years Covered</th>
<th>Obs/Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>X</td>
<td></td>
<td>INE – Census</td>
<td>More than 10 workers</td>
<td>1979-1986</td>
<td>4,700</td>
</tr>
<tr>
<td>India</td>
<td>X</td>
<td></td>
<td>CMIE (Prowess) – Balance Sheet</td>
<td>Large Firms</td>
<td>1989-2003</td>
<td>2,047</td>
</tr>
<tr>
<td>Mexico</td>
<td>X</td>
<td></td>
<td>SEC-OFI – Sample</td>
<td>Medium/Big Plants</td>
<td>1984-1990</td>
<td>3,026</td>
</tr>
<tr>
<td>Romania</td>
<td>X</td>
<td></td>
<td>BvD Amadeus – Tax Records</td>
<td>No</td>
<td>1999-2007</td>
<td>19,444</td>
</tr>
<tr>
<td>Slovenia</td>
<td>X</td>
<td></td>
<td>Statistical Office – Census</td>
<td>No</td>
<td>1994-2000</td>
<td>4,151</td>
</tr>
</tbody>
</table>

Note: The X refers to which unit of observation the specific data records. Datasets can comprise both firm- and plant-level data if the plant-level data contains firm identifiers. For the U.S., Obs/Year is plant observations per year. The Obs/year is the average number of firms/plants per year calculated from the total firm/plant-year observations and the number of years covered.
Table 2: Summary Statistics Across Tier 1 Datasets

<table>
<thead>
<tr>
<th>Country</th>
<th>Workers</th>
<th>$\Delta s$</th>
<th>$\Delta \omega$</th>
<th>Medians</th>
<th>Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Disp. MRPK</td>
<td>Disp. $k$</td>
<td>Disp. $\omega$</td>
<td>Volatility</td>
</tr>
<tr>
<td>U.S.$\dagger$</td>
<td>111</td>
<td>0.01</td>
<td>0.00</td>
<td>0.98</td>
<td>1.78</td>
</tr>
<tr>
<td>Chile</td>
<td>19</td>
<td>0.02</td>
<td>0.00</td>
<td>1.22</td>
<td>1.92</td>
</tr>
<tr>
<td>France</td>
<td>8</td>
<td>0.02</td>
<td>0.02</td>
<td>1.28</td>
<td>2.04</td>
</tr>
<tr>
<td>India</td>
<td>n.a.</td>
<td>0.06</td>
<td>0.04</td>
<td>1.13</td>
<td>1.61</td>
</tr>
<tr>
<td>Mexico</td>
<td>141</td>
<td>0.02</td>
<td>0.02</td>
<td>1.40</td>
<td>2.13</td>
</tr>
<tr>
<td>Romania</td>
<td>5</td>
<td>0.01</td>
<td>0.01</td>
<td>1.38</td>
<td>2.05</td>
</tr>
<tr>
<td>Slovenia</td>
<td>4</td>
<td>0.07</td>
<td>0.03</td>
<td>1.56</td>
<td>2.51</td>
</tr>
<tr>
<td>Spain</td>
<td>8</td>
<td>0.03</td>
<td>0.01</td>
<td>1.48</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Note: Dispersion MRPK is given by $\text{Std}(\text{MRPK}_{it})$, and volatility is $\text{Std}(\omega_{it} - \omega_{it-1})$ – i.e., we compute dispersion across the entire dataset. $\dagger$ Median computed for the U.S. Census data as the average of plants between the 48th and 52nd percentile.
Table 3: Dispersion MRPK and volatility

<table>
<thead>
<tr>
<th>Country</th>
<th>Coefficient</th>
<th>$R^2$</th>
<th>Industry-Year Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. [Plants]</td>
<td>0.76***</td>
<td>0.47</td>
<td>4,037</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. [Firms]</td>
<td>0.68***</td>
<td>0.44</td>
<td>4,037</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chile</td>
<td>0.54*</td>
<td>0.13</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>1.03***</td>
<td>0.28</td>
<td>167</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>0.19**</td>
<td>0.07</td>
<td>296</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>0.61**</td>
<td>0.28</td>
<td>279</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Romania</td>
<td>0.44***</td>
<td>0.21</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.53**</td>
<td>0.09</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.56*</td>
<td>0.35</td>
<td>181</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All I</td>
<td>0.55***</td>
<td>0.67</td>
<td>5,326</td>
</tr>
<tr>
<td>(unweighted)</td>
<td>(0.15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All II</td>
<td>0.74***</td>
<td>0.50</td>
<td>5,326</td>
</tr>
<tr>
<td>(weighted)</td>
<td>(0.03)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: We report the coefficient of a regression of $\text{Std}_{\text{st}}(MRPK)$ against volatility, defined as $\text{Std}_{\text{st}}(\omega_{it} - \omega_{it-1})$, including year dummies. Standard errors are clustered at the industry level. *, **, and *** denote significance at the 10%, 5% and 1% levels respectively. ‘All I’ refers to the unweighted regression, whereas ‘All II’ refers to a weighted regression with the weights the number of producers in a country-industry-year observation. These cross-country-industry-year regressions include year and country dummies, and report standard errors clustered at the country level. Table OA.15 in the Online Appendix reports the regression coefficients for the US using only variation across industries. This is directly related to Figure 2.
Table 4: Dispersion of MRPK and volatility of TFPR: robustness

<table>
<thead>
<tr>
<th>Country</th>
<th>Std$<em>t$[(\omega</em>{it} - \omega_{i,t-1})]</th>
<th>AR(1)</th>
<th>AR(1)FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.82***</td>
<td>0.86***</td>
<td>1.24***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.07)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Chile</td>
<td>1.48*</td>
<td>2.10***</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.65)</td>
<td>(0.65)</td>
<td>(1.48)</td>
</tr>
<tr>
<td>France</td>
<td>1.73***</td>
<td>1.75***</td>
<td>2.55***</td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.41)</td>
<td>(0.61)</td>
</tr>
<tr>
<td>India</td>
<td>1.31***</td>
<td>1.75***</td>
<td>2.75***</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(0.39)</td>
<td>(0.55)</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.39*</td>
<td>0.41**</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.17)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Romania</td>
<td>0.76***</td>
<td>0.94***</td>
<td>1.38*</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.36)</td>
<td>(0.72)</td>
</tr>
<tr>
<td>Slovenia</td>
<td>2.73***</td>
<td>2.47***</td>
<td>3.47***</td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.41)</td>
<td>(0.69)</td>
</tr>
<tr>
<td>Spain</td>
<td>1.24***</td>
<td>1.46***</td>
<td>2.55***</td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td>(0.44)</td>
<td>(0.59)</td>
</tr>
</tbody>
</table>

Note: We report the coefficient of a regression of Std$_t$ (MRPK) against alternative measures of volatility, defined in the text. Standard errors are clustered at the industry level. *, **, and *** denote significance at the 10%, 5% and 1% levels respectively.
Table 5: Additional Predictions: MRPK against shocks to TFPR

<table>
<thead>
<tr>
<th>Country</th>
<th>Shock</th>
<th>Shock AR1</th>
<th>Shock AR1-FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.29***</td>
<td>1.26***</td>
<td>1.13***</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Chile</td>
<td>1.42***</td>
<td>1.42***</td>
<td>1.04***</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>France</td>
<td>1.37***</td>
<td>1.37***</td>
<td>1.26***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>India</td>
<td>1.32***</td>
<td>1.32***</td>
<td>1.16***</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.07***</td>
<td>1.07***</td>
<td>0.67***</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Romania</td>
<td>1.31***</td>
<td>1.31***</td>
<td>1.12***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Slovenia</td>
<td>1.65***</td>
<td>1.64***</td>
<td>1.48***</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Spain</td>
<td>1.28***</td>
<td>1.28***</td>
<td>0.69***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

Note: We run, by country, ln(MRPK) against the TFPR shock, capital, lagged TFPR and year and industry fixed effects. The TFPR shock is given by $\xi_{it} \equiv \omega_{it} - E(\omega_{it}|I_{it-1})$, where $I_{it-1}$ is the information set of producer $i$ at time $t-1$; depending on the TFPR process we consider this contains lagged TFPR and producer and year fixed effects. We suppress the coefficients on capital and lagged TFPR (they are significant with negative and positive signs, respectively, everywhere) and also suppress the fixed effects on year and industry. The standard errors are clustered at the firm/plant-level to account for serially correlation and heteroskedasticity. *** denotes significance at the 1 percent level.
Table 6: Aggregate Implications

<table>
<thead>
<tr>
<th>Aggregate Moment</th>
<th>Coefficient</th>
<th>$R^2$</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std$_{st}$(Δ$MRPK$) [U.S. only]</td>
<td>1.03***</td>
<td>0.63</td>
<td>4,039</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std$_{st}$(Δ$MRPK$) [excl U.S.]</td>
<td>0.56***</td>
<td>0.64</td>
<td>1,289</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std$_{st}$(Δ$MRPK$) [All]</td>
<td>0.89***</td>
<td>0.68</td>
<td>5,326</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std$_{st}$(Δ$k$) [U.S. only]</td>
<td>0.13***</td>
<td>0.31</td>
<td>4,037</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std$_{st}$(Δ$k$) [excl U.S.]</td>
<td>0.07***</td>
<td>0.62</td>
<td>1,182</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std$_{st}$(Δ$k$) [All]</td>
<td>0.12***</td>
<td>0.76</td>
<td>5,219</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std$_{st}$(Δ$k$) [U.S. only] ×{&gt;Median Vol.}</td>
<td>-0.03**</td>
<td>0.63</td>
<td>1,182</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std$_{st}$(Δ$k$) [excl U.S.] ×{&gt;Median Vol.}</td>
<td>-0.09</td>
<td>0.09</td>
<td>5,219</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std$_{st}$(Δ$k$) [All] ×{&gt;Median Vol.}</td>
<td>-0.04**</td>
<td>0.76</td>
<td>5,219</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The coefficients are obtained by regressing each aggregate moment against volatility using country-industry-year variation, where we include year and country fixed effects. Standard errors are clustered by country when pooled, and by industry when using U.S. data.
Table 7: Comparing dispersion of MRPK to other inputs’ MRPs

<table>
<thead>
<tr>
<th>Country</th>
<th>Capital</th>
<th>Labor</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.81</td>
<td>0.63</td>
<td>0.54</td>
</tr>
<tr>
<td>Chile</td>
<td>1.22</td>
<td>0.93</td>
<td>0.48</td>
</tr>
<tr>
<td>France</td>
<td>1.25</td>
<td>0.79</td>
<td>0.87</td>
</tr>
<tr>
<td>India</td>
<td>1.01</td>
<td>0.87</td>
<td>0.55</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.19</td>
<td>0.85</td>
<td>0.51</td>
</tr>
<tr>
<td>Romania</td>
<td>1.40</td>
<td>1.17</td>
<td>0.67</td>
</tr>
<tr>
<td>Slovenia</td>
<td>1.54</td>
<td>0.98</td>
<td>0.54</td>
</tr>
<tr>
<td>Spain</td>
<td>1.45</td>
<td>0.93</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Note: We compute the standard deviation of the MRP of each input by industry-year, and we report the average across industry-years by country.

Table 8: Adjustment Cost Estimates and Moments by Country

<table>
<thead>
<tr>
<th>Country</th>
<th>Adjustment Costs</th>
<th>Data Moments on Change in Log Capital</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Convex</td>
<td>Fixed</td>
<td>Less than 5%</td>
</tr>
<tr>
<td>U.S.</td>
<td>8.80</td>
<td>0.09</td>
<td>0.39</td>
</tr>
<tr>
<td>Chile</td>
<td>4.10</td>
<td>0.07</td>
<td>0.19</td>
</tr>
<tr>
<td>India</td>
<td>3.46</td>
<td>0.12</td>
<td>0.29</td>
</tr>
<tr>
<td>France</td>
<td>0.21</td>
<td>0.00</td>
<td>0.13</td>
</tr>
<tr>
<td>Spain</td>
<td>0.74</td>
<td>0.00</td>
<td>0.20</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.15</td>
<td>0.22</td>
<td>0.08</td>
</tr>
<tr>
<td>Romania</td>
<td>0.66</td>
<td>0.03</td>
<td>0.08</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.35</td>
<td>0.00</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Notes: Standard errors were computed using the usual formula for minimum-distance estimators. However, due to the large size of the datasets we employ, the standard errors are of the order of $1 \times 10^{-3}$ or smaller and so we do not report them. Adjustment costs for Slovenia are based on a model with production function coefficients set to the mean U.S. coefficients (see the discussion in Section 5.2).
Table 9: Dispersion in MRPK, $S^2$ measures of model fit by specification

<table>
<thead>
<tr>
<th>Country</th>
<th>Specification</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td></td>
<td>0.223</td>
<td>0.806</td>
<td>0.806</td>
<td>0.643</td>
<td>0.820</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td>0.892</td>
<td>0.702</td>
<td>0.899</td>
<td>0.944</td>
<td>0.651</td>
</tr>
<tr>
<td>Chile</td>
<td></td>
<td>0.994</td>
<td>0.983</td>
<td>0.987</td>
<td>0.963</td>
<td>0.785</td>
</tr>
<tr>
<td>India</td>
<td></td>
<td>0.984</td>
<td>0.941</td>
<td>0.964</td>
<td>0.976</td>
<td>0.596</td>
</tr>
<tr>
<td>Mexico</td>
<td></td>
<td>0.879</td>
<td>0.813</td>
<td>0.883</td>
<td>0.908</td>
<td>0.634</td>
</tr>
<tr>
<td>Romania</td>
<td></td>
<td>0.983</td>
<td>0.923</td>
<td>0.817</td>
<td>0.702</td>
<td>0.846</td>
</tr>
<tr>
<td>Slovenia</td>
<td></td>
<td>0.967</td>
<td>0.774</td>
<td>0.967</td>
<td>0.984</td>
<td>0.683</td>
</tr>
<tr>
<td>Spain</td>
<td></td>
<td>0.718</td>
<td>0.627</td>
<td>0.600</td>
<td>0.530</td>
<td>0.495</td>
</tr>
<tr>
<td>All (ex U.S.)</td>
<td></td>
<td>0.879</td>
<td>0.777</td>
<td>0.820</td>
<td>0.800</td>
<td>0.640</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td>0.674</td>
<td>0.786</td>
<td>0.816</td>
<td>0.748</td>
<td>0.696</td>
</tr>
</tbody>
</table>

Specification details:

- All U.S. adj. costs X X
- Own country adj. costs X
- All 2x U.S. adj. costs X
- 1 period time-to-build only X
- U.S. avg. $\beta$’s X
- Industry-country $\beta$’s X X X X X

Note: The unit of observation is the country-industry. Specifications are: (1) All countries have the U.S.’s estimated adjustment costs and production coefficients equal to the U.S. averages across industries; (2) Industry-country specific production coefficients (except for Slovenia see Section 3.2), country specific adjustment costs, industry-country specific AR(1); (3) as for (2), but with the U.S.’s estimated adjustment costs for all countries; (4) as for (3), but with twice the U.S.’s estimated adjustment costs for all countries; and, (5) as for (3), but with zero adjustment costs (other than the one period time-to-build) for all countries. In all specifications, the AR(1) is estimated using TFPR computed using the production coefficients used in the model specification.
Figure 1: MRPK dispersion and volatility: Model simulation

Notes: Values used in this simulation are: $\epsilon = -4, \delta = 10\%, \beta = \frac{1}{1+0.065}, \beta_K = 0.12, \beta_M = 0.40, \beta_L = 0.23, C_F^K = 0.09, C_Q^F = 8.8, \lambda = 1, \mu = 0, \rho \in \{0.65, 0.85, 0.94\}$ (corresponding to the lines from bottom (0.65) to top (0.94)), $\sigma \in [0.1, 1.4]$. We use the means in the U.S. Census Data to get our $\beta$’s and use estimates of adjustment costs for the United States discussed in Section 5.
Figure 2: Volatility and the dispersion in MRPK: U.S. plant data 1972-97

Notes: The unit of observation is the industry. The line is generated by an OLS regression on 188 industries, in which the estimated slope is 0.73 (0.08) and the constant is 0.57 (0.03), and the $R^2 = 0.3$, where the standard errors are in parenthesis.
Figure 3: Model Simulation: Dispersion in the change in Capital and Volatility

(a) Adj. costs: \((C^F_K, C^Q_K) = (0.09, 8.8)\)

\[
\begin{align*}
\text{Productivity volatility (}\sigma) & \\
0 & 0.1 0.19 0.28 0.37 0.46 0.55 0.64 0.73 0.82 0.91 1 1.09 1.18 1.27
\end{align*}
\]

\[
\begin{align*}
\text{Stdst}(\epsilon_K) & \\
0 & 0.2 0.4 0.6 0.8 1 1.09 1.18 1.27
\end{align*}
\]

(b) Adj. costs: \((C^F_K, C^Q_K) = (0.09, 8.8)\) and \((0, 0)\)

\[
\begin{align*}
\text{Productivity volatility (}\sigma) & \\
0 & 0.1 0.19 0.28 0.37 0.46 0.55 0.64 0.73 0.82 0.91 1 1.09 1.18 1.27 1.36
\end{align*}
\]

\[
\begin{align*}
\text{Stdst}(\epsilon_K) & \\
0 & 0.1 0.19 0.28 0.37 0.46 0.55 0.64 0.73 0.82 0.91 1 1.09 1.18 1.27 1.36
\end{align*}
\]

Notes: Parameters are as for Figure 1. \(\rho \in \{0.65, 0.85, 0.94\}\) (corresponding to the lines from bottom (0.65) to top (0.94), when \(\sigma = 0.3\))
Figure 4: Country-level Static misallocation and TFPR volatility

(a) Tier 1 Data

(b) Tier 2 Data (WBES)

Note: Circles indicate countries, where circle size for Tier 2 data (Panel (b) is increasing in the number of firms per country. The bold straight line is the line–of–best–fit (computed using OLS with a constant term). The horizontal axis indicates the value of the standard deviation of $\omega_{it} - \omega_{it-1}$. The vertical axis indicates the standard deviation in MRPK. The regression line for Panel (a) is given by: $1.01 (0.23)+1.02(0.66)*vol$ with a $R^2$ of 0.28. The regression line for Panel (b) is given by: $0.78 (0.10)+0.67 (0.21)*vol$ with $R^2$ of 0.31, where standard errors are given in parentheses, and “vol” denotes our measure of volatility.
Figure 5: Country-level MRPK dispersion: Data vs model simulation

Notes: The vertical axis is data, while the horizontal axis is the model prediction. The unit of observation is the country. Predictions are computed at the industry-country level and then aggregated to the country level. Black dots are tier 1 countries. Unfilled circles are countries in the WBES data. All predictions use industry-country specific production coefficients, a country-level AR(1) process and the adjustments costs estimated for the U.S. in Section 5. The $S^2$ for the World Bank countries is 0.802 and is 0.906 for the Tier 1 countries. The solid line is the 45° line.
Appendix

A Proof of Invariance to Fixed Effects in Productivity Process

Theorem 1 Consider the dynamic optimization problem described by the Bellman equation:
\[
V(\Omega, K) = \max_{I, M, L} S(\Omega, K, L, M) - p_L L - p_M M - C(I, K, L, M, \Omega) + \beta \int_{\Omega'} V(\Omega', \delta K + I) f(d\Omega'|\Omega).
\] (21)

Let \( f(\Omega'|\Omega) \) be described by one of the following processes:
(A) \( \omega_{it+1} = \mu_i + \rho \omega_{it} + \sigma \epsilon_{it}; \) and
(B) \( \tilde{\omega}_{it+1} = \tilde{\mu}_i + \rho \tilde{\omega}_{it} + \sigma \epsilon_{it}. \)

Then, for any \( \mu_i \) and \( \tilde{\mu}_i, \)
\begin{enumerate}
\item (s\(_{it} - x_{it}|\mu_i) = (s\(_{it} - x_{it}|\tilde{\mu}_i); \) and
\item (x\(_{it} - x_{it-1}|\mu_i) = (x\(_{it} - x_{it-1}|\tilde{\mu}_i), \) where \( x \in \{l, m, k\}. \)
\end{enumerate}

Proof. The proof proceeds by: First, showing that changing the constant in the AR(1) amounts to a level shift in the AR(1) process; then, Second, showing that the entire problem is homogenous of degree 1; then, Third, using this to show that changing the AR(1) constant results in a level shift in the inputs; Lastly, we note that these level shifts get cancelled out when computing differences at the firm level. We use a series of lemmas to develop this reasoning.

Lemma 1 Consider two processes (A) and (B), above. Process (B) is a level shift of process (A). That is, conditional on initial conditions and the history of \( \epsilon_{it}, \) \( \tilde{\omega}_{it} = \omega_{it} + \log \Lambda \) where \( (1 - \rho) \log \Lambda = \tilde{\mu}_i - \mu_i. \)

Proof. Starting with process (A), increase \( \omega_{it} \) by \( \log \Lambda. \) Now, consider the evolution of process (B) from \( \omega_{it} + \log \Lambda: \)
\[
\tilde{\omega}_{it+1} = \tilde{\mu}_i + \rho (\omega_{it} + \log \Lambda) + \sigma \epsilon_{it} \\
= \mu_i + (1 - \rho) \log \Lambda + \rho (\omega_{it} + \log \Lambda) + \sigma \epsilon_{it} \\
= \mu_i + \log \Lambda + \rho \omega_{it} + \sigma \epsilon_{it} \\
= \omega_{it+1} + \log \Lambda
\]

Hence, process (B) is a level shift of process (A). \( \blacksquare \)

Lemma 2 A process determining the evolution of \( \tilde{\Omega}, \) where \( \log \tilde{\Omega} = \tilde{\omega}, \) described by (B) is isomorphic, in terms of realizations of random variables, to a process determining \( \Lambda \Omega \) where the process describing the evolution of \( \Omega \) is (A).

\(^{42}\)In both the theorem and proof, unless noted otherwise, variable definitions and notation follows that used in the paper.
Proof. This is a corollary of Lemma 1. ■

The rest of the proof employs a transformation of the problem. Let
\[
G_{it}^{1-a-b-c} = \Omega_{it}, \quad \text{and} \quad (1 - a - b - c) g_{it} = \omega_{it}
\]
since this is a bijective mapping, we can rewrite the TFPR process as
\[
(1 - a - b - c) g_{it+1} = \mu_{t} + \rho (1 - a - b - c) g_{it} + \sigma \epsilon_{it}
\]
and the sales function as
\[
S_{it} = G_{it}^{1-a-b-c} K_{it}^{a} L_{it}^{b} M_{it}^{c}
\]
This transformation will allow us to exploit homogeneity properties in a transparent manner. To keep notation consistent, but distinct, let
\[
\lambda^{1-a-b-c} = \Lambda.
\]

Lemma 3 (Sales) \[\lambda S_{it} (G_{it}, K_{it}, L_{it}, M_{it}) = S_{it} (\lambda G_{it}, \lambda K_{it}, \lambda L_{it}, \lambda M_{it})\]

Before proceeding to the homogeneity of the value function, it is helpful to establish that the static inputs, \(L\) and \(M\), under processes (A) and (B) are (multiplicative) level shifts of each other. This makes it easier to state subsequent Lemmas and manipulate the value function.

Lemma 4 If \(L_{it}^{*}\) and \(M_{it}^{*}\) are solutions to the system of first order conditions of static inputs, given \(G_{it}\) and \(K_{it}\); then, given \(\lambda G_{it}\) and \(\lambda K_{it}\), \(\lambda L_{it}^{*}\) and \(\lambda M_{it}^{*}\) are solutions.

Proof. It is sufficient to show that this is true for labor. As established in the paper, the first order condition is
\[
\frac{b S_{it} (G_{it}, K_{it}, L_{it}^{*}, M_{it}^{*})}{L_{it}^{*}} = p L
\]
Now, we need to show that, given \(\lambda G_{it}\) and \(\lambda K_{it}\), \(\lambda L_{it}^{*}\) and \(\lambda M_{it}^{*}\) solve the first order condition.
\[
\frac{b S_{it} (\lambda G_{it}, \lambda K_{it}, \lambda L_{it}^{*}, \lambda M_{it}^{*})}{\lambda L_{it}^{*}} = \frac{b \lambda S (G_{it}, K_{it}, L_{it}^{*}, M_{it}^{*})}{\lambda L_{*}} = \frac{b S_{it} (G_{it}, K_{it}, L_{it}^{*}, M_{it}^{*})}{L_{*}} = p L
\]
where the first equality follows from Lemma 3, and the last from equation (23). Hence, \(\lambda L_{it}^{*}\) and \(\lambda M_{it}^{*}\) solve the first order condition. ■

Lemma 4 allows us to express everything that follows as functions of \(G\) and \(K\) (and \(I\)), noting that, where relevant, a proportional increase in both leads to an equivalent proportional increase in \(L\) and \(M\). Note, in particular, that we can re-write the Bellman equation as
\[
V(G, K) = \max_{I} \pi(G, K) - C(G, K, I) + \beta \int_{G'} V(G', \delta K + I) \phi(dG'|G).
\]

\[\text{Bloom (2009) employs a similar transformation (at footnote 25).}\]
We now turn to establishing the homogeneity properties of the various components of the Bellman equation, stated in Theorem 1.

**Lemma 5 (Period Profits)** Given \( \pi(G_{it}, K_{it}) = S_{it}(G_{it}, K_{it}, L_{it}^*(G_{it}, K_{it}), M_{it}^*(G_{it}, K_{it})) - p_L L_{it}^*(G_{it}, K_{it}) - p_M M_{it}^*(G_{it}, K_{it}) \), then, \( \pi(\lambda G_{it}, \lambda K_{it}) = \lambda \pi(G_{it}, K_{it}) \).

**Lemma 6 (Capital Transition)** \( \lambda K_{it+1} (K_{it}, I_{it}) = K_{it+1}(\lambda K_{it}, \lambda I_{it}) \)

**Lemma 7 (Adjustment Costs)** \( \lambda C_{it}(G_{it}, K_{it}, I_{it}) = C_{it}(\lambda G_{it}, \lambda K_{it}, \lambda I_{it}) \)

**Lemma 8 (TFPR Transition)** Let process (B) be written in terms of \( G \) such that

\[(1 - a - b - c) g_{it+1} = \mu_i + (1 - \rho) (1 - a - b - c) \log \lambda + \rho (1 - a - b - c) g_{it} + \sigma \epsilon_{it} \]

and let the associated distribution describing the transitions of \( G \) be \( \phi_{(B)}(G_{it+1}|G_{it}) \). Similarly, let (A) be written as in equation 22 and let the associated distribution describing the transitions of \( G \) be \( \phi_{(A)}(G_{it+1}|G_{it}) \). Then, fixing \( G_{it} \) and \( G_{it+1} \),

\[\phi_{(B)}(\lambda G_{it+1}|\lambda G_{it}) = \phi_{(A)}(G_{it+1}|G_{it})\]

**Proof.** This follows from Lemma 1 and 2, noting that \( \lambda^{1-a-b-c} = \Lambda \). ■

We now turn to the value function, as defined in equation (24). Let \( V_{(A)}(G, K) \) be the value function when the TFPR process is described by (A). Similarly, let \( V_{(B)}(G, K) \) be the value function when the TFPR process is described by (B). That is,

\[V_{(B)}(G, K) = \max_I \pi(G, K) - C(G, K, I) + \beta \int_{G'} V_{(B)}(G', \delta K + I) \phi_{(A)}(dG'|G)\]

**Lemma 9 (Value Function)** For any \( G \) and \( K \), \( V_{(B)}(\lambda G, \lambda K) = \lambda V_{(A)}(G, K) \)

**Proof.**

We begin by defining \( I^*_{(A)}(G, K) \) as the optimal investment policy corresponding to \( V_{(A)}(G, K) \). We next define \( W_{(A)}(G, K, I) \) as the choice specific value function under process (A). That is, \( W_{(A)}(G, K, I) \) is the value generated when investment in the current period is set at \( I \), rather than \( I^*_{(A)}(G, K) \). So,

\[W_{(A)}(G, K, I) = \pi(G, K) - C(G, K, I) + \beta \int_{G'} V_{(A)}(G', \delta K + I) \phi_{(A)}(dG'|G)\]  \hspace{1cm} (25)

\( W_{(B)}(G, K, I) \) is defined analogously.

The proof proceeds by assuming that the future value function, \( V_{(B)}(G', \delta K + I) \), satisfies the Lemma, and showing that this implies that \( W_{(B)}(G, K, I) \) has the same property. We then show that this, in turn, implies that the present value function, \( V_{(B)}(G, K) \), satisfies the Lemma. Hence, in a stationary context, the proof exploits an inductive argument.
First, assume $V_{(B)}(\lambda G', \lambda (\delta K + I)) = \lambda V_{(A)}(G', \delta K + I)$. Now,

$$W_{(B)}(\lambda G, \lambda K, \lambda I) = \pi(\lambda G, \lambda K) - C(\lambda G, \lambda K, \lambda I) + \beta \int_{\lambda G'}^{G'} V_{(B)}(\lambda G', \delta \lambda K + \lambda I)\phi_{(B)}(d\lambda G'|\lambda G)$$

next, from Lemma 8:

$$= \pi(\lambda G, \lambda K) - C(\lambda G, \lambda K, \lambda I) + \beta \int_{G'}^{G'} V_{(B)}(\lambda G', \delta \lambda K + \lambda I)\phi_{(B)}(dG'|G)$$

then, from Lemmas 5, 6, and 7, and the maintained assumption:

$$= \lambda \pi(G, K) - \lambda C(G, K, I) + \lambda \beta \int_{G'}^{G'} V_{(A)}(G', (\delta K + I))\phi_{(A)}(dG'|G).$$

$$= \lambda W_{(A)}(G, K, I)$$

Next, we show that this implies that the present value function, $V_{(B)}(G, K)$, satisfies the Lemma. First note that if $I^*_A(G, K) = \arg \max_I W_{(A)}(G, K, I)$ then $\lambda I^*_A(G, K)$ solves $\arg \max_I W_{(B)}(\lambda G, \lambda K, I)$ since $W_{(B)}(\lambda G, \lambda K, I) = \lambda W_{(A)}(G, K, I)$. Next,

$$V_{(B)}(\lambda G, \lambda K) = \max_I W_{(B)}(\lambda G, \lambda K, I)$$

$$= W_{(B)}(\lambda G, \lambda K, \lambda I^*_A(G, K))$$

$$= \lambda W_{(A)}(G, K, I^*(G, K))$$

$$= \lambda V_{(A)}(G, K)$$

Thus $V_{(B)}(\lambda G, \lambda K) = \lambda V_{(A)}(G, K)$. ■

**Lemma 10** Let $\{\epsilon_{it}\}_{t=0}^{\infty}$ be a path of realizations of $\epsilon_t$ and let $g_0$ and $K_0$ be the initial conditions of $g$ and $K$ under process (A) and $g_0 + \lambda$ and $\lambda K_0$ be the initial conditions under process (B). Then, if $\{K_{it}\}_{t=0}^{\infty}$ is the path of capital under process (A) then $\{\lambda K_{it}\}_{t=0}^{\infty}$ is the path under process (B).

**Proof.** This follows from Lemmas 1 and 8, and Lemma 9. As before, let $I^*(G, K)$ be the investment policy under process (A). Now, consider the optimal investment problem under process (B) with capital state $\lambda K$.

From Lemma 9 we know that $\lambda I^*_A(G, K) = I^*_B(\lambda G, \lambda K)$. That is, if $I^*_A(G, K)$ is the solution when $\lambda = 0$ (i.e. process (A)), then $I^*_A(G, K)$ is the solution when $\lambda > 0$ (i.e. process (B)). Hence, under process (B), the path of the capital stock is a level shift of that under process (A). That is, if $\{K_{it}\}_{t=0}^{\infty}$ is the path of capital under process (A) then $\{\lambda K_{it}\}_{t=0}^{\infty}$ is the path under process (B). ■

Together, Lemmas 3, 4 and 10 allow us to compare $\text{std}(s_{it} - x_{it})$ and $\text{std}(x_{it} - x_{it-1})$ under processes (A) and (B). Holding all else constant, if $s_{it}$, $x_{it}$ and $x_{it-1}$ are the realizations under (A), then $s_{it} + \log(\lambda)$, $x_{it} + \log(\lambda)$ and $x_{it-1} + \log(\lambda)$ are the realizations under (B). Since constants will be cancelled out in the computing of differences, the theorem is established. ■
B Data Appendix

We employ multiple datasets in our analysis. We classify these datasets into two tiers, shown in Table 1. Tier 1 consists of country-specific producer-level data from eight countries: the United States, Chile, France, India, Mexico, Romania, Slovenia, and Spain. Each of these data sets has been used extensively in the literature; most commonly in the analysis of productivity. Tier 2 consists of the World Bank Enterprise Survey (WBES). We discuss the details of each dataset below. For a description of the measurement of productivity see Section 3.2.

B.1 United States

The data for the United States comes from the U.S. Census Bureau’s Research Data Center Program. We use data on manufacturing plants from the Census of Manufacturers (henceforth, CMF), and the Annual Survey of Manufacturers (henceforth, ASM) from 1972 to 1997. The CMF sends a questionnaire to all manufacturing plants in the United States with more than 5 employees every five years, while the ASM is a four-year rotating panel with replacement, sent to approximately a third of manufacturing plants, with large plants being over-represented in the sampling scheme.

Labor is measured using the total number of employees at the plant. Materials are measured using total cost of parts and raw materials.

Capital is constructed in two ways. For the majority of plants, including all plants in the CMF, capital is measured using a question on total assets – be they machines or buildings – at the plant. For the remaining observations, capital is constructed using the perpetual inventory method, using industry-specific depreciation rates and investment deflators from the Bureau of Economic Analysis and the National Bureau of Economic Research. Capital, materials and sales are deflated using the NBER-CES industry-level deflators into 1997 dollars.

The original dataset has approximately 3 million plants. However, only 1.8 million of these have sufficient; i.e. – non-zero and non-missing, data on sales, labor, capital and materials, required to construct productivity. Out of these, we keep plant-years for which we have observations in consecutive years, which allow us to measure changes in productivity. There are several industries (measured by the four-digit SIC code), which have a small number of plants. We drop industries which either: a) have less than 50 plants in any given year, or b) with less than 1,000 plants over the entire sample period. The omission of these small-plant-number industries has little effect on our estimates, and they represent a limited number of plants in the data; but dropping these small plant-number industries is essential for the disclosure of our results. The final dataset has 735,342 plants over a 26 year period.

B.2 Chile

Annual plant-level data on all manufacturing plants with at least ten workers were provided by Chile’s Instituto Nacional de Estadistica (INE). These data, which cover the period 1979-1986, include production, employment, investment, intermediate input, and balance-sheet variables. The data were prepared for analysis by INE; standardization of variable definitions across years, identification of entering and exiting plants


\footnote{We use a version of these files that has been processed for productivity analysis by the staff at the Center for Economic Studies at the U.S. Census Bureau, and more information on the construction of this data can be found in the productivity database files at Census.}
and adjustment for inflation distortions, and construction of capital stock variables. Industries are classified according to the four digit ISIC industry code.

Output and input price indices are constructed at the three digit industry and obtained directly from average price indices produced by the Central Bank of Chile. Data on nominal and real values of the various capital goods are reported, including buildings, machinery, furniture, vehicles and others, and allow the construction of price deflators. We directly observe total number of employees, total real value of production, total real intermediate input, total real book-value of fixed assets, total real salaries. In total there are 37,600 plant-year observations reporting employment, with a minimum of 4,205 plants in 1983 and 5,814 plants in 1979.

The data were generously provided by Jim Tybout through a license at the International Economics Section of Princeton University. See Pavcnik (2002) for a productivity study using these data.

B.3 France, Romania and Spain

Annual firm-level data on manufacturing firms for France, Romania and Spain are obtained from Bureau Van Dijck’s (BvD) Amadeus dataset and cover firms reporting to the local tax authorities and/or data collection agencies for the period 1999-2007. We selected three relatively large European countries at different stages of economic development. The coverage for all three countries is substantial in that we cover approximately 90 percent of economic activity in each of the three manufacturing sectors. For example, for France, in 2000, we record total sales of 739 billion Euros, whereas the OECD reports total sales to be 768 billion Euros. This implies coverage of 96 percent of total economic activity in manufacturing. For Spain we find, using the same coverage calculation, coverage of 88 percent. The collection protocol of BvD is consistent across countries. We focus on the manufacturing sector to facilitate the measurement of productivity.

The data include standard production data where we observe Total Operating Revenue (production), Total Number of Employees (employment), Total Material Costs (intermediate input), Total Costs of Employees (wagebill), Total Fixed Assets and all the subcomponents of the capital stock such as Buildings, Furniture, Vehicles, Equipment and Others, as well as other standard income statement and balance-sheet variables. The data also provide information on the firm’s legal status, whether the firm is active and its consolidation code. We use this information to make sure we only include firms actively producing in a specific industry and only use their unconsolidated accounts to for instance avoid including total sales of a multinational across affiliates located in different countries. This data is known to slightly under-represent small firms due to the threshold on either firm size or total number of employees (see Table 1 above).46

Industries are classified according to the two digit NACE Rev 1.1. code for all three countries. Our data covers sectors firms primarily active in sectors NACE Rev 1.1. 15 to 36.

The manufacturing sector in each country leaves us with 391,422, 174,435 and 457,934 firm-year observations for France, Romania and Spain. Two digit NACE rev.1.1. industry producer prices are used to deflate all nominal values and are downloaded from EUROSTAT’s online statistics database.47

Access to Bureau Van Dijck’s Amadeus was obtained through Princeton University’s Library license. For recent work drawing on the AMADÉUS data see Bloom, Draca, and Van Reenen (2011) and the discussion therein.

46 In Table OA.5 in the Online Appendix we verify that our results are invariant to imposing a common threshold across all our datasets.

47 See http://appsso.eurostat.ec.europa.eu/mui/setuModifyTableLayout.do. The data are found under “Industry, trade and services> Short-term business statistics > Producer prices in industry”.

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B.4 India

Annual firm-level data on manufacturing firms were provided by Prowess, and are collected by the Centre for Monitoring the Indian Economy (CMIE). Prowess is a panel that tracks firm performance over time. These data cover the period 1989-2003 and contain mainly medium and large Indian firms.

Industries are classified according to the NIC classification code (India’s industrial classification system) and firms report the principal industry activity at the four digit PNIC level.

These data include various production, employment, investment, intermediate input, and balance-sheet variables. In particular we observe Total Sales, Total Material Costs, Total Fixed Assets and Total Wage-bill. The data reports both product-level sales and total sales. We aggregate product-level sales to the firm level. The Indian data does not report the wage-bill separate from the number of workers. We do, however, take care to appropriately deflate the wage-bill. All nominal values are converted to real values using a two digit producer prices. In total there are 30,709 firm-year observations reporting a wage-bill, and there are 4,154 firms active throughout the sample period.

The data are used in Goldberg, Khandelwal and Pavcnik (2011) and were bought under a license by Goldberg, Khandelwal and Pavcnik. For recent work using the same data in the context of production function estimation see De Loecker et al. (2012), and more details on the data are discussed therein.

B.5 Mexico

Annual plant-level data on manufacturing plants are recorded by Mexico’s Annual Industrial Survey and are provided by Mexico’s Secretary of Commerce and Industrial Development (SEC-OFI). The sample of plants (the 3200 largest manufacturing firms) represents approximately eight percent of total output, where the excluded plants are the smallest ones. For each plant and year we observe the usual data on production, input use, investment, inventories, and costs, as well as industry codes and plant identity codes that allow us to track establishments over time.

Industries are classified according to the Mexican Industrial Classification (a four digit industrial classification system).

These data, which cover the period 1984-1990, include production, employment, investment, intermediate input, and balance-sheet variables. In particular we use Total Value of Output, Total Employment, Total Material Costs and Total Fixed Assets. SECOFI also provided price indices at the industry level for output and intermediate inputs, and sector-wide deflators for machinery and equipment, buildings, and land, which we used to convert all nominal values to real values. In total there are 21,180 plant-year observations reporting employment, with a minimum of 2,958 plants in 1989 and 3,175 plants in 1984.

The data were generously provided by Jim Tybout through a license at IES Princeton University. Tybout and Westbrook (1995) contains more details and contains an application to productivity analysis.

B.6 Slovenia

The data are taken from the Slovenian Central Statistical Office and are the full company accounts of all firms operating in the manufacturing sector between 1994 and 2000. The original accounting data for the period between 1994 and 2002 was provided by AJPES (Agency of the Republic of Slovenia for Public Legal Records and Related Services).

We have information on 7,915 firms: an unbalanced panel with information on production, employment, investment, intermediate input, and balance-sheet variables. In particular we observe: Total Sales, Total Material Costs, Total Fixed Assets, Total
Cost of Employees and Total number of employees. All monetary variables are recorded in Slovenian Tolars and have been deflated using the consumer price index (for data relating to capital stock) and a producer price index (at the 2-digit NACE industry level). In total there are 29,058 firm-year observations reporting employment, with a minimum of 3,355 in 1994 and a maximum of 4,788 firms in 2000. The sharp increase in the number of firms, unlike in datasets with thresholds on firm size, reflects the sharp growth of Slovenia and the manufacturing sector in particular. See for example De Loecker and Konings (2006) for a discussion on the entry of de novo firms during the transition period – which is covered in our sample period.

Industries are classified according to the two digit NACE Rev 1.1 code for all three countries. Our data covers sectors firms primarily active in sectors NACE Rev 1.1 15 to 36.

We would like to thank Joze Damijan at Ljubljiana University for sharing the data. We refer the reader to De Loecker and Konings (2006) and De Loecker (2007) for more on the data, and an application to production function estimation.

B.7 World Bank Data

The World Bank Enterprise Research Data were collected by the World Bank across 41 countries and many different industries between 2002 and 2006. Standard output and input measures are reported in a harmonized fashion. In particular, we observe sales, intermediate inputs, various measures of capital, and employment, during (and covering up to) a three-year period, which allows us to compute changes in TFPR and capital. Out of the 41 countries in the data, 33 have usable firm-level observations. This is primarily because, for many years and countries, the World Bank did not collect multi-year data on capital stock. Table B.1 lists the countries we are able to use, together with the number of observations on each country. The data are available from http://www.enterprisesurveys.org, accessed on December 15th, 2010. Extensive documentation is available from the same website.

The survey documentation describes the sampling universe as follows: “6. The population of industries to be included in the Enterprise Surveys and Indicator Surveys, the Universe of the study, includes the following list (according to ISIC, revision 3.1): all manufacturing sectors (group D), construction (group F), services (groups G and H), transport, storage, and communications (group I), and subsector 72 (from Group K). Also, to limit the surveys to the formal economy the sample frame for each country should include only establishments with five (5) or more employees. Fully government owned establishments are excluded as the Universe is defined as the non-agricultural private sector.” 48

panel-data aspect of these data, relating to activity in year $t - 1$, comes from the recollections and records of managers in year $t$.

To construct data on TFPR and the change in TFPR we need two years of information on sales, assets, intermediate inputs and employment. 5,558 firms across our 33 countries meet this criterion. For some of the countries in the World Bank Enterprise Data, a number of issues emerged in the calculation of TFPR. In particular, labor use is typically reported as the number of employees or a wage bill converted to the number of employees with no correction for hours worked. Moreover, sales and gross output data are not corrected for inventories, and the capital stock is based on book values. These are standard data restrictions researchers face using this type of data.

Sales are directly measured in the data. Hence, for many firm-years in the data, we can compute TFPR directly. However, for some firm-years, we observe only the firm’s wage bill and not the number of workers. To address this issue, we use the median country-industry wage, $\bar{w}$, (imputed from observations with both the wage bill and the number of workers) as a deflator and apply it to the wage bill to give a measure of labor. That is, to compute $L_{it}$ we use $L_{it} = \frac{w_{it}}{\bar{w}}$. In what is presented in this paper, we use this measure for all firm-year observations. Finally, we rely on the book value of capital as measured by either total assets or net book value. We experimented with both measures and our results are invariant. When we consider a measure of value added, we compute it by netting the sales variable from the use of intermediate inputs.

Finally, we convert all relevant variables into real values using detailed producer price and input price deflators where available. For the 33 countries covered in the World Bank data, we rely on the World Bank deflators to convert all monetary variables into USD. To do this, we use the World Bank’s measure of purchasing power parity (PA.NUS.PPP). Note that we account for differences in the rate of inflation across countries by using a year-specific measure of PPP. Since TFPR is a ratio, these PPP conversions get netted out in many specifications, but they are useful when, for instance, we use controls for firm size.

While there are over 41,000 observations in the data, only 5,558 have information on capital over several years, which is needed to compute TFPR volatility. Table B.2 presents summary statistics of the data, where for each variable, the first line refers to the data that we use, while the second presents the data that we dropped due to insufficient information to compute changes in TFPR. The dropped observations are usually smaller firms with lower sales and fewer employees. However, changes in inputs (such as changes in capital or labor) are comparable across the data we did and did not use. Notice that the dispersion of TFPR is similar between the two data sets, with a standard deviation of 1.0 (our data) versus 1.2 (dropped data), as well as the dispersion of the sales to capital ratio which is 1.1 (our data) versus 1.3 (dropped data). Thus, the sampling bias will slightly understate the level of TFPR and MRPK dispersion, but this effect is small relative to the large differences in dispersion across countries.

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49 We also drop countries with fewer than 25 observations. This has little effect on our results.

50 Summary statistics, analogous to Table 2, can be found in Table OA.1 in the Online Appendix.
Table B.1: Countries in the World Bank data sample

<table>
<thead>
<tr>
<th>Region</th>
<th>Country</th>
<th>Std.(MRPK)</th>
<th>Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Africa</td>
<td>Morocco</td>
<td>0.75</td>
<td>376</td>
</tr>
<tr>
<td>Sub-Saharan Africa</td>
<td>Benin</td>
<td>0.81</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>Ethiopia</td>
<td>1.31</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>Madagascar</td>
<td>0.93</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>Malawi</td>
<td>1.03</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>Mauritius</td>
<td>1.49</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>South Africa</td>
<td>1.29</td>
<td>199</td>
</tr>
<tr>
<td></td>
<td>Tanzania</td>
<td>1.65</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>Zambia</td>
<td>0.82</td>
<td>157</td>
</tr>
<tr>
<td>Central Asia</td>
<td>Kyrgyzstan</td>
<td>0.53</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>Tajikistan</td>
<td>0.87</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>Uzbekistan</td>
<td>0.89</td>
<td>92</td>
</tr>
<tr>
<td>Middle East</td>
<td>Syria</td>
<td>1.13</td>
<td>55</td>
</tr>
<tr>
<td>South Asia</td>
<td>Bangladesh</td>
<td>1.28</td>
<td>134</td>
</tr>
<tr>
<td></td>
<td>Sri Lanka</td>
<td>0.96</td>
<td>114</td>
</tr>
<tr>
<td>South East Asia</td>
<td>Indonesia</td>
<td>1.53</td>
<td>426</td>
</tr>
<tr>
<td></td>
<td>Philippines</td>
<td>1.06</td>
<td>278</td>
</tr>
<tr>
<td></td>
<td>Thailand</td>
<td>0.75</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>Vietnam</td>
<td>0.95</td>
<td>448</td>
</tr>
<tr>
<td>Central America</td>
<td>Costa Rica</td>
<td>1.22</td>
<td>273</td>
</tr>
<tr>
<td></td>
<td>Ecuador</td>
<td>1.51</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>El Salvador</td>
<td>0.95</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td>Guatemala</td>
<td>0.95</td>
<td>162</td>
</tr>
<tr>
<td></td>
<td>Honduras</td>
<td>1.10</td>
<td>203</td>
</tr>
<tr>
<td></td>
<td>Nicaragua</td>
<td>1.14</td>
<td>222</td>
</tr>
<tr>
<td>South America</td>
<td>Brazil</td>
<td>1.00</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>Chile</td>
<td>1.40</td>
<td>745</td>
</tr>
<tr>
<td></td>
<td>Guyana</td>
<td>2.37</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>Peru</td>
<td>0.85</td>
<td>31</td>
</tr>
<tr>
<td>Europe</td>
<td>Moldova</td>
<td>0.94</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>Lithuania</td>
<td>1.37</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>Poland</td>
<td>0.58</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>Turkey</td>
<td>1.87</td>
<td>36</td>
</tr>
</tbody>
</table>
Table B.2: Selection Bias due to Missing Data in World Bank Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Sales</td>
<td>7.0</td>
<td>3.1</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>6.7</td>
<td>3.3</td>
<td>51043</td>
</tr>
<tr>
<td>Log Value Added</td>
<td>6.0</td>
<td>3.1</td>
<td>4719</td>
</tr>
<tr>
<td></td>
<td>5.9</td>
<td>3.3</td>
<td>42230</td>
</tr>
<tr>
<td>Log Materials</td>
<td>6.4</td>
<td>3.3</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>5.2</td>
<td>3.5</td>
<td>46642</td>
</tr>
<tr>
<td>Log Capital</td>
<td>6.9</td>
<td>3.1</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>7.5</td>
<td>3.0</td>
<td>12728</td>
</tr>
<tr>
<td>Log Labor</td>
<td>5.2</td>
<td>2.9</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>3.1</td>
<td>23696</td>
</tr>
<tr>
<td>Workers</td>
<td>284</td>
<td>874</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>1010</td>
<td>50891</td>
</tr>
<tr>
<td>Productivity</td>
<td>2.3</td>
<td>1.0</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>2.4</td>
<td>1.2</td>
<td>4750</td>
</tr>
<tr>
<td>Sales to Capital Ratio</td>
<td>0.1</td>
<td>1.1</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>1.3</td>
<td>12528</td>
</tr>
<tr>
<td>Sales to Labor Ratio</td>
<td>2.9</td>
<td>2.2</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>3.1</td>
<td>3.2</td>
<td>37918</td>
</tr>
<tr>
<td>Change in Capital</td>
<td>0.1</td>
<td>0.5</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.5</td>
<td>11268</td>
</tr>
<tr>
<td>Change in Labor</td>
<td>0.2</td>
<td>0.7</td>
<td>4626</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.6</td>
<td>14360</td>
</tr>
<tr>
<td>Change in the Sales to Capital Ratio</td>
<td>0.0</td>
<td>0.7</td>
<td>5579</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.7</td>
<td>11017</td>
</tr>
</tbody>
</table>

Note: The first row shows the data used in the paper, and the second row indicates data that we dropped due to some missing observation.
C Model Computation

The parameters we use are found in Table C.1. Parameters for the elasticity of demand, depreciation rate, and discount rate follow those adopted by Bloom (2009). The last set of parameters we need to fix are the \( \sigma \), \( \rho \) and \( \mu \) terms in the AR(1) process, which governs the evolution of productivity over time. We compute the model for values of \( \sigma \) between 0.1 and 1.4, which covers the range we observe in data. For \( \rho \) we pick three values, 0.65, 0.85 and 0.94. Lastly, we set \( \mu = 0 \). We also implicitly normalize the prices of non-capital inputs by setting \( \lambda = 1 \). More precisely, what we are normalizing is \( \lambda \), a function of these non-capital input prices. The functional form of \( \lambda \) puts structure on the relative prices of non-capital inputs. Subject to this structure, normalizing \( \lambda \) is equivalent to a normalization of one of the non-capital input prices.

Table C.1: Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon = -4 )</td>
<td>} Values also used in Bloom (2009).</td>
</tr>
<tr>
<td>( \delta = 10% )</td>
<td>}</td>
</tr>
<tr>
<td>( \beta = \frac{1}{1+6.5%} )</td>
<td>}</td>
</tr>
<tr>
<td>( \beta_K = 0.12 )</td>
<td>} Mean values in U.S. Census Data.</td>
</tr>
<tr>
<td>( \beta_M = 0.40 )</td>
<td>}</td>
</tr>
<tr>
<td>( \beta_L = 0.23 )</td>
<td>}</td>
</tr>
<tr>
<td>( C_F^0 = 0.09 )</td>
<td>} Estimated using U.S. Census Data, see Section 5.1.</td>
</tr>
<tr>
<td>( C_Q^0 = 8.8 )</td>
<td>}</td>
</tr>
<tr>
<td>( \rho \in {0.65, 0.85, 0.94} )</td>
<td>} Selected to fall within range of estimated values for the U.S. Census.</td>
</tr>
<tr>
<td>( \sigma \in [0.1, 1.4] )</td>
<td>}</td>
</tr>
<tr>
<td>( \lambda = 1 )</td>
<td>Scaling parameter that normalizes the price of non-capital inputs.</td>
</tr>
<tr>
<td>( \mu = 0 )</td>
<td>Normalization that has no effect on computed moments, by Theorem 1.</td>
</tr>
</tbody>
</table>

We compute the optimal investment policies for the value function in equation (9). We solve this model using a discretized version of the state space \( (\Omega_{it}, K_{it}) \). Specifically, we use a grid of capital states ranging from log capital 3 to log capital equal to 20, in increments of 0.03. Moreover, we use a grid of productivity with 30 grid points, whose transition matrix and grid points are computed using Tauchen (1986)'s method. The model is solved using policy iteration with a sparse transition matrix (since there are 17,000 states). Using the computed optimal policies, we simulate the evolution of a country, or industry, for 10,000 firms over 1,000 periods. We use the output from the 1,000th and 988th periods to compute the reported results (corresponding to years \( t \) and \( t - 1 \); recall that we interpret a period as a month).