

# Online Appendix: A Study of the Internal Organisation of a Bidding Cartel

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## **Abstract**

This appendix explores in more detail issues only dealt with briefly in the paper *A Study of the Internal Organisation of a Bidding Cartel*. It examines the claim that a multiplicatively separable structure for bidders' valuations is more appropriate given the data. It then examines the implications of restricting attention to two-bidder knockouts and the spectre of sample selection that is so raised.

Keywords: Auction, Bidding, Ring, Cartel, Damages

JEL Codes: D44, K21,L41,L12

## 1 Multiplicative Separability

The valuation of bidder  $i$  in knockout auction  $k$ ,  $u_{ik}$ , is modelled as

$$u_{ik} = \Gamma(x_k)(v_{ik}\varepsilon_k) \quad \text{where } \Gamma(x_k) = e^{x_k\gamma}$$

where  $x_k$  collects variables on which auctions are observed to differ;  $\varepsilon_k$  is auction-level heterogeneity observed by bidders but not the econometrician; and  $v_{ik}$  is the private value of the bidder. Assuming this multiplicatively separable form allows both the mean and variance of the value distribution to vary with observed and unobserved auction characteristics, albeit in a tightly parametrized fashion. This is attractive, as it reflects patterns observed in the data. Here I explain the sense in which this seems to fit the data nicely.

An alternative specification would be to make  $u_{ik}$  additively separable. That is,

$$u_{ik} = \Gamma(x_k) + v_{ik} + \varepsilon_k$$

with some appropriate choice of the function  $\Gamma(\cdot)$ . It is straightforward to show that an analog of lemma 3 applies for this additive case.

An immediate implication of lemma 3 (and its analog for the additive case) is that, in the multiplicatively separable case, as the mean bid for an object increases so should the standard deviation of bids. In the additively separable case, the standard deviation of bids should stay constant as the mean bid increases. Figures OA1 and OA2 show the within-knockout mean bid plotted against the within-knockout standard deviation, for 2- and 3-bidder knockouts respectively. As can be seen, the standard deviation does vary with the mean, as predicted by the multiplicative model.

Figures OA1 and OA2 Here

Figure OA2 is particularly relevant to the paper's focus on two-bidders knockouts. It is not possible to prove lemma 3 for the three bidder case. However, an additive version can be established. Despite this, the implications of an additive specification are clearly inappropriate given the data shown in OA2.<sup>1</sup>

## 2 Selection Issues

The main paper focuses attention on the auctions associated with two-bidder knockouts, for the reasons outlined above. An assumption made in doing so is that ring bidders know that they are

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<sup>1</sup>An additive model was estimated on the three-bidder data anyway and found to yield results that could only be interpreted as the result of severe model misspecification, as suggested by the data represented in Figure OA2.

only faced with one other ring bidder. It is plausible that for some of these two bidder knockouts there is some uncertainty as to the number of ring bidders who will actually bid. Where such uncertainty exists ring bidders will place a bid that is a weighted average of the optimal bids in the two-, three- (and so on) knockouts that they think might be possible.

To the extent that some two-bidder knockouts have this uncertainty associated with them, this raises the concern that the sample is selected in a way that may give rise to misleading inference due to model mis-specification. This section examines this concern. It does this by estimating a propensity score (for the event that the auction has two bidders) for each auction based on auction observables. It then selects out the two-bidder knockouts with a propensity score that places them in the top 50% of two-bidder knockouts. Then the model in the main paper is re-estimated on this sample.

With the full model (accounting for unobserved heterogeneity), damage estimates using the selected sample tend to be higher than those estimated using the full sample. However, each point estimate, derived from either data set, lies within the corresponding 90% confidence interval derived from the other data set. In the same vein, the results have the same qualitative features. Hence, while this selection issue may be a valid concern it appears not to have a significant effect on the economic findings.

The rest of this section provides additional detail on the estimation of the selected sample and its treatment in the estimation procedure outlined in the main paper. Then, brief comments are provided on the results.

## 2.1 Estimation Details

The first step is to select those two-bidder knockouts that are more likely to have two-bidders, based on observables. This is done by taking the set of all auctions, assigning a dummy variable equal to one to those that have two bidders and using a logistic regression to project this dummy onto auction observables. The output of this logistic regression is shown in Table OA1. The coefficients that are estimated are used to construct the propensity score of each auction (i.e.  $p = f(x\beta)$ ). The comparison between the two-bidder knockouts and all other knockouts are shown in Table OA2 and Figure OA3. Figure OA3 reveals that the the propensity scores of two bidder knockouts have a density which lies to the right of the density of the propensity scores of other knockouts. Further, it lacks the sharp peaks in the left end of the distribution that are present in the density of non-two-bidder knockouts.

Tables OA1 and 2 Here and Figure OA3 Here

These propensity scores are used to select a subsample of two-bidder auctions that have less uncertainty surrounding the number of ring members active in them. This is done by selecting those knockouts with a propensity score greater than the median. This yields 182 knockouts to analyze (as compared to 366 in the full sample).<sup>2</sup>

Having obtained this selected sample estimation proceeds using exactly the same procedure as described in the main paper. The only difference is that the smoothing parameters need to be re-selected. Thus  $T_n$  for  $\ln(\varepsilon)$ ,  $\ln\beta(v_s)$ ,  $\ln\beta(v_w)$  and  $\ln(r)$  are 20, 6.3, 2.0 and 7.3 respectively. The lower values for the smoothing parameters reflect the smaller sample size requiring greater smoothing.

## 2.2 Results

Tables OA3-6 mirror Tables 7-10 in the main paper. In this selected sample average naive damages in the data are equal to \$95.7 (conditional on the ring winning) and the proportion of target auctions won by the ring equal 37.6%. Thus, in the raw data the naive damage estimate has risen considerably from the \$67 in the full sample. Despite this, as noted above, the qualitative nature of the results are the same and the magnitudes indicated by point estimates from either sample are not sufficiently different so as to lie outside estimated confidence intervals from the other sample.

Tables OA3 through 6 Here

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<sup>2</sup>The extreme outlier bid in the two bidder knockouts is dropped after the selection is done (this observation is also dropped in the full sample). This is why there is 182 not 183 auctions in the selected sample.

Figure OA1: Within-Knockout Mean vs Standard Deviation of Bids:  
Raw data, Number of bidders = 2

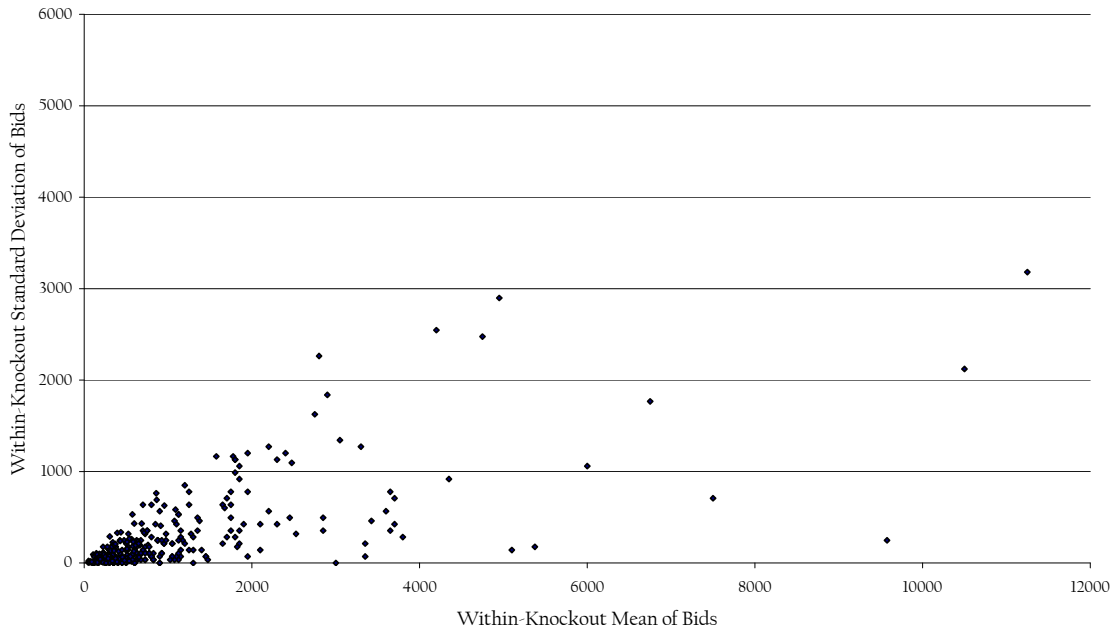
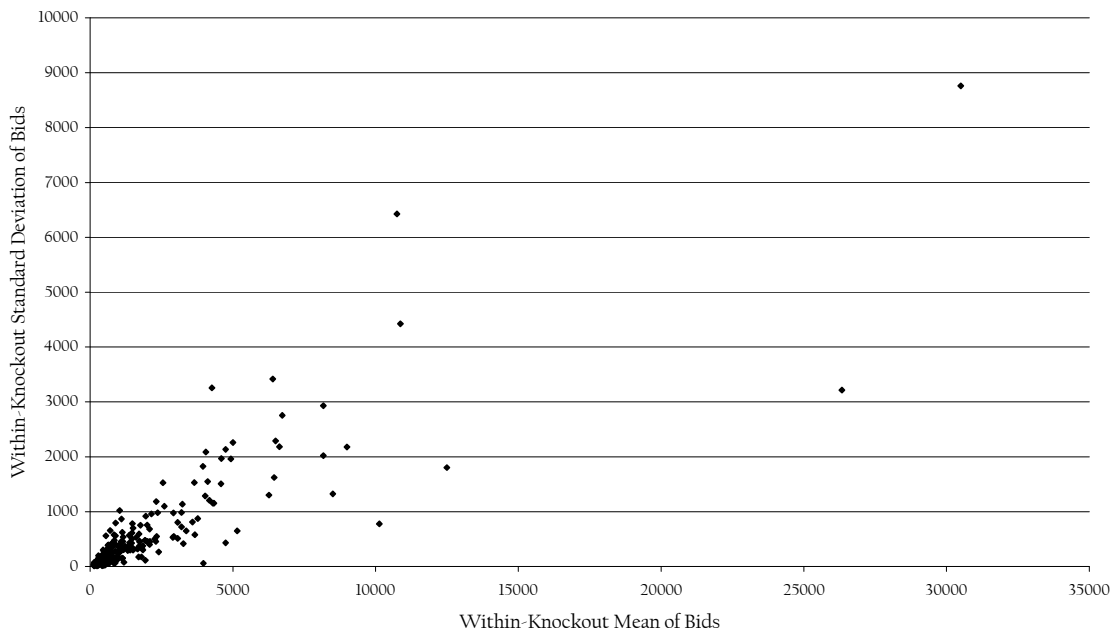


Figure OA2: Within-Knockout Mean vs Standard Deviation of Bids:  
Raw data, Number of bidders = 3



**Table OAI: Determinants of the 2-bidder knockout auctions**

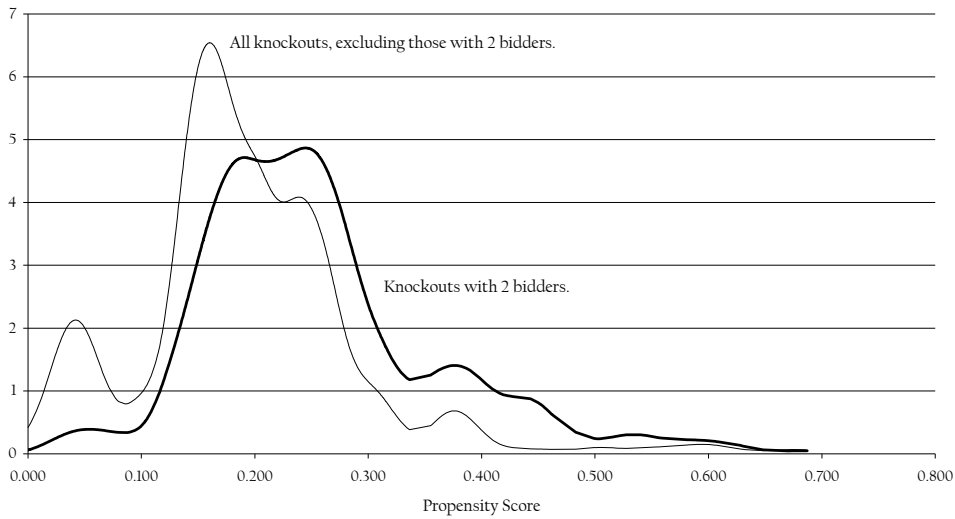
Specification	Point Estimates	
	Coeffs.	Std Err.
Constant	0.077	(0.638)
$(E_{min} + E_{max})/2$	-0.305	(0.195)
$[(E_{min} + E_{max})/2]^2$	-0.008	(0.020)
$(E_{max} - E_{min})$	0.655	(0.617)
$[(E_{max} - E_{min})]^2$	0.135	(0.162)
Catalog Price	0.025	(0.037)
Catalog Price <sup>2</sup>	-0.0004	(0.001)
$(\text{Grade Min} + \text{Grade Max})/2$	-0.824**	(0.364)
$[(\text{Grade Min} + \text{Grade Max})/2]^2$	0.141***	(0.048)
$(\text{Grade Min} - \text{Grade Max})$	0.464**	(0.201)
$[(\text{Grade Min} - \text{Grade Max})]^2$	-0.118**	(0.051)
No Grade	1.501***	(0.234)
Exclusively US	-0.591	(0.613)
No Value	-0.617	(1.063)
House HRH	-0.697**	(0.307)
House DK	-0.911**	(0.430)
House IM	-0.334	(0.343)
House MB	-2.377***	(0.428)
House RS	-0.283	(0.306)
House S	0.063	(0.361)
House SA	-0.541	(0.401)
Pseudo R-squared	0.064	
Observations	1781	

Notes: The dependent variable is a dummy equal to 1 if there were two bidders in the knockout. Estimation is conducted using Logistic Regression. The omitted auction house is Christies. Estimated Minimum ( $E_{min}$ ), Estimated Maximum ( $E_{max}$ ) and Catalog Price are all divided by 1000  
\*, \*\* and \*\*\* indicate significance at the 10%, 5% and 1% levels respectively.

**Table OA2: Properties of estimated propensity scores**

Propensity Score p	% of Knockouts with Propensity Score greater than or equal to p		Conditional on propensity score exceeding p, proportion of all knockouts that have exactly 2 bidders	Number of 2 bidder knockouts with propensity score greater than or equal to p
	2-Bidder	All Others		
0.100	0.973	0.868	0.225	357
0.120	0.965	0.842	0.229	354
0.140	0.946	0.802	0.234	347
0.160	0.856	0.614	0.266	314
0.180	0.798	0.529	0.281	293
0.200	0.668	0.404	0.300	245
0.220	0.569	0.324	0.313	209
0.240	0.520	0.262	0.340	191
0.260	0.365	0.153	0.382	134
0.280	0.297	0.115	0.402	109
0.300	0.243	0.091	0.410	89
0.320	0.199	0.067	0.435	73
0.340	0.183	0.061	0.438	67
0.360	0.172	0.059	0.432	63
0.380	0.128	0.040	0.452	47
0.400	0.101	0.028	0.481	37
0.420	0.098	0.025	0.507	36
0.440	0.068	0.024	0.424	25
0.460	0.046	0.022	0.354	17
0.480	0.038	0.021	0.318	14
0.500	0.038	0.020	0.333	14
0.520	0.030	0.017	0.314	11
0.540	0.025	0.016	0.281	9
0.560	0.016	0.012	0.261	6
0.580	0.014	0.012	0.227	5
0.600	0.014	0.008	0.313	5

**Figure OA3: Densities of Propensity Scores by Number of Bidders in Knockout Auction**



**Table OA3: Damages to the seller**

Model:		With unobserved auction heterogeneity			No unobserved auction heterogeneity		
	Assumption	Point estimate	90% Confidence interval:		Point estimate	90% Confidence interval:	
			Lower bound	Upper bound		Lower bound	Upper bound
Mean naïve damages (\$)		108.50	52.82	164.79	138.77	73.23	197.15
Mean damages (\$)	U. B.	50.43	29.87	94.81	90.45	35.01	132.79
	L. B.	38.57	23.17	86.87	76.54	17.85	124.03
Mean damage ratio	U. B.	0.94	0.90	0.97	0.91	0.88	0.96
	L. B.	0.96	0.91	0.98	0.94	0.89	1.00
Proportion of auctions with $Pr > Pc$	U. B.	0.00	0.00	0.00	0.00	0.00	0.00
	L. B.	0.17	0.04	0.20	0.11	0.05	0.21
Mean damage ratio ( $Pr > Pc$ )	L. B.	1.08	1.02	1.14	1.22	1.05	1.34
Proportion of auctions with $Pr < Pc$	U. B.	0.32	0.19	0.41	0.23	0.17	0.37
	L. B.	0.32	0.19	0.41	0.23	0.17	0.37
Mean damage ratio ( $Pr < Pc$ )	U. B.	0.82	0.72	0.87	0.64	0.62	0.78
	L. B.	0.82	0.72	0.87	0.64	0.62	0.78
Proportion of auctions with $Pr = Pc$	U. B.	0.68	0.59	0.81	0.77	0.63	0.83
	L. B.	0.51	0.45	0.72	0.66	0.51	0.70
Proportion of target auctions won		0.39	0.09	0.53	0.51	0.26	0.58
Simulated auctions		100000			100000		

Notes: Damage ratio is the ratio of the price received with the ring to the price received with competitive bidding. All means are over target auctions that the ring won (unless further conditioned as noted). L. B. = Lower Bound, U. B. = Upper Bound. Pr refers to the price sellers receive with the ring, Pc is the price with competitive bidding. Confidence intervals are bootstrapped with 5,000 iterations.

**Table OA4: Damages to the non-ring bidders**

Model:		With unobserved auction heterogeneity			No unobserved auction heterogeneity		
		Point estimate	90% Confidence interval:		Point estimate	90% Confidence interval:	
			Lower bound	Upper bound		Lower bound	Upper bound
Damages due to misallocation:							
	Proportion of target auctions ring won	0.39	0.09	0.53	0.51	0.26	0.58
	Proportion of target auctions ring won with damages	0.17	0.04	0.20	0.11	0.05	0.21
	Mean damages (conditional on ring winning target auction, \$)	11.87	1.10	15.15	13.91	3.00	29.59
Damages due to price inflation:							
	Mean damages (conditional on ring not winning target auction, \$)	124.23	66.23	147.90	213.21	104.03	260.86
	# Simulated auctions	10000			100000		

Notes: All estimates obtained using the lower bound assumption. Confidence intervals are bootstrapped with 5,000 iterations.



**Table OA5: Impact on market efficiency**

Model:	With unobserved auction heterogeneity			No unobserved auction heterogeneity		
	Point estimate	90% Confidence interval:		Point estimate	90% Confidence interval:	
		Lower bound	Upper bound		Lower bound	Upper bound
Mean efficiency loss (\$)	12.03	0.34	15.61	13.91	3.00	29.59
Mean proportional efficiency losses:						
Ring active	0.005	0.00003	0.006	0.009	0.0012	0.017
No ring bidders	0.11	0.02	0.14	0.19	0.08	0.24
Only ring bidders	0.27	0.17	0.41	0.27	0.21	0.45
Proportion of target auctions won	0.39	0.09	0.53	0.51	0.26	0.58
# Simulated auctions	100000			100000		

Notes: Means are conditional on the ring winning. The mean proportional efficiency losses are averages over all auctions, not just those won by the ring. Confidence intervals are bootstrapped with 5,000 iterations.

**Table OA6: Returns to the ring**

Model:	With unobserved auction heterogeneity			No unobserved auction heterogeneity		
	Point estimate	90% Confidence interval:		Point estimate	90% Confidence interval:	
		Lower bound	Upper bound		Lower bound	Upper bound
Mean naïve return (equiv. damages, \$)	108.50	52.82	164.79	138.77	73.23	197.15
Proportion of ring wins that harmed ring	0.17	0.04	0.20	0.11	0.05	0.21
Mean return to ring (harm, \$)	-11.87	-15.54	-1.13	-13.91	-29.61	-3.01
Mean return to ring (benefit, \$)	50.27	29.54	95.41	90.45	34.98	132.79
Mean return to ring (net, \$)	38.40	22.81	87.71	76.54	17.80	123.98
Mean proportional price discount	0.94	0.90	0.97	0.91	0.88	0.96
# Simulated auctions	100000			100000		

Notes: All means are over target auctions that the ring won. Confidence intervals are bootstrapped with 5,000 iterations.