Lectures on Auction Empirics, Collusion and Bidding Rings, Part 1: Modern Structural Auction Empirics

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1 Preliminaries: Auctions

This handout is intended to give you a map to what will be covered in the 3 hrs or so I will be talking about modern structural auction empirics. Basically, what I want to do is to give you an introduction to the core technical tricks currently used to analyze auctions and (perhaps more importantly) give you the tools to access current research in the area.

Likely, there is more material here than we will be able to go through so I might be selective.

Here are some useful sources/core things to be familiar with:

- Guerre, Perrigne and Vong (2000), Optimal Nonparametric Estimation of First Price Auctions, E'metrica, 68, 525

If you were to read anything following class, pp.525-532 of this article would be a good start.

- Li and Vong (1998), Nonparametric Estimation of the Measurement Error Model Using Multiple Indicators, Journal of Multivariate Analysis 65, 139-165

Probably not a journal you have spent a lot of time reading, but these techniques are key to many of the identification results in auctions. If you were to read another thing following class, pp.139-145 of this article would be a good.

- Asker (2008), A Study of the Internal Organization of a Bidding Cartel, American Economic Review 2010.

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The reason I will talk about this is because it is a good example of a pure application of the technique. Much of that literature is at least partially motivated by a desire to explore econometric methods, and it is helpful to be reminded that this doesn't need to be the case to right a paper people will pay attention to. It also speaks directly to topic 2, which is collusion.

- Haile, Phil and Elie Tamer (2003), Inference with an Incomplete Model of English Auctions,JPE, 111, 1-52

This paper is very different from the above three in terms of setting and technique. It uses a partially identified model to sidestep a bunch of thorny issues in the estimation of English auctions. It is also a beautiful piece of research.

- Athey, Susan and Phillip Haile (2005a), Non-Parametric Approaches to Auctions at http://www.econ.yale.edu/˜pah29/hbk.pdf

  This is a handbook chapter now I think in the Handbook of Econometrics V6A. Anyway, this is a great survey to look at to see how stuff is done and also the bibligraphy is super useful.

- Pagan and Ullah, *Nonparametric Econometrics.* This is the most useful econometrics book to give you a foundation in density estimation etc. Harry Paarsch also has a very useful text called *An Introduction to the Structural Econometrics of Auction Data* which has a particularly useful appendix that goes through heaps of the toolbox technique with a useful focus on auction problems. The last econometric reference worth mentioning in an introduction is Joel Horowitz's Bootstrap chapter in the handbook of econometrics v5.

2 Auction Empirics: Preliminary Comments

2.1 What is the point of empirical work on auctions?

The next three or so hours will be on recent empirical work on auctions. There are several reasons for teaching this vibrant area of research:

- it introduces the ideas behind identification in structural models in a formal yet digestible way

- asymmetric information is one of the most fertile areas for empirical work: auctions present an easily understandable environment for examining the impact of assym. info issue (ie we can see the rules of the game, understand the strategy set etc)

- my suspicion is that the tools developed in the auction literature could be adapted to other environments (e.g. flick through Laffont and Tirole for some examples of theory models in this spirit - whether they have direct empiricial application is another question)

- auctions can be thought of as a kind of monopoly model (the auctioneer has monopoly power, but less than perfect information and limited capacity, for a similar view see Bulow and Klemperer)

- something like 10% of GDP is transacted through auction markets and their design and application has been, and continues to be, a topic of ongoing interest to both policy makers and business.

Basically auctions are more central to IO than most economists think.

The pattern of these lectures will go something like

1. what is the point of empirical work on auctions? + themes

2. review of the theory

3. basic empirical results in first price auctions

4. basic empirical results in ascending auctions

5. extensions: auction heterogeneity & bidder heterogeneity

There are several reasons for empirical work on auctions

- Validating basic assumptions: the theory makes a big deal of the role of asymmetric information. To be confident that our models mean anything we should send some time making sure that asymmetric information does, indeed, matter. This is how I think of much of the contributions made by Hendricks and Porter

- Testing theory: Theory makes some pretty specific predictions about how model primitives map to outcomes. These seem worth testing. Experimental work has been successful here. Read the Handbook of Experimental Economics Chapter for an introduction to this fruitful area of experimental work.

- Evaluating policy: the optimality of design decisions usually depends on the properties on the underlying primitives we can divide these into two areas:
  - Uncovering the specific distribution of private information: in a FPSB IPV single unit auction the reserve price depends on the specific distribution of private information
  - Uncovering the properties of the structure of private information: in a single unit auction, the attractiveness of the FPSB auction depends on whether we are in the IPV or common value environment or some other private info structure.

The recent work has been on the last dot point mostly. I will talk mostly about the work that seeks to evaluate policy by uncovering the underlying distribution of private information. Testing private values vs common values tends to build on this anyway, often using auxiliary data.

2.2 What are we really talking about?

The problem is: given bid data and observable characteristics of the auction and bidders, what can we say about the private information possessed by the bidders when they make bids?

Let's start by defining an auction:

The standard simple auction model has

- N bidders
- one indivisible good for sale
- each bidder has some private information represented by a parameter $\theta \in \mathbb{R}$. This sets either values or costs depending on whether we are in a procurement setting or not.
- the auctioneer has a utility function which is given by

$$V = v - p \qquad \text{or} \qquad V = p - v$$

  again depending on whether we are in a procurement setting or not (procurement is first)

- the bidders have a utility function which looks like

$$U_n = p - \theta \qquad \text{or} \qquad U_n = \theta - p$$

  It should be obvious that having bidders as buyers or sellers is just a sign change. Since each are equally relevant empirically, I will use whichever is most appropriate for the application.

- we need a joint distribution for the bidders' private information so let $\{\theta_1 ... \theta_N\}$ be drawn from the joint CDF $F \in \mathbf{F}\cdot$

- the auction also needs some rules for allocating the winner and payments and saying what permitted bids are. Let $\mathbf{G}$ denote the set of all distributions over the space of permitted bids.

You should have covered the equilibrium theory of auctions in Micro last year. If you are hazy on that, look through Krishna's excellent text book which I recommend buying as a reference. I will cover it very fast today.

The equilibrium theory gives us a way to map from the private information to bids (which may or may not be the same as the prices (transfers) arising from the auction). Lets call this mapping from the theory $\gamma \in \Gamma$ where $\gamma : \mathbf{F} \rightarrow \mathbf{G}$.

2.2.1 Identification

The key idea of these lectures is identification. In most contexts what this means is that given an observed distribution of bids we can say identify a distribution of private information that is "most likely" or has the "best fit" or slightly more formally minimizes the loss function of your choice.

I find it useful in structural modeling to draw the distinction between model identification and identification in data

Lets be formal about model identification:

Definition 1 (Identification)**.** A model $(\mathbf{F}, \Gamma)$ is identified if for every $\left(F, \widetilde{F}\right) \in \mathbf{F}^2$ and $(\gamma, \widetilde{\gamma}) \in \mathbf{\Gamma}^2$, $\gamma\left(F\right) = \widetilde{\gamma}\left(\widetilde{F}\right)$ implies $(F, \gamma) = \left(\widetilde{F}, \widetilde{\gamma}\right)$

This gives us some hope that with real data we can invert the mapping provided by the bid function to get the the private information and thus estimate F. Often we will take a stand on γ which will make the identification of F much more tractable.

Lastly, bear in mind the difference between this view of identification, which is, in a sense, asymptotic, and the practical issue of identification in small samples which is the problem you always face when confronted with data (identification in data).

It is always possible that a model is identified in theory but places demands that the data at hand cannot meet. Always think about this when considering your own research and that of others. This often comes down to judgement but there is some science that can help: things you can do to develop some intuition about your situation

- run monte carlo simulations

- plot the data to examine the available variation in exogenous variables

- look at what other researchers were able to do with similar technology and data

3 Review of Theory

Notation

- random variables in upper case, realisations in lower case
- vectors in bold
- latent variable CDF is $F_Y(\cdot)$
- observed variable CDF is $G_Y(\cdot)$
- where order statistics are used let:
  - $Y^{k:n}$ be the k^{th} order statistic from the sample $\{Y_{1,\ldots,}Y_n\}$ with $Y_1 < Y_n$
  - $F_Y^{k:n}(\cdot)$ is the CDF of this order statistics

Basics

- we consider the sale of a single indivisable good to one of n bidders where $n \geq 2$
- bidders are risk neutral
- $\mathcal{N}$ is the set of bidders (where bidders are ex ante symmetric this is not so as important as $n = |\mathcal{N}|$ will be a sufficient statistic for our purposes)
- $\mathcal{N}_{-i}$ are the opponents of bidder i

Private Information

bidders' private information (or type) is a scalar random variable X_i, $\mathbf{X} = \{X_1, ..., X_n\}$

- we have a scalar signal X_i with realisation x_i
- we assume signals are informative in that

$$\frac{\partial E\left[U_i \left| X_i = x_i, X_{-i} = x_{-i}\right.\right]}{\partial x_i} > 0 \qquad \forall x_{-i}$$

  so no matter what the signal of the other guys are, my signal always has an impact on the expectation of my utility. Among other things this rules out any one player having a perfect signal in, say, a mineral rights model.

- We often use U_i and X_i interchangeably as one is just a monotonic transformation of the other. (note that in some procurement settings this will not make sense, but everything I talk about here works this way)

- Lastly, $\mathcal{N}$, $F_Y(\cdot)$ are common knowledge

Auction Terminology

The auction literature has been around for a long time and a terminology has grown up around it that you should know. So lets run through it.

Definition 2 (Private Values)**.** Bidders have private values if

$$E[U_i | X_i = x_i, X_{-i} = x_{-i}] = E[U_i | X_i = x_i] \qquad \forall x_j \text{ and } U_i$$

Definition 3 (Common Values)**.** Bidders have common values if

$$E[U_i | X_i = x_i, X_{-i} = x_{-i}]$$

is strictly increasing in x_j for all i, j and x_j

- This division is not exhaustive, but is the space in which most people work. Note in particular that it does not include the work on auctions with externalities by Phillippe Jehiel, Benny Moldovanu and Ennio Stacchetti

- Common values can have the winners curse. The precise meaning of this differs depending on the literature you are reading. In most economics it means the possibility that people have some ex post regret from winning the auction. However, it can also refer to the behavioral phenomenon of people bidding too high relative to the equilibrium and making a loss in expectation.

- A further breakdown is between the independence of signals and affiliation of signals

- Independance means that
$$f_{X_i,X_j} = f_{X_i} f_{X_j}$$
where f_Z is the marginal distribution of Z

- Affiliation is defined as follows

Definition 4 (Affiliation)**.** A set of random variables $\mathbf{Y} = \{Y_1, ..., Y_n\}$ is said to affiliated if for all $\mathbf{y}$ and $\widehat{\mathbf{y}}$ (which are realisations of $\mathbf{Y}$)

$$f_{\mathbf{Y}}\left(\mathbf{y} \vee \widehat{\mathbf{y}}\right) f_{\mathbf{Y}}\left(\mathbf{y} \wedge \widehat{\mathbf{y}}\right) \geq f_{\mathbf{Y}}\left(\mathbf{y}\right) f_{\mathbf{Y}}\left(\widehat{\mathbf{y}}\right)$$

where $\vee$ denotes the component-wise maximum and $\wedge$ denotes the component-wise minimum. (See Milgrom and Weber 1982 for more info)

Affiliation means, very loosely, that the higher is my signal, the the more likely it is that yours is high.

Example:

Let

$$\begin{aligned} \mathbf{Y} &= \left\{ \begin{array}{cc} 1 & 1 \\ 1 & 2 \\ 2 & 1 \\ 2 & 2 \end{array} \right\} \\ f_{\mathbf{Y}} &= \left\{ \begin{array}{c} \frac{1}{4} + \varepsilon \\ \frac{1}{4} - \varepsilon \\ \frac{1}{4} - \varepsilon \\ \frac{1}{4} + \varepsilon \end{array} \right\} \end{aligned}$$

$\mathbf{Y}$ is affiliated if $\varepsilon \geq 0$. You may notice that affiliation is equivalent to imposing a lattice-like structure on the structure of private information.

- So auctions that we often talk about are:
  - IPV: PV with U_i being independant
  - Symmetric IPV: $U_i \sim iid$
  - Affiliated PV (APV)
  - Pure Common Values: $U_i = U_o$
  - Mineral Rights: pure common rights with signals iid conditional on U_o

3.0.2 Applications

Think about how these models apply to the following applications:

1. OCS Oil Drilling Rights Auctions

2. Timber Auctions

3. Treasury Bill Auctions

4. Highway construction contract Auctions

5. Auctions of bus routes

4 Empirical Example: Reduced form work prior to modern structural approach

Let's look at Henricks and Porter's classic paper 'An Empirical Study of an Auction with Assymetric Information' in the AER 1988. It is a mineral rights model.

- Setting: Drainage leases in OCS 1959-69 - leases next to tract in which a deposit has been discovered.
- Symmetry/assymtery of information is important for qualitative predictions in CV auctions. In common value auction, want to consider the precision of signals in thinking about assymtery. Drainage vs Wildcat: drainage is adjacent to known deposit, wildcat is not
- Research Question: Does the bidding behavior look consistent with a CV model that reflects institutions? is there evidence of bidding coordination?

TABLE 1—SELECTED STATISTICS ON WILDCAT AND DRAINAGE TRACTS[a]

	Wildcat	Drainage
Number of Tracts	1056	144
Number of Tracts Drilled	748	124
Number of Productive Tracts	385	86
Average Winning Bid	2.67	5.76
	(0.18)	(1.07)
Average Net Profits	1.22	4.63
	(0.50)	(1.59)
Average Tract Value	5.27	13.51
	(0.64)	(2.84)
Average Number of Bidders	3.46	2.73

[a]*Source:* Kenneth Hendricks, Robert Porter, and Bryan Boudreau (1987). Dollar figures are in millions of \$1972. The numbers in parentheses are standard deviations of the sample means.

more than twice the average value of wildcat tracts. Yet, there was less competition, and profit was roughly four times higher on drainage tracts than on wildcat tracts. The profit differential was even greater when measured in dollars per acre, since drainage tracts were typically half the size of wildcat tracts.[1] The government captured 77 percent of the value of wildcat tracts, but only 66 percent of the value of drainage tracts. Thus, even though drainage tracts were lower risk investments and yielded a significantly higher rate of return, firms were less likely to participate in these auctions. What can explain these facts?

The main difference between wildcat and drainage auctions is the distribution of information. Information in a wildcat auction is essentially symmetric, since the precision of seismic survey information is not likely to vary much across firms. This is not true in drainage auctions. Firms which own neighbor tracts obtain information about the drainage tract from their drilling activities on adjacent tracts. Non-neighbor firms derive their information from private seismic surveys, and observable production on adjacent tracts. The latter sources of information are imperfect substitutes for the information that on-site drilling on adjacent tracts can reveal. Consequently, neighbor firms are likely to be better informed than non-neighbor firms, which, if true, would give them an advantage in bidding against the latter. Non-neighbor firms would have to bid cautiously, if at all, since they would have to worry that their bids will win only if the neighbors' estimate is low. (This affliction is often called the "Winner's Curse.")

We find that the data strongly support this hypothesis. Conditional on publicly available information, the participation and bidding decisions of neighbor firms are significantly better predictors of tract profitability than the participation and bidding decisions of non-neighbor firms. Neighbor firms won most of the profitable drainage tracts, and their average share of the value of drainage tracts is about 44 percent. By contrast, non-neighbor firms earned approximately zero profits.

A naive theory of bidding in a drainage auction with one neighbor firm might predict that non-neighbor firms will not bid, on the grounds that they can never make money against a better-informed neighbor firm. However, such reasoning requires firms to hold incorrect beliefs about the bidding behavior of their rivals. If non-neighbor firms choose not to participate, and the neighbor firm correctly anticipates this strategy, its optimal response is to bid the reservation price when it is worthwhile. But, in that case, the non-neighbor can bid slightly more, win the auction, and earn positive profits on average. Thus, for the firms' behavior to be consistent with an equilibrium model of bidding, non-neighbor firms must behave strategically, and participate in such a manner that the neighbor firm is forced to consider the possibility that it will lose the tract if it bids too low.

We find that the data are consistent with the predictions of the Bayesian Nash equilibrium model of bidding in first-price, sealed bid auction with asymmetric information. Non-neighbor firms were relatively cautious in their bidding, but at least one non-neighbor firm bid in 69 percent of the auctions. The number of non-neighbor bids was more

[1]Walter Mead et al. (1984) obtain similar results, in that the internal rates of return they calculate are higher on drainage tracts. They also note that these returns are higher for firms owning neighbor tracts.

- **THE MODEL (FPSB)**
- Consider each tract in isolation
- 1 Neighbor (informed) firm [NFirms]
- N-1 non-neighbor firms [NNFrims]
- X - private signal on V , Z - public signal
- Neighbor firm sees X, non-neighbors do not
- Assume z (realization of Z) is fixed. Essentially, the assumption is that the neighbor firm sees everything that the non-neighbors see. This is a stretch, but models involve simplifying assumptions...
- Stratgey of NNFirm is $G_i(.)$ which maps z to R^+
- Information of NFirm is $H = E[V|X, z]$
- H is distributed $F(.|z)$ with mean $\bar{H}$
- Strategy of neighbor firm is σ mapping H to R^+, hence: $\sigma(h) \in [R, \infty]$ where R is reserve price
- $G(b)$ = distribution function of max of NN bids, so profit of N firm is $G(\sigma(h))(h - \sigma(h))$
- profit of NN firm is

  $$E[H - b - c|\tau(b) > h; z] \times F(\tau(b)|z)Pr(\max NN)$$

  c is a cost from transaction cost in negotiating pool with a neighbor

  τ is a inverse of strategy
- Solve by looking for BNE

- The paper proceeds by characterizing equilibrium. The characterization is ugly and so I will skip it (it is also a very specific model, so little technical dividend from going through it). Instead I will list the comparative statics that come out of the model that are taken to data:

1. The event that no neighbor firm bids occurs less frequently than the event that no non-neighbor firm bids.
2. The neighbor firm wins at least one-half of the tracts.

3. Expected profits to non-neighbor firms are zero. They are negative on the set of tracts where no neighbor firm bids, and positive on the set of tracts where the neighbor firm bids.
4. Expected profits to the neighbor firm incorporates an information premium which makes its earnings above "average."
5. If c is equal to zero, the *ex ante* bid distributions (i.e., prior to the realization of X) are approximately the same.
6. The bidding strategy of the neighbor firm is independent of the number of non-neighbor firms.
7. The bidding strategy of the neighbor firm is an increasing function of the public signal, when a larger signal is "good news."

- **DATA**

- 144 auctions of drainage tracts

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II. Data and Estimation Methods

Our study focuses on the federal lands off the coasts of Louisiana and Texas which were leased between 1959 and 1969. During this period, the government held 8 drainage sales, in which it auctioned off 144 tracts. This number does not include 25 drainage tracts on which the high bid was rejected, for which we have no data.

In our sample, each lease is sold via a first-price, sealed bid auction. A bid is a dollar figure which the firm promises to pay to the government at the time of the sale if it is awarded the tract. This payment is called the *bonus*. The terms of the lease are that, if no exploratory work is done after five years have elapsed, then ownership of the lease reverts to the government. If oil and/or gas is discovered in sufficient quantities so that the firm begins production, the lease is automatically renewed for as long as it takes the firm to extract the hydrocarbons. A fixed fraction of the revenues from any oil and/or gas extracted, one-sixth throughout our sample, accrues to the government. This sum is paid on an annual basis and is called the *royalty* payment. A nominal rental fee ($3 per acre on wildcat tracts, and $10 per acre on drainage tracts) is paid by the firm each year until either the lease expires or production begins.

The government may enter the auction as a bidder in two ways. In our sample, it announced a reservation price of $25 per acre on most drainage leases. (The reservation prices varied from sale to sale.) In addition, it retains the right to reject the high bid on a tract if it believes the bid is too low. The usual basis on which it makes this judgment is its private estimate of the value of the tract. These estimates may be based in part upon the geological and seismic reports which the firms are required to submit. For sales in our sample, the high bid was rejected on 7 percent of the wildcat tracts, and on 15 percent of the drainage tracts.

Our data set contains the following information for each tract: the date it was sold; its location and acreage; which firms bid and the value of their bids; the number and date of any wells that were drilled; and annual production through 1980 if any oil or gas was extracted. The drilling and production data were used, together with the annual survey of drilling costs conducted by the American Petroleum Institute, to calculate *ex post* discounted revenues and costs for each tract. Real wellhead prices in the United States were virtually constant from 1950 until 1973, and we assume that the expectations of the bidders in our sample would be that this pattern would continue. Accordingly, future production paths were converted into revenues by using the real wellhead prices at the date of sale, and discounted to the auction date at a 5 percent per annum rate. See our previous paper with Boudreau for further detail.

From the original sample of 144 drainage tracts, we selected 114 tracts which were adjacent to previously leased federal tracts. (The remaining tracts were adjacent to state tracts, about which we have no information.) For each drainage tract, we then designated neighboring tracts as those which had previously been sold and were adjacent to it, and designated firms as neighbors if they had purchased the rights to one of these tracts. By the same methods we used for the drainage tracts, we computed discounted revenues and costs for each of the neighboring tracts.

The tracts are typically in a square grid pattern, but can vary in size. Wildcat tracts are usually either 5,000 or 5,760 acres,

- Data are:
  - Date
  - Location and size
  - Who bid and how much
  - Number and date of wells
  - Annual production through 1980
  - Cost estimates of production

    (note ex post data allows for actually realization to be used)

	0	1	2	3	4	5	6
No. of tracts	0	40	43	21	5	4	1

FIGURE 1. NUMBER OF NEIGHBOR FIRMS

	0	1	2	3	4	5	6
No. of tracts	19	79	15	0	0	1	0

FIGURE 2. NUMBER OF NEIGHBOR BIDS

TABLE 2—DEFINITION OF VARIABLES[a]

	Mean	Standard Deviation
B_I: maximum bid by neighbor	3.78	11.52
B_U: maximum bid by non-neighbor	3.60	9.57
N_I: number of neighbor bids	1.00	0.67
N_U: number of non-neighbor bids	1.69	2.09
N: number of neighbor tracts	3.01	1.98
NF: number of neighbor firms	2.06	1.08
π: *ex post* tract gross profitability	8.75	20.83
V: *ex post* gross profits of adjacent tract	14.51	20.16
A: tract acreage	2.679	1.533

[a]Dollar figures are in millions of $1972. Tract acreage is in thousands of acres.

and drainage tracts are often 2,500 acres or less. Consequently, the number of possible neighbor tracts is never less than eight, and is sometimes larger. The actual number is usually much less, and the number of neighbor firms is even smaller, since one firm frequently owned more than one neighbor tract. The frequency distribution of neighbor firms per drainage tract is given in Figure 1.

There were 74 tracts with more than one neighbor firm. However, in most of these cases, only one of the neighbor firms bid. The frequency distribution for the number of neighbor bids per drainage tract is given in Figure 2.

The fact that only 16 of the 74 tracts received more than one neighbor bid suggests that neighbor firms may have coordinated their participation and bidding decisions. Such behavior was not prohibited by the federal government in offshore oil auctions during the sample period. In fact, neighbor firms may have formed a joint venture prior to the sale in order to manage production from the common pool. This would also have provided neighbor firms with a mechanism for distributing the benefits from cooperation. Neighbor firms which did not bid could have received transfer payments through the allocation of production shares.

In what follows, we shall assume that the neighbor firms coordinated their bid decisions and submitted one serious bid on tracts which were considered worthwhile. The alternative hypothesis of competitive bidding among neighbor firms is examined in Section IV.

Table 2 lists the empirical analogues of the theoretical variables. The largest neighbor bid is denoted by B_I, or the reservation price in the event that no neighbor firm bid. Similarly, the largest non-neighbor bid is denoted by B_U, or the reservation price in the event that no non-neighbor firm bid. The number of non-neighbor bids is given by N_U, the number of neighbor bids by N_I, and the number of neighbor firms by NF. Our proxy

TABLE 3—SAMPLE STATISTICS ON TRACTS WON BY EACH TYPE OF FIRM[a]

	Wins by Neighbor Firms		Wins by Non-Neighbor Firms		
	A	Total	B	C	Total
No. of Tracts	35	59	19	36	55
No. of Tracts Drilled	23	47	18	33	51
No. of Productive Tracts	16	36	12	19	31
Average Winning Bid	3.28	6.04	2.15	6.30	4.87
	(0.56)	(2.00)	(0.67)	(1.31)	(0.92)
Average Gross Profits	10.05	12.75	−0.54	7.08	4.45
	(3.91)	(3.21)	(0.47)	(2.95)	(1.99)
Average Net Profits	6.76	6.71	−2.69	0.78	−0.42
	(3.02)	(2.69)	(0.86)	(2.64)	(1.76)

[a]Dollar figures are in millions of $1972. The numbers in parentheses are the standard deviations of the sample means. Column A refers to tracts which received no non-neighbor firm bid, column B refers to tracts which received no neighbor bid, and column C to those in which a neighbor firm bid, but a non-neighbor firm won the tract.

neighbor firms. The number of tracts which received no neighbor bid is 19, and the number of tracts which received no non-neighbor bid is 35. Therefore, at least one neighbor firm participated in 83 percent of the auctions, and at least one non-neighbor firm participated in 68 percent of the auctions. This is consistent with the theoretical model.

Table 3 gives sample statistics on the tracts won by each type of firm. Column A refers to tracts which received no non-neighbor firm bid, column B refers to tracts which received no neighbor bid, and column C to those in which a neighbor firm bid, but a non-neighbor firm won the tract.

The evidence is consistent with the model. The neighbor firm won 62 percent of the tracts that it bid on. As we calculated in our previous paper, its share of the tract value was approximately 44 percent, which was considerably higher than the 23 percent average firm share on wildcat tracts. The average net profit of non-neighbor winners was virtually zero. It was positive on tracts which received a neighbor bid, and it was significantly negative on tracts which received no neighbor bid. By contrast, the participation decisions of the non-neighbor firms had no effect on the earnings of neighbor firms. Based on these return figures, it appears as if the neighbor firm was better able to identify which drainage tracts were more likely to contain oil, and was able to exploit this knowledge to obtain above average profits.

We found no evidence of a mass point at the reservation price in the distribution of bids of the neighbor firm. One possible explanation for this result is that firms were afraid that the government would reject reservation price bids. Recall that the government rejected the high bid on 25 drainage tracts.[3]

The existence of a positive reservation price provides an explanation for why non-neighbor firms drilled tracts on which no neighbor firm bid. The lack of participation by the neighbor firm implies that its expectation of net profit is less than R. Since R is positive, and the bid is a sunk cost, it may still have been rational for the non-neighbor firm to drill its lease. Drilling outcomes were not inconsistent with this belief, since the average gross profit of tracts which received no neighbor bid was not significantly different from zero at conventional confidence levels.

An indirect test of our assumption that neighbor firms coordinated their bidding decisions is to compare the bidding behavior of neighbor firms and their net profits on single neighbor tracts to that on multiple neighbor

[3]We study the issue of a random reservation price, and its empirical implications, in a subsequent paper.

TABLE 4—THE EFFECT OF NEIGHBOR FIRM COMPETITION ON NEIGHBOR FIRM PARTICIPATION AND PROFITS[a]

	Single Neighbor Tracts	Multiple Neighbor Tracts No. of Neighbor Bids		
		1	≥ 2	Total
No. of Tracts	40	48	15	74
No. of Tracts with No Neighbor Bid	8	–	–	11
No. of Wins	19	29	11	40
Average Winning Bid of Neighbor Firm	4.795	2.615	17.193	6.624
	(1.444)	(0.697)	(9.953)	(2.885)
Average Gross Profits of Neighbor Firm	13.601	4.670	32.597	12.350
	(5.608)	(2.148)	(11.506)	(3.965)
Average Net Profits of Neighbor Firm	8.806	2.055	15.404	5.725
	(4.762)	(1.690)	(10.963)	(3.297)

[a]Dollar figures are in millions of $1972. The numbers in parentheses are the standard deviations of the sample means.

tracts. If our assumption is correct, they should not be significantly different across the two categories.

The statistics reported in Table 4 are consistent with this prediction. In each category, the neighbor firm won approximately one-half of the tracts. The net profits on the single neighbor tracts won by neighbor firms is somewhat higher than on the multiple neighbor tracts won by neighbor firms, but the difference is not statistically significant. In both categories, net profits were significantly positive, and quite large. High value multiple neighbor tracts tended to attract more than one neighbor bid, but average net profits were substantially *higher* on these tracts than on the multiple neighbor tracts with one neighbor bid. This may be an indication of "shadow" bidding, in which the neighbor firms submit more than one bid in order to convince the government that bidding is competitive.

A. *Information Structure*

An important assumption of our theoretical model, both in terms of its predictive consequences and its influence on our empirical formulation, is that non-neighbor firms have access only to publicly available information. We assume that they have no private signals which are useful in predicting tract profitability, and are not also observed by the neighbor firm.

While we cannot observe the firms' private information signals, it is possible to test this assumption indirectly. In particular, we conducted the following predictive exercise. We first regressed the gross profitability of a given drainage tract on indices of whether a neighbor firm bid (the dummy variable $D_I = 1$ if so), on the number of neighbor bids, on tract acreage, on the number of neighbor tracts, and on a second-order polynomial in the maximum neighbor firm bid and the value of the neighboring tract. We then supplemented this set of regressors with an index of whether any non-neighbor firms bid ($D_U = 1$ in these cases) and, if so, how many do so. We also included the maximum uninformed bid as an additional variable in the second-order polynomial expression. We then performed the same exercise with the roles of the neighbor and non-neighbor reversed. If non-neighbor firms had access to informative private signals, then one would expect their participation and bidding decisions to have some predictive content. If not, then the assumption that no payoff-relevant private information was available to non-neighbor firms may be correct. We have included public information such as acreage, number of neighbor tracts, and adjacent tract value, since the relationship between profitability and bids depend on these values in equilibrium.

In Table 5 we report three regression equations, corresponding to the three sets of

TABLE 6—JOINT DISTRIBUTION OF BIDS CONDITIONAL ON PUBLIC INFORMATION[a]

Independent Variable	Unrestricted: Dependent Variable $\log(B_I/R)$	Unrestricted: Dependent Variable $\log(B_U/R)$	Restricted: Dependent Variable $\log(\text{BID}/R)$
Constant	1.98068	2.05437	−1.99365
	(3.44)	(2.70)	(3.96)
V	0.07391	0.00523	0.04966
	(3.42)	(0.19)	(2.52)
V^2	−0.00073	−0.00009	−0.00050
	(−2.92)	(−0.30)	(−2.17)
A	−0.11092	0.13285	−0.02499
	(−0.82)	(0.74)	(−0.21)
N	−0.08226	−0.28903	−0.14763
	(−0.74)	(−1.97)	(−1.51)

	Unrestricted	Restricted
$\begin{bmatrix} \sigma_I & \\ \rho_{IU} & \sigma_U \end{bmatrix}$	$\begin{bmatrix} 2.0151 & \\ (11.7) & \\ 0.1034 & 2.6596 \\ (0.94) & (12.7) \end{bmatrix}$	$\begin{bmatrix} 2.0528 & \\ (11.5) & \\ 0.0638 & 2.6785 \\ (0.57) & (12.8) \end{bmatrix}$
	$\text{Log } L = -428.895$	$\text{Log } L = -434.184$

[a]Asymptotic t-statistics are in parentheses. They are computed from the analytic second derivatives. They are not appreciably different from the Eicker-White t-statistics.

mated correlation coefficient, ρ_{IU}, is 0.10 and not significantly different from zero. This suggests that there are no omitted variables which might significantly affect both bidding equations. In particular, although tract profitability is correlated with the informed firm's private signals, and so also with the informed firm bid, it appears that this additional information is only weakly correlated with the unexplained component of the maximum uninformed bid.

The theoretical model also predicts that, conditioning on public information and tract profitability, the distributions of the informed bid and the maximum uninformed bid should differ. More precisely, tract profitability should be highly correlated with the neighbor bid, and orthogonal to the maximum non-neighbor bid. To examine this implication of asymmetric information, we computed the maximum likelihood estimates of the joint distribution of bids conditional on tract gross profits and that variable squared, and the public information variables listed above.

The estimation results, which are reported in Table 7, are consistent with the above predictions. The χ^2 statistic of the null hypothesis that the two distributions are the same is 19.84, which is rejected at size 0.01. The coefficients for the tract profitability variables are highly significant in the equation for the neighbor firm bid, and insignificant in the equation for the maximum non-neighbor firm bid. Non-neighbor firms do not appear to have access to information, other than the number and value of neighbor tracts and tract acreage, which is correlated with tract profitability.

The estimates presented in Table 8 can be given a more structural interpretation: they are the maximum likelihood estimates of the coefficients of the informed firm's bid equation and the maximum uninformed firm bid equation, accounting for the truncation of these two variables and the sample selection rule. The maximum uninformed bid is taken to be a function of publicly available information: acreage, number of neighbor tracts, adjacent tract value, and that value squared. The informed bid is a function of these variables, together with our proxies for its private information: actual tract profitability and that figure squared. We also include the number of uninformed firms in both equations. According to the theory, this number

TABLE 7—JOINT DISTRIBUTION OF BIDS CONDITIONAL ON PUBLIC INFORMATION AND TRACT PROFITABILITY[a]

Independent Variable	Unrestricted: Dependent Variable $\log(B_I/R)$	Unrestricted: Dependent Variable $\log(B_U/R)$	Restricted: Dependent Variable $\log(\text{BID}/R)$
Constant	1.86237	2.07435	1.88962
	(4.15)	(2.42)	(4.63)
π	0.09102	0.02765	0.07532
	(4.30)	(0.79)	(3.93)
π^2	−0.00053	−0.00026	−0.00046
	(−2.12)	(−0.62)	(−2.09)
V	0.04428	−0.00323	0.03361
	(2.55)	(−0.12)	(2.05)
V^2	−0.00045	−0.00001	−0.00036
	(−2.25)	(−0.03)	(−1.80)
A	−0.20962	0.10221	−0.13419
	(−1.95)	(0.58)	(−1.34)
N	−0.00888	−0.25858	−0.06645
	(−0.10)	(−1.81)	(−0.83)
$\begin{bmatrix} \sigma_I & \\ \rho_{IU} & \sigma_U \end{bmatrix}$	$\begin{bmatrix} 1.5996 & \\ (11.5) & \\ 0.0492 & 2.6162 \\ (0.46) & (13.1) \end{bmatrix}$		$\begin{bmatrix} 1.6379 & \\ (11.3) & \\ -0.0216 & 2.8014 \\ (-0.20) & (11.8) \end{bmatrix}$
	Log $L = -409.0028$		Log $L = -418.9243$

[a]Asymptotic t-statistics are in parentheses. They are computed from the analytic second derivatives.

should have no explanatory power in the informed bid equation, unless it serves as a proxy for omitted public information variables which might affect uninformed firm bidding. Since the maximum uninformed bid is an order statistic whose distribution depends on N_U, it should be significant in this equation. However, it is properly viewed as endogenous and its coefficient has no structural interpretation.

The estimates in Table 8 indicate that the informed bid is an increasing function of tract profitability and the value of the adjacent tract over the range of values encountered in our sample, and it is essentially independent of the number of neighbor tracts and the number of uninformed bids. The lack of correlation between the maximum informed bid and the number of uninformed bids provides further evidence that our list of public information variables is adequate. If it were not, then unproxied public information (omitted elements of Z) would influence both N_U and B_I.

By contrast, the maximum uninformed bid is not significantly correlated with tract profitability or the value of the adjacent tract, and it is a decreasing function of the number of neighbor tracts. As expected, there is a strong positive correlation between the maximum uninformed bid and the number of uninformed bids.

The sign and magnitudes of the coefficients for the number of neighbor tracts variable in the bid equations are consistent with the maintained hypothesis that neighbor firms do not compete against each other. Under this assumption, the number of neighbor tracts is a proxy for the amount of information which neighbor firms possess. Therefore, non-neighbor firms should bid less aggressively on tracts with a larger number of neighbor tracts, and, in response, the neighbor firm should shade its bid downward.

The signs of the coefficients for the other two public information variables that possess some incremental explanatory power,

TABLE 8—BID EQUATIONS[a]

Independent Variable	Equation (1) Dependent Variable $\log(B_I/R)$	Equation (1) Dependent Variable $\log(B_U/R)$	Equation (2) Dependent Variable $\log(B_I/R)$	Equation (2) Dependent Variable $\log(B_U/R)$	Equation (3) Dependent Variable $\log(B_I/R)$	Equation (3) Dependent Variable $\log(B_U/R)$
Constant	1.86973	2.13073	1.64933	2.15018	1.67785	0.064395
	(−4.19)	(2.90)	(3.52)	(2.96)	(3.66)	(1.14)
π	0.08967		0.08505		0.08501	
	(4.26)		(4.09)		(4.08)	
π^2	−0.00051		−0.00047		−0.00047	
	(−2.04)		(−1.88)		(−1.88)	
V	0.04452	0.00257	0.04814	0.00120	0.04757	0.02083
	(2.58)	(0.10)	(2.82)	(0.04)	(2.79)	(1.08)
V^2	−0.00045	−0.00006	−0.00047	−0.00005	−0.00046	−0.00011
	(−2.25)	(−0.21)	(−2.47)	(−0.18)	(−2.42)	(−0.58)
A	−0.20738	0.12154	−0.25435	0.12908	−0.25713	−0.22645
	(−1.95)	(0.68)	(−2.32)	(0.74)	(−2.38)	(−1.71)
N	−0.01001	−0.27341	0.03228	−0.27116	0.03506	0.03029
	(−0.12)	(−1.92)	(0.36)	(−1.93)	(0.41)	(0.28)
N_U			0.13505		0.11312	0.83705
			(1.26)		(1.42)	(8.48)
$\begin{bmatrix}\sigma_I & \\ \rho_{UI} & \sigma_U\end{bmatrix}$	$\begin{bmatrix}1.5956 & \\ (11.5) & \\ 0.0453 & 2.6238 \\ (0.43) & (13.0)\end{bmatrix}$		$\begin{bmatrix}1.5664 & \\ (11.3) & \\ -0.0782 & 2.6101 \\ (-0.62) & (13.0)\end{bmatrix}$		$\begin{bmatrix}1.5663 & \\ (11.5) & \\ -0.0576 & 1.8769 \\ (-0.56) & (13.0)\end{bmatrix}$	
	Log $L = -409.3745$		Log $L = -408.6295$		Log $L = -378.5628$	

[a]Asymptotic t-statistics are displayed in brackets. They are computed from the analytic second derivatives.

namely, the value of the adjacent tract and that value squared, are the same in the informed and maximum uninformed bid equations. They are significant only in the bid equation of the informed firm. This is consistent with the prediction of the theoretical model that the bids of the non-neighbor firms are much "noisier" than the bids of the neighbor firms. Finally, note that the estimated standard error of the residuals in the informed bid equation is much lower than that of the maximum uninformed bid, although the informed bid itself has a higher standard deviation. (See Table 2.) We can explain a much higher percentage of the variation in the informed firm bids.

IV. A Competitive Bidding Model

In this section, we consider an alternative to the coordination model of neighbor firm bidding. We shall estimate a bidding model under the assumption that neighbor firms act independently and competitively. Our objective is to determine whether estimation under this behavioral hypothesis leads to implications which are not consistent with the theory of competitive bidding.

In the competitive bidding model, each neighbor firm observes a private signal on the value of the drainage tract, which, conditional on the value of the tract, is independently distributed across firms. The precisions of the signals are assumed to be identical, that is, information is symmetrically distributed among the neighbor firms. We shall continue to assume that neighbor firms view the bids of non-neighbor firms as uninformative random variables, and care only about the distribution of the maximum uninformed bid.[4]

[4]This assumption is somewhat *ad hoc*. In his analysis of an auction with asymmetric information in which uninformed firms have noisy, but private estimates of the informed firm's valuation, Wilson (1975) shows that firms which have access only to public information should never bid. If they do, they will lose money.

5 Structural vs Reduced Form vs Model Free

Different questions suggest different approaches. Quantification often requires more modeling than testing comparative statics. Purely descriptive work probably requires the least. Think about what the Hendricks Porter can do - e.g. evidence suggestive of cooperative bidding. And what it can't do: e.g. quantify inefficiency if any.

5.1 Structural Work on Common Values

This is pretty pessimistic, or at least less developed. Recall what identification is:

Definition 5 (Identification)**.** A model $(\mathbf{F}, \Gamma)$ is identified if for every $\left(F, \widetilde{F}\right) \in \mathbf{F}^2$ and $(\gamma, \widetilde{\gamma}) \in \mathbf{\Gamma}^2$, $\gamma(F) = \widetilde{\gamma}\left(\widetilde{F}\right)$ implies $(F, \gamma) = \left(\widetilde{F}, \widetilde{\gamma}\right)$

So we want to be able to map from observed behavior to primitives. Primitives, in this case, are a joint distribution of the signals and the underlying value. For identification, we want this mapping to be unique.

Consider the most standard data in which we see a bunch of data from a set of auctions with a fixed number of bidders. Let's make it a FPSB auction, so we see everyone's bids. Now consider a simplification of the common value setting in which there is a common value for everyone U_o like the Hendricks Porter setup.

Now consider what you want to uncover - the joint distribution of signals and the U_o. So with N bids, you want to understand the behavior of $N+1$ random variables. That's going to be tough without more structure.

Basically, the extra structure that makes this easy is the Private Values assumption. Which we explore below. I'll show you how to relax that a little if we have time, but basically, the results on structural identification of common value models tend to be negative due to the above problem. Where progress has been made, it is through imposing extra structure, focusing on questions that on require identification of a partial model, or through being able to bring in auxiliary data (e.g. like the Hendricks Porter exploitation of ex post data of profitability, or changes in bidders or information structure that is plausibly exogneous). For an Example see Ali Hortascu and Jakub Kastl's paper on Treasury Auctions in Canada. Here they test for CV exploiting auxiliary data from a particular information structure.

Anyway, due to the implication of this problem, most structural work has focused on the private values case, to which we now turn.

6 Private Values: Equilibrium Theory

Here I will run through what we need for the empirical models we want to explore.

– we divide auctions into the following formats depending on the rules governing who wins etc:

  * · FPSB
    · SPSB
    · Ascending
    · Clock (or Dutch)
    · Other (various multi-unit or combinatorial variants)

6.0.1 FPSB Auctions

– We use Perfect Bayesian Nash equilibrium in pure strategies

– denote the equilibrium bid as B_i, with realisation b_i. Let $\mathbf{B} = \{B_1, ..., B_n\}$

– assume affiliation for this section and also take no stand on private vs common values

– we assume some regularity (see Athey and Haile for details). Notably that values lie on some compact subset of the real line

– we would like some existence and uniqueness results to proceed: Here is what exists

– Existence:

  * $\exists$ an equilibrium with strictly increasing strategies except in some CV auctions with asymmetric bidders

– Uniqueness:

  * $\exists$ a unique equilibrium in strictly increasing strategies in PV auctions as long as we have independence or symmetry (or both)

– In CV formats we tend to assume what we need. Note that we don't really need uniqueness, we just need everyone to be playing the same strictly increasing equilibrium.

Equilibrium a bidder with signal $X_i = x_i$ solves

$$\max_{\widetilde{b}} \left(E\left[U_i \middle| X_i = x_i, \max_{j \in \mathcal{N}_{-i}} B_j \leq \widetilde{b} \right] - b \right) \Pr\left(\max_{j \in \mathcal{N}_{-i}} B_j \leq \widetilde{b} \,| X_i = x_i \right) \tag{1}$$

that is, you set your bid to maximise the expected profit if you win (conditional on your signal) times the probability of winning (again, conditional on your signal)

Let

$$\widetilde{v}\left(x_i, m_i; \mathcal{N}\right) = E\left[U_i \middle| X_i = x_i, \max_{j \in \mathcal{N}_{-i}} B_j = m_i\right]$$

and

$$v\left(x_i, y_i; \mathcal{N}\right) = E\left[U_i \middle| X_i = x_i, \max_{j \in \mathcal{N}_{-i}} B_j = \beta_i\left(y_i; \mathcal{N}\right)\right]$$

this expression will turn out to be important as $v\left(x_i, x_i; \mathcal{N}\right)$ is the expected value of winning when i is pivotal.

Let

$$G_{M_i|B_i}\left(m_i \mid b_i; \mathcal{N}\right) = \Pr\left(\max_{j \neq i} B_j \leq m_i \mid B_i = b_i, \mathcal{N}\right)$$

this is the CDF of the maximum bid of the opposing bidders given i's bid and $\mathcal{N}$

Let $g_{M_i|B_i}\left(m_i \mid b_i; \mathcal{N}\right)$ be the corresponding conditional density

Given this notation we can rewrite (1) as

$$\max_{\widetilde{b}} \int_{-\infty}^{\widetilde{b}} \left(\widetilde{v}\left(x_i, m_i; \mathcal{N}\right) - \widetilde{b}\right) g_{M_i|B_i}\left(m_i \mid \beta_i\left(x_i; \mathcal{N}\right); \mathcal{N}\right) dm_i \tag{2}$$

Some technical stuff is required here to prove that this is differentiable a.e. See Athey and Haile, and Krishna's text for technical comments on these points.

We can differentiate (2) w.r.t. $\widetilde{b}$ to get

$$v\left(x_i, x_i; \mathcal{N}\right) = b_i + \frac{G_{M_i|B_i}\left(b_i \mid b_i; \mathcal{N}\right)}{g_{M_i|B_i}\left(b_i \mid b_i; \mathcal{N}\right)} \tag{3}$$

- This is the key equation from theory use for estimation purposes. Note the following
  * It is pretty nice - linear structure is simple (this is from risk neutrality)
  * LHS is the latent variable we are interested in
  * RHS has stuff that this observed (bids) or functions related to observed stuff, this gives hope for estimation.

6.0.2 Ascending Auctions

- Clock or button auctions are the framework used in most theory
- Unattractive for empirical work of very few ascending auctions work like this

- Even if this framework is OK we have to deal with the fact that we never see the value at which the highest bidder would drop out.
- However, we would see all but the first order statistic, which is a lot of information really. That said, you have to be very careful: order statistics loose their independence if you do not observe all of them and, in an ascending auction, but construction you do not observe them all (see the Athey Haile survey paper for more). So overcoming the mechanical loss of independence due to the data generating process is a central issue. Sometimes you can get around this, more often it is a technical issue that can kill a paper. Compare to Haile Tamer to follow.
- The key thing from theory, regardless of approach used is that bidders never bid more than they are willing to pay, and never let anyone win with a price they are willing to beat.

7 Identification and Estimation of FPSB PV Auctions

- Early work by Harry Paarsch and others in the early 1990s explored ways of estimating using bid functions and imposing parametric forms on the distribution of private information.
- Other work by Laffont, Ossard and Vong (in various combinations), and Pat Bajari tried other approaches using different theoretical 'handles'
- The literature seems to have converged on the following basic line of attach due to Guerre, Perrigne and Vong (2000).

Basic idea:

in equilibrium each bidder is acting optimally against the distribution of behaviour of other bidders

How does this help?

- it means that the distribution of opponents bids in (3) can be inferred from the data without parametric restriction.
- ie: "given what others are doing you are doing the best you can *and* given what you are doing they are doing the best they can" - we are taking both parts of this statement very seriously

Lets be formal recall:

$$u_i = b_i + \frac{G_{M_i|B_i}(b_i \mid b_i; \mathcal{N})}{g_{M_i|B_i}(b_i \mid b_i; \mathcal{N})}$$

– first; note that this is the inverse of the bid function

– second; note that the b_i is observed

– third; note that $\frac{G_{M_i|B_i}(b_i|b_i;\mathcal{N})}{g_{M_i|B_i}(b_i|b_i;\mathcal{N})}$ is a ratio of properties of the joint distribution of opponents bids.

The invertability of the bid function is the key thing for identification (i.e. we rely on the assumption that the bidders use a strictly increasing bid function)

Guerre, Perrigne and Vong (2000), Li, Perrigne and Vong (2002) and Campo, Perrigne and Vong (2003) work all this out for various scenarios.

Summarizing: in the APV model, if symmetric just need all the bids, if not, then need the identities of the bidders

7.1 Estimation

How do you operationalize this?

What I want to do is take you through the standard non-parametric approach and then follow up with some comments.

Map of Approach

Say the data is from T auctions. All auctions are the same, although there is some variation in the number of bidders. The estimation proceeds in the following steps:

1. (a) Estimate the $\frac{G_{M_i|B_i}(b_i|b_i;\mathcal{N})}{g_{M_i|B_i}(b_i|b_i;\mathcal{N})}$ from bid data

   (b) Use this to compute $u_i = b_i + \frac{G_{M_i|B_i}(b_i|b_i;\mathcal{N})}{g_{M_i|B_i}(b_i|b_i;\mathcal{N})}$

   (c) Use the psuedo-sample of u_i to estimate F_U

There is an econometric issue that we need to confront: How do you estimate a density or CDF from data?

7.1.1 Aside: Kernel Estimation

- There are several ways to estimate a density from data. Kernels are the most commonly used method in this application. A good reference on this and other approaches is Pagan and Ullah (1999)
- A kernel estimator is basically an adaption of the idea of a histogram to the case of data that has a support that is an interval of the real line (or for higher dimensional kernels, $\mathbb{R}^n$, where n should be fairly small or else you hit curse of dimensionality problems)
- Discrete data: say you have a random variable x such that the support of x is the set $\{1, 2, 3\}$. To get the density of x you would count the number of times it falls into each bin and then divide these counts by the number of observations. This is consistent etc.
- Data with a Continuous Support
  * This is where kernels come in
  * The idea of a kernel is that we can learn about the value of the density function at a point v, $f(v)$, by looking at how common it is to see realizations of the random variable near this point.

Details

- Imagine we had data on the realizations of a random variable, drawn from a unknown distribution $F(\cdot)$ and want to know the value of the density $f(\cdot)$ at a point x. The data is $\{x_1, ..., x_n\}$
- The closest thing to constructing a histogram we might do is to pick an interval around the point $\widetilde{x}$, and count how many observations fall in this interval.
- Our estimator would look like

$$\widehat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} \mathbb{I}\left(x - \frac{h}{2} \leq x_i \leq x + \frac{h}{2}\right)$$

  where $\mathbb{I}(\cdot)$ is an indicator function and h is a parameter setting the bandwidth. We can rewrite this as

$$\begin{aligned} \widehat{f}(x) &= \frac{1}{nh} \sum_{i=1}^{n} \mathbb{I}\left(-\frac{1}{2} \leq \frac{x_i - x}{h} \leq \frac{1}{2}\right) \\ &= \frac{1}{nh} \sum_{i=1}^{n} K(\psi) \quad \text{where} \quad \psi = \frac{x_i - x}{h} \end{aligned}$$

This is our first kernel estimator. The function $K(\cdot)$ is the kernel, which in this case is a indicator function which equals when when ψ lies in the interval $\left[\frac{-1}{2}, \frac{1}{2}\right]$.

- This kernel is not attractive as it steps, that is, it is not smooth. It has the advantage that it integrates to one, however. Smoothness is generated by choosing a kernel such that
$$\int_{\mathbb{R}} K(\psi)\, d\psi = 1$$

Some examples of commonly used kernels are:

1. standard normal: $K(\psi) = (2\pi)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\psi)^2\right]$
2. epanechnikov: $K(\psi) = \frac{3}{4}\left(1-\psi^2\right)$ if $|\psi| < 1$, zero otherwise (this minimizes Integrated Mean Squared Error)

Many others exist.

A few things about small sample properties:

Kernel estimates are typically biased. The bias is a function of the bandwidth, the kernel and the density to be estimated. Bias can generally be reduced by choosing a smaller bandwidth at the cost of increasing variance in the estimates. Procedures for generating bias-reducing kernels exist but require that we allow the kernel to take negative values.

How to select bandwidth:

There are two ways to proceed: by plotting the density implied from the density and seeing if it looks "good". Using the data to select the optimal bandwidth by imposing some criteria (like integrated square error[1] or its expectation). Both are easy to implement, for details see Pagan and Ullah.

Asymptotic Properties

Under regularity conditions and some assumptions on the DGP kernel estimates are CAN. See P&U for these assumptions

The key points are that

1. i.i.d draws are useful but not required (you can handle time series type issues)
2. convergence is different from a parametric estimator which means you have to be a bit careful when using them with parametric estimators in a multi-step procedure.

[1]ISE is $\int \left(\widehat{f}(x) - f(x)\right)^2 dx$

3. The asymptotic variance generates a 95% confidence interval that looks like

$$f \pm 1.96\,(nh)^{-\frac{1}{2}} \left[f(x) \int K^2(\psi)\, d\psi \right]^{\frac{1}{2}}$$

4. bootstraps are usually used for small-sample confidence intervals.

7.1.2 Application to the FPSB PV Problem

lets start with the simplest case: FPSB with a symmetric IPV specification

1. (a) $F_{\mathbf{U}} = \prod_{i=1}^{n} F_{U_i}$ This makes step (c) easier

   (b) due to independence:

   $$\begin{aligned} G_{M_i|B_i}(m_i \mid b_i; \mathcal{N}) &= G_{M_I}(m_i \mid n) \\ &= \Pr\left(\max_{j \neq i} B_j \leq m_i \mid n \right) \end{aligned}$$

   (c) this changes the estimated equation to

   $$u = b + \frac{G_B(b \mid n)}{(n-1)\, g_B(b \mid n)}$$

   where notation has been abused a little bit

   i. $G_B(b \mid n)$: marginal distribution of equilibrium bids in n bidder auctions

   ii. $g_B(b \mid n)$: the associated density

   (d) This formulation makes estimation straightforward using kernels

   $$\widehat{G}_B(b \mid n) = \frac{1}{nT_n} \sum_{t=1}^{T} \sum_{i=1}^{n} \chi\{b_{it} \leq b, n_t = n\}$$

   $$\widehat{g}_B(b \mid n) = \frac{1}{nT_n h_g} \sum_{t=1}^{T} \sum_{i=1}^{n} K\left(\frac{b - b_{it}}{h_g}\right) \chi\{n_t = n\}$$

   so this gives us step (a)

   (e) then

   $$\widehat{u} = b + \frac{\widehat{G}_B(b \mid n)}{(n-1)\, \widehat{g}_B(b \mid n)}$$

   gives us step (b)

   (f) then

   $$\widehat{f}(u) = \frac{1}{T_n h_f} \sum_{t=1}^{T} \frac{1}{n_t} \sum_{i=1}^{n} K\left(\frac{u - \widehat{u}_{it}}{h_f}\right)$$

- the assymptotic distribution of these estimators are "largely unresolved".
- the easiest way to side-step the issue is use the bootstrap (although always check that you are ok to use it: depending on the dgp you may need to make some adjustments)

7.1.3 Algorithm for Estimation

1. take each observed b_o
2. estimate $\widehat{G}_B(b_o \mid n)$ and $\widehat{g}_B(b_o \mid n)$ using the entire data set
3. infer $\widehat{u}(b_o)$
4. once the above is done for each bid, estimate $\widehat{f}(u)$ and draw a picture of it
5. then do whatever policy stuff you want

7.1.4 Other issues

- * for the theory to work we need a compact support of u_i. Issues arise when the estimator is near the boundary of the support - consistency is no longer assured. Li, Perrigne and Vong (2002, p 180 on) sort this out
  * while nonparametric estimation is very attractive ex ante it may be that the data set you are facing works better with the extra structure imposed by a parametric specification. That is more structure may give you more power. In considering whether to go parametric think about what you want to use the estimates for, where identification is coming from and whether any auxiliary data can but used to justify the parametric assumption.
  * there is still a lot of structure being imposed on the data here, particularly in stages (b) and (c). Take some time to think about how much work the functional form assumptions are doing in these stages.
  * lastly, and most importantly, note the big assumptions on auction heterogeneity, bidder heterogeneity etc. More on this later...

7.1.5 Dealing with Bidder Asymmetry

- The asymmetry that we deal with here is differences in the distributions of private information (as opposed to covariates observed by other bidders, which we deal with later)

- with symmetry the markdown term was

$$\frac{G_{M_i|B_i}\left(b_i \mid b_i; n\right)}{g_{M_i|B_i}\left(b_i \mid b_i; n\right)}$$

  without symmetry it is

$$\frac{G_{M_i|B_i}\left(b_i \mid b_i; \mathcal{N}\right)}{g_{M_i|B_i}\left(b_i \mid b_i; \mathcal{N}\right)}$$

- the natural apporach, building on what we have done before is to use a kernel estimator such that

$$\widehat{G}_{M,B}\left(m, b; \mathcal{N}_t\right) = \frac{1}{T_{\mathcal{N}} h_g} \sum_{s=1}^{T} K\left(\frac{b - b_{is}}{h_g}\right) \chi\left\{m_{it} \leq b, \mathcal{N}_t = \mathcal{N}_s\right\}$$

$$\widehat{g}_{M,B}\left(m, b; \mathcal{N}_t\right) = \frac{1}{T_{\mathcal{N}} h_g^2} \sum_{s=1}^{T} K\left(\frac{b - b_{is}}{h_g}, \frac{b - m_{is}}{h_g}\right) \chi\left\{\mathcal{N}_t = \mathcal{N}_s\right\}$$

- this allows each bidder to have their own distribution from which they draw their private information.
- If you think about this approach for just a second you will realize that it requires a bucket load of data. You need to see the same set of bidders interacting again and again...
- Often we can group bidders together into say class A and class B. Now we just need to make sure that the auctions we use to estimate each density have the same number of class A and B bidders in them. The rest of the adjustments should, by now, be straightforward.
- A good example of this approach is the Campo, Perrigne and Vong (2003) paper - lets have a look at it

involves two or three firms. A number of arguments for joint bidding have been given in the literature. For instance, joint bidding can weaken financial constraints, reduce costs by pooling cartel members' information and capital through the joint venture and spread risks among firms. See e.g. DeBrok and Smith (1983), Millsaps and Ott (1985), Gilley *et al*. (1985) and Hendricks and Porter (1992). As noted by many economists, however, joint bidding may have introduced some *ex ante* asymmetry among bidders.

Because joint bidding is negligible in the 1950s–1960s, our study focuses on auctions held between December 1972 and 1979.[12] Because of data requirements explained subsequently, we consider auctions with two bidders who can be either joint or solo. This gives a total of 227 auctions from which 55 auctions have two solo bids, 60 auctions have two joint bids and 112 auctions have one solo bid and one joint bid. Among the latter, 63 auctions are won by the joint bidder. Using a normal approximation, the ratio 63/112 is greater than 1/2 at the 10% significance level in a one-sided test, where 1/2 would be the expected ratio if the two participants have equal chance of winning.[13] Thus joint bidding has increased the probability of winning suggesting some *ex ante* asymmetry among participants.

For each wildcat auction, we know the date, the acreage of the tract, the number of bidders, their bids in constant 1972 dollars and whether the bid is a solo or a joint bid. Table I gives some summary statistics in $ per acre for the 454 bids considered in our empirical study as well as on solo and joint bids separately, whether the opponent's bid is of the same type or of a different type.

A first feature revealed by the means displayed in Table I is that joint bids tend to be higher on average than solo bids, as a number of empirical studies have found. Moreover, joint bidders tend to bid higher when they face a joint bidder than when they face a solo bidder. Likewise, though their bids are lower than those of joint bidders, solo bidders tend to bid on average higher when they face a joint bidder than when they face another solo bidder. This suggests that the bidding strategy of each type of bidder depends on the type of their opponent. This could arise from bidders taking into account some possible asymmetry in their bidding strategies. For instance, a test of the equality of means for solo bids versus joint bids in the 112 auctions with one bidder of each type gives a t-statistic equal to 1.66, which (weakly) rejects their equality. It is also interesting to note that the within variability of solo versus solo bids is much smaller than the within variability

Table I. Summary statistics on bids

Variable	# Obs	Mean	STD	Minimum	Maximum	Within STD
All bids	454	687.30	1,431.31	19.51	20,751.32	1,258.91
Joint bids	232	837.32	1,717.54	21.46	20,751.32	—
Solo bids	222	532.53	1,033.20	19.51	11,019.08	—
Joint vs joint	120	875.13	2,056.12	33.94	20,751.32	2,011.99
Joint vs solo	112	796.83	1,266.32	21.46	6,377.94	—
Solo vs joint	112	603.28	1,226.61	19.51	11,019.08	—
Solo vs solo	110	456.45	788.19	20.80	7,009.10	747.43

[12] We exclude auctions after 1979 since the rules of the auction mechanism have changed somewhat after this date. We also exclude the unique sale held in 1970 and the first sale in 1972 because the water depth of the tracts sold at these sales was much greater than usual.
[13] Hereafter, all tests are conducted at the 10% significance level.

private values than solo bidders with a relatively more important variability for the former. As pointed out in Section 3.2, these differences can be explained by unobserved tract heterogeneity and differences between joint and solo bidders. This issue is further investigated below.

As Figure 2 does not provide information on the affiliation between private values within the same auction whether they are both joint or solo, it is useful to test for their independence. We use the non-parametric test proposed by Blum *et al.* (1961) (BKR hereafter), which is consistent and distribution free. For two variables X and Y, the test statistic is equal to $(1/2)\pi^4 B$, with $B = N^{-4}\sum_{\ell=1}^{N}(N_1(\ell)N_4(\ell) - N_2(\ell)N_3(\ell))^2$, with N the number of observations and $N_1(\ell), N_2(\ell), N_3(\ell), N_4(\ell)$ the numbers of points lying respectively in the regions $\{(x,y)|x \le X_\ell, y \le Y_\ell\}$, $\{(x,y)|x > X_\ell, y \le Y_\ell\}$, $\{(x,y)|x \le X_\ell, y > Y_\ell\}$ and $\{(x,y)|x > X_\ell, y > Y_\ell\}$. To impose symmetry among bidders of the same type, we duplicate the observations so that $N = 2 \times L$. We find a test statistic equal to 6.57 using observed bids and to 4.69 using trimmed private values for joint bidders. For solo bidders, we obtained a test statistic equal to 9.52 using observed bids and equal to 6.92 using trimmed private values. The null hypothesis of independence is clearly rejected in all cases.

Case $(n_1, n_0) = (1,1)$

The potentially asymmetric case is estimated using the 112 auctions with both types. Because there is only one bidder of each type, (4) and (5) simplify as B_1^* and B_0^* are void. In particular, their denominators reduce to the conditional densities $g_{b_0|b_1}(b_1|b_1)$ and $g_{b_1|b_0}(b_0|b_0)$. Hence, (4) and (5) reduce to

$$v_1 = \xi_{10}(b_1) = b_1 + G_{b_0|b_1}(b_1|b_1)/g_{b_0|b_1}(b_1|b_1) \tag{18}$$

$$v_0 = \xi_{01}(b_0) = b_0 + G_{b_1|b_0}(b_0|b_0)/g_{b_1|b_0}(b_0|b_0) \tag{19}$$

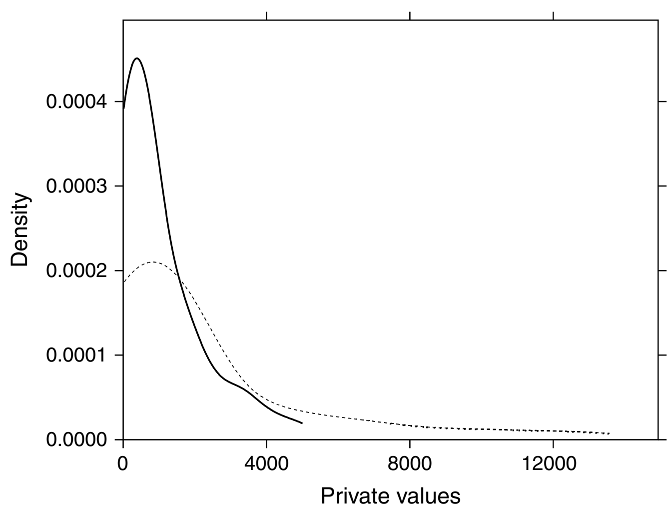

Figure 2. Marginal densities of private values

$2195.14 per acre with a standard deviation equal to $3024.10 and a range of [$33.94;$13 605.86]. For the (0,2) case, these numbers are $1027.90, $1170.15 and [$26.89; $5031.39], respectively from the 98 trimmed private values.

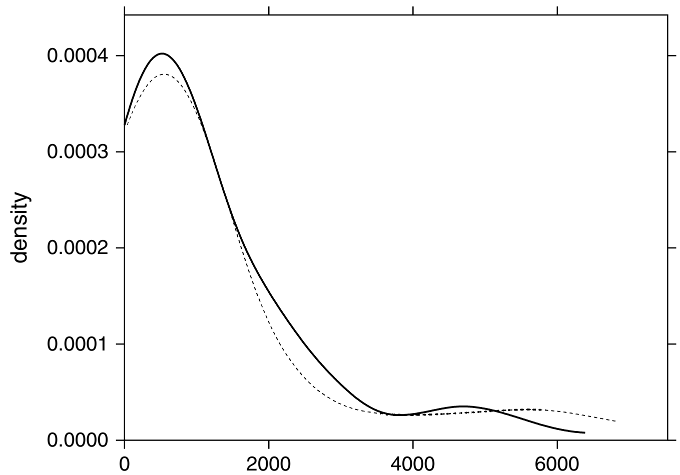

Figure 4. Marginal densities of private values

asymmetry though weak between joint and solo bidders as the empirical cumulative distribution functions slightly differ with a single crossing.[22]

Such an asymmetry leads the solo bidders to shade less their private values than joint bidders, as found in Figure 3, so as to increase their probability of winning the auctions. See also Maskin and Riley (2000a) and Pesendorfer (2000). However, the shading effect does not counterbalance fully the asymmetry in terms of valuation distributions, as indicated by the bid averages for joint versus solo and solo versus joint in Table I and the empirical probability of winning. As is well known, the aggressiveness of the weak bidder relative to the strong bidder may introduce some inefficiency in the auction in the sense that the winner of the auction has the lowest valuation. It turns out that this does not happen in our data set, which can be explained by the relatively weak asymmetry and the important variability of private values within each auction.

It is interesting to compare these results to the first two cases where bidders are of the same type. Figure 5 displays the inverse bidding strategies for a joint bidder when facing a joint bidder ($\hat{\xi}_{11}(\cdot)$) and when facing a solo bidder ($\hat{\xi}_{10}(\cdot)$), the former being to the right of the latter. Given a same tract valuation, a joint bidder will bid more aggressively when facing a joint bidder than when facing a solo bidder. For, the joint bidder faces less 'competition' when facing a solo bidder who is more likely to draw a lower private value. Figure 6 displays the inverse bidding strategies for a solo bidder when facing a solo bidder ($\hat{\xi}_{00}(\cdot)$) and when facing a joint bidder ($\hat{\xi}_{01}(\cdot)$), the former being to the left of the latter. Thus, a solo bidder will bid slightly more aggressively when facing a joint bidder than when facing a solo bidder to compensate for his lower private value. These results confirms the descriptive statistics of Table I. Both figures indicate that bidders have integrated the type of their opponents in their bidding strategies.

[22] The graph is available upon request from the authors. A Kolmogorov–Smirnov test does not reject the equality of the c.d.f.s on either private values or bids. Note, however, that the Kolmogorov–Smirnov test is based on the independence of the two samples. This is not the case as joint and solo private values (or bids) are affiliated, which decreases the power of the test.

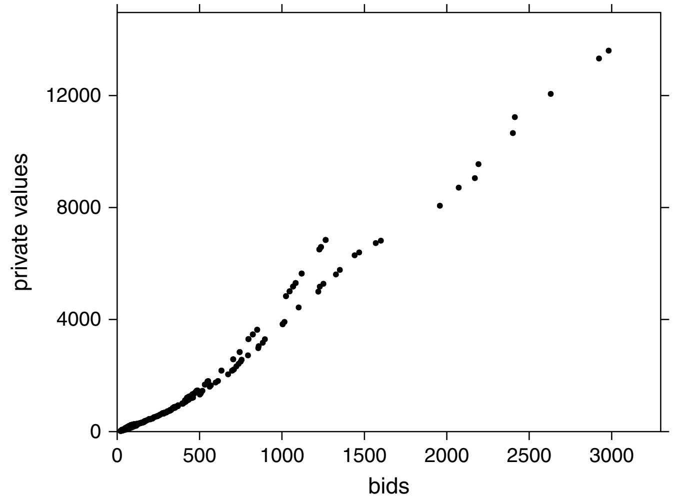

Figure 5. Inverse bidding strategies of joint bidders

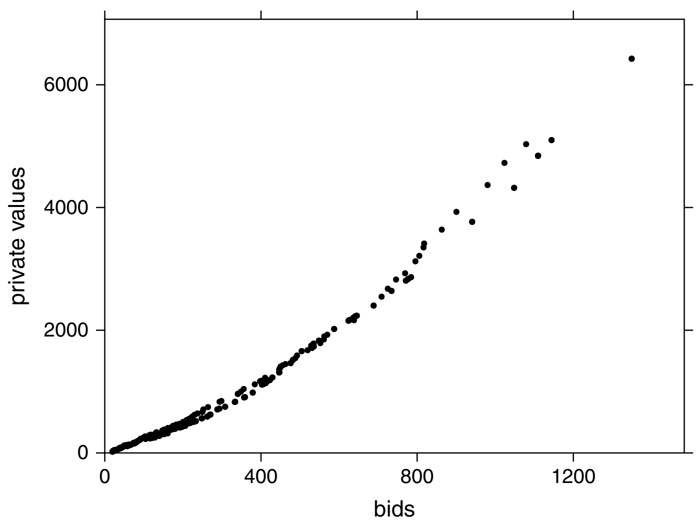

Figure 6. Inverse bidding strategies of solo bidders

Lastly, as indicated in Section 3.2, the comparisons of $\hat{f}_1^{(2,0)}(\cdot)$ and $\hat{f}_1^{(1,1)}(\cdot)$ as well as of $\hat{f}_0^{(0,2)}(\cdot)$ and $\hat{f}_0^{(1,1)}(\cdot)$ provide some information about unobserved tract heterogeneity. A Kolmogorov–Smirnov test gives a test statistic equal to 0.1917 and 0.0965 for the former and latter comparisons, respectively. This represents a clear rejection of $\hat{f}_1^{(2,0)}(\cdot) = \hat{f}_1^{(1,1)}(\cdot)$. Under the assumptions of Section 3.2, this means that there is some unobserved tract heterogeneity as tracts attracting two joint bidders differ significantly from tracts attracting one joint bidder and one solo

7.1.6 Incomplete data:

- * Often you just observe the transaction data from FPSB auctions and little else.
  * Some results exist on identification suggesting that a feasible estimator may exist. Indeed a simple approach may be to exploit the properties of order statistics. Athey and Haile has a little discussion of this.
  * To my knowledge no-one has really implemented an estimator with incomplete data to get at a substantive issue.

8 Ascending Auctions

- Ascending auctions are often modeled as button auctions
- these are pretty poor descriptors of what the data generating process really looks like since the information transmission in a button auction is too good.
- Haile and Tamer investigate a bounds approach to which is elegant and likely to have application in other asymmetric information problems where the data generating process is not perfectly modeled.
- They start with the following two assumptions

1. (a) i. [A1] Bidders do not bid more than they want to pay
   ii. [A2] Bidders do not let someone win at a price they are willing to beat

- * note: this allows jump bidding, bids $\neq$ valuations, bidders that merely watch the action without entering a bid etc
  * the idea here is to use these restrictions to provide partial identification of valuation. That is, provide bounds.

Formally:

1. Assume we see the bids made by all the bidders in the data set.
2. [A1] is equivalent to $b_i \leq u_i \quad \forall i$
3. it follows that in an n-bidder auction $b^{(i;n)} \leq u^{(i;n)}$ [this is easy but not immediate]
4. it follows that

$$G_B^{(i;n)}(u) \geq F_U^{(i;n)}(u) \;\; \forall i, u, n \tag{4}$$

5. now we need to do a little statistics:
6. the distribution of an order statistic from an iid sample of size n from an arbitrary distribution $F(\cdot)$ has distribution

$$F^{(i:n)}(s) = \frac{n!}{(n-i)!\,(i-1)!}\int_0^{F(s)} t^{i-1}(1-t)^{n-i}\,dt$$

7. since the RHS is strictly increasing in $F(\cdot) \in [0,1]$, $F^{(i:n)}(s)$ uniquely determines a value for $F(s)\ \forall s$
8. this is a useful result since it allows us to define the following function, ϕ, implicitly. ϕ is just the mapping between $F^{(i:n)}(s)$ and $F(s)$

$$H = \frac{n!}{(n-i)!\,(i-1)!}\int_0^{\phi} t^{i-1}(1-t)^{n-i}\,dt \qquad H \in [0,1]$$

so that

$$F_U(u) = \phi\left(F_U^{i:n}(u); i, n\right) \tag{5}$$

9. since $\phi : [0,1] \to [0,1]$ is strickly increasing (4) and (5) give

$$\phi\left(G_B^{i:n}(u); i, n\right) \geq F_U(u)$$

so, given an estimate of $G_B^{i:n}(u)$ we can get an upper bound on $F_U(u)$. Lets set the estimation of $G_B^{i:n}(u)$ aside for one moment.

10. We have several versions of $G_B^{i:n}(u)$, so we have a bunch of upper bounds to choose from. What we do is choose the least upper bound (the most informative bound)

$$F_U^+(u) = \min_{i,n} \phi\left(G_B^{i:n}(u); i, n\right)$$

11. The lower bound is similar, although we have slightly less data to work with...
12. [A2] implies that all losing bidders have valuations less than $b^{n:n} + \Delta$, where Δ is the minimum bid increment
13. this implies

$$u^{n-1:n} < b^{n:n} + \Delta$$

14. this gives

$$G_\Delta^{n:n}(u) \leq F_U^{n-1:n}(u) \qquad \forall n, u$$

where $G_\Delta^{n:n}(u)$ is the distribution of $B^{n:n} + \Delta$

15. now things proceed as above, but for the fact we only have $|\{\underline{n}, ..., \overline{n}\}|$ lower bounds. Also we should look for the greatest lower bound.

Comments

- * This seems a pretty flexible approach: do not really need all the bids, just the transaction price.
  * Also, note the importance of knowing the number of bidders. Without this it is a bit hard to exploit the order statistics.
  * It might be possible to proceed if you had a distribution over the possible number of bidders.

8.0.7 Estimation

- for this to work we need estimates of $G_B^{i:n}(u)$ and $G_\Delta^{n:n}(u)$
- as before we use non-parametric estimates which, given these are CDFs, are actually pretty trivial
- the estimators are

$$\begin{aligned} G_B^{i:n}(b) &= \frac{1}{T_n}\sum_{t=1}^{T} \varkappa\left\{n_t = n, b^{i:n_t} \leq b\right\} \\ G_\Delta^{n:n}(b) &= \frac{1}{T_n}\sum_{t=1}^{T} \varkappa\left\{n_t = n, b^{n_t:n_t} + \Delta_t \leq b\right\} \end{aligned}$$

- after these estimates have been taken, we plug it into the formulae above to get the bounds (there is a little bit of computation to be done here)
- the asymptotic distribution of the final estimates is a little weird due to the min and max operators but Haile and Tamer show that the bootstrap is able to be applied here.
- there is a problem in that in finite samples the upper and lower bounds may overlap. H&T discuss this and propose a solution that basically involves taking weighted averages rather than the min or max.

8.0.8 So What?

- The key issue is whether these bounds allow you to say anything useful about the world.
- The authors show that careful inspection of the basic auction model allows to say things about reserve prices (bound the optimal reserve price)
- An adaptation of Manski and Tamer allows them to say things about how valuations are affected by covariates

- The extent to which you can make useful inference about policy variables will depend on your application and model
- Lets have a look at the H&T results

TABLE 2
Gaps Between First- and Second-Highest Bids

Quantiles	High Bid	Gap	Minimum Increment	Gap ÷ Increment
10%	9,151	30	4.1	1.2
25%	22,041	92	10.1	6.9
50%	55,623	309	23.4	14.8
75%	127,475	858	52.1	20.0
90%	292,846	2,048	110.5	76.4

dollars) on the median tract.[23] Forest Service officials report that jump bidding is common. Table 2 provides some support, showing a gap between the highest and second-highest bid of several hundred dollars (roughly 10–20 times the minimum increment) in the majority of auctions. Since the cost of jump bidding—the risk that one wins with the jump bid and pays too much—is highest at the end of the auction, jump bidding is likely to be more significant early in the auctions. However, these gaps themselves are generally quite small relative to the total bid, suggesting that we may be able to obtain tight bounds.

B. *Reserve Price Policy*

The Forest Service's mandated objective in setting a reserve price is to ensure that timber is sold at a "fair market value," defined as the value to an "average operator, rather than that of the most or least efficient" (U.S. Forest Service 1992). Many observers have argued that Forest Service reserve prices fall short of this criterion and are essentially nonbinding floors (see, e.g., Mead, Schniepp, and Watson 1981, 1984; Haile 1996; Campo et al. 2000). Bidders, for example, claim that the reserve prices never prevent them from bidding on a tract (Baldwin et al. 1997). As discussed above, for our purposes it is sufficient to assume only that the actual reserve prices are below the profit-maximizing reserve prices.

There is an ongoing controversy over so-called below-cost sales—sales generating revenues insufficient to cover even the costs to the Forest Service of administering the contract (see, e.g., U.S. General Accounting Office 1984, 1990, 1991; U.S. Forest Service 1995). Obviously, this is possible only with reserve prices below profit-maximizing levels. However, reserve prices are not set with the goal of profit maximization nor

[23] Forest Service rules actually require only that total bids rise as the auction proceeds, although local officials often specified discrete increments. In the time period we consider, the 5 cent increment was a common practice in this region. Sometimes increments of 1 cent per MBF were used, and many sales used no minimum increment. We use the 5 cent increment since this results in a more conservative bound, although variations of this magnitude have very little effect on the results: 5 cents represents about 0.05 percent of the average bid in our sample.

TABLE 3
Summary Statistics

	Mean	Standard Deviation	Minimum	Maximum
Number of bidders	5.7	3.0	2	12
Year	1985.2	2.6	1982	1990
Species concentration	.68	.23	.24	1.0
Manufacturing costs	190.3	43.0	56.7	286.5
Selling value	415.4	61.4	202.2	746.8
Harvesting cost	120.2	34.1	51.1	283.1
Six-month inventory*	1,364.4	376.5	286.4	2,084.3
Zone 2 dummy	.88		0	1

* In millions of board feet.

are quite tight. The shape of the true distribution suggested by these bounds resembles a lognormal distribution, which has been used in several prior studies.

To construct estimates of bounds on the optimal reserve price, an estimate of v_0, the cost of allowing the harvest of the tract, is needed. We consider a range of possible values based on Forest Service estimates (U.S. Forest Service 1995; U.S. General Accounting Office 1999).[27] Table 4 shows the results of simulations used to evaluate the trade-offs between net revenues and the probability that a tract goes unsold with alternative reserve prices. Values of v_0 between \$20 and \$120 are considered and the implied bounds on the optimal reserve prices calculated. For each value of v_0, we consider three possible reserve prices: $\hat{p}_L$, $\hat{p}_U$, and the average of the two. The table reports simulated gains in profit per MBF relative to actual profits, using each value of v_0 as the measure of costs. This is done both assuming $F(\cdot|\mathbf{X}) = \hat{F}_L(\cdot|\mathbf{X})$ and assuming $F(\cdot|\mathbf{X}) = \hat{F}_U(\cdot|\mathbf{X})$, providing estimated bounds on the profit gains (losses) from using each reserve price considered. Note that lemma 4 enables us to use equilibrium bids in a second-price sealed-bid auction to obtain revenue predictions.

As foreshadowed by our simulations, despite the tightness of the bounds on $F(\cdot)$ in figure 8, the bounds on the optimal reserve price for each v_0 are fairly wide. Because the bounds on $F(\cdot)$ are tight, however, our estimates of the expected revenues obtained with reserve prices

[27] For sales in region 6 in 1993, the Forest Service estimated that costs of the timber sales program were between \$85 and \$113 per MBF (U.S. General Accounting Office 1999). On the basis of sales in 1990–92, nationwide cost-based reserve prices between \$18 and \$47 per MBF were suggested as appropriate (U.S. Forest Service 1995), depending on which timber sales program costs are to be covered by auction revenues. Both calculations include some costs that are sunk at the time of the auction and, therefore, should be excluded from v_0. However, other costs, such as forgone return on investment and adverse environmental impacts, are excluded. Obtaining more precise estimates of v_0, ideally as a function of tract characteristics, would be an important step toward a more definitive analysis of reserve price policies.

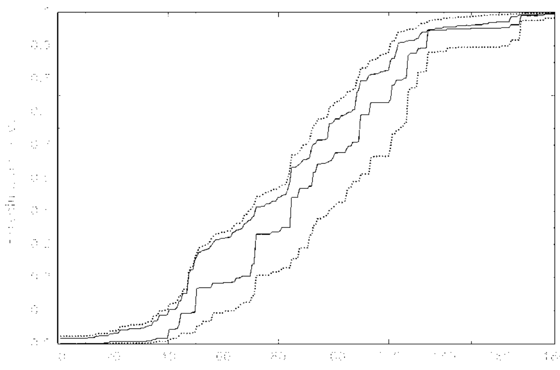

FIG. 10.—U.S. Forest Service timber auctions. Solid curves are estimated bounds, and dotted curves are bootstrap confidence bands.

between $\hat{p}_L$ and $\hat{p}_U$ differ little, with v_0 held fixed. The calculated bounds on the optimal reserve prices provide strong support for the assumption that the actual reserve price (around \$54) is well below the optimum. Even with $v_0 = 0$, the estimated lower bound on p^* is still slightly larger than the average actual reserve price. These results also suggest that, at least on average tracts in our sample, reserve prices could be raised considerably without causing many tracts to go unsold. Even if $F(\cdot) = \hat{F}_U(\cdot)$, a reserve price nearly twice the actual average would be required to drive the probability that a tract will go unsold past 15 percent—a key threshold given a Forest Service policy of ensuring that at least 85 percent of all offered timber volume is actually sold (U.S. Forest Service 1992).

The potential gains in profit from raising reserve prices obviously depend on v_0. With $v_0 = \$20$, for example, we estimate that gains would be less than 10 percent (and not necessarily positive) even when $F(\cdot) = F_L(\cdot)$.[28] With $v_0 = \$80$, however, the potential gains are much larger. In that case, the Forest Service might achieve net gains of \$10 per MBF or more, which would represent more than an 80 percent increase in profits. With opportunity costs above the average gross revenues of \$92.08 per MBF, sales typically lead to a net loss. Hence, for costs of \$100 or \$120, substantial gains (reductions in losses) from im-

[28] Note that, in general, revenues need not be higher with a given reserve price between p_L and p_U given one particular CDF between $F_L(\cdot)$ and $F_U(\cdot)$. However, if $\Delta = 0$ or if Myerson's regularity condition is assumed, then lemma 4 implies that we can rule out the optimality of reserve prices that yield a (statistically significant) reduction in expected revenues when $F(\cdot) = F_L(\cdot)$ is assumed. This follows from the fact that a rightward shift in $F(\cdot)$ raises expected revenues at any reserve price. In our simulations, reductions in expected revenues appear for a few reserve prices, but only when $F(\cdot) = F_U(\cdot)$ is assumed.

TABLE 4
SIMULATED OUTCOMES WITH ALTERNATIVE RESERVE PRICES

	Reserve Price					
	p_L		$(p_L + p_U)/2$		p_U	
	Distribution of Valuations					
	F_L	F_U	F_L	F_U	F_L	F_U
Reserve price when v_0 = \$20	62.40		86.02		109.65	
Change in profit	6.96	−2.78	6.67	−2.74	1.74	−18.57
Pr(no bids)	.00	.02	.07	.12	.19	.41
Reserve price when v_0 = \$40	74.93		92.29		109.65	
Change in profit	7.64	−.61	7.61	−1.14	6.30	−10.04
Pr(no bids)	.03	.05	.11	.18	.19	.41
Reserve price when v_0 = \$60	85.67		103.39		121.11	
Change in profit	9.23	1.92	12.04	3.14	7.21	−6.05
Pr(no bids)	.07	.12	.15	.28	.35	.58
Reserve price when v_0 = \$80	98.20		112.34		126.48	
Change in profit	13.65	7.63	15.03	6.82	10.44	.96
Pr(no bids)	.13	.24	.28	.46	.46	.72
Reserve price when v_0 = \$100	111.09		122.54		134.00	
Change in profit	20.09	15.94	21.65	16.87	17.00	14.30
Pr(no bids)	.28	.45	.45	.60	.67	.80
Reserve price when v_0 = \$120	144.74		156.01		167.29	
Change in profit	32.06	31.31	33.72	31.64	31.56	28.87
Pr(no bids)	.84	.86	.84	.89	.88	.97

NOTE.—Profit and reserve price figures are given in 1983 dollars per MBF. See text for additional details.

posing higher reserve prices would be obtained by selling only tracts receiving unusually high bids. While revenue maximization is not the objective of the Forest Service timber sales program, these estimates suggest the magnitudes of revenues and costs that must be weighed against other objectives in determining optimal policy.

To evaluate the effects of auction observables on bidder valuations, we estimate the simple semiparametric model

$$v_{it} = \mathbf{X}_t \boldsymbol{\beta} + \epsilon_{it}$$

assuming $\text{med}[\epsilon_{it} \mid X_{it}] = 0$. Table 5 presents estimated bounds on the parameter vector $\boldsymbol{\beta}$. Following Manski and Tamer (2002), we construct confidence intervals using the bootstrap. Since zero lies outside the 95 percent confidence interval for each coefficient, we can reject the hypothesis that any of these conditioning variables has no effect on valuations. The implied signs are as expected: larger inventories, higher harvesting costs, or higher manufacturing costs reduce valuations. Greater species concentration and higher selling value of end products

9 Extensions to the basic framework

There are many extensions to this basic framework. I want to deal with the two that seem to me to be crucial to pretty much any empirical investigation you might wish to conduct. Mainly I discuss the FPSB auction.

These are:

- Auction Heterogeneity (both observed and unobserved)
- Bidder Heterogeneity (both observed and unobserved)

9.1 Observed Auction Heterogeneity - FPSB Auctions

- Basically the news here is good: All the identification results we have used before go through.
- Here I show how to handle observed heterogeneity in empirical implementation
- Let $\mathbf{Z}$ be a vector of auction covariates
  * Now the variables we have been playing with become:
  * $\beta_i\left(\cdot;\mathcal{N},\mathbf{Z}\right), F_{\mathbf{U}}\left(\cdot\,|\mathbf{Z}\right), G_{M_i|B_i}\left(b \mid b;\mathcal{N},\mathbf{Z}\right), g_{M_i|B_i}\left(b \mid b;\mathcal{N},\mathbf{Z}\right)$
  * and the bidding function we use for estimation changes accordingly
- One approach to the previous estimation is to use standard kernel smoothing over covariates
  * This can also be used in the ascending auction application (see Haile and Tamer)
  * However as is often the case with kernels this approach is vulnerable to curse of dimensionality problems
- An alternative suggested by Haile, Hong and Shum (2003) [and applied in Krasnokutskaya (2004), Bajari and Tadelis (2004), and Shneyerov (2005)] is as follows:
- We exploit the fact that additive separability is preserved by equilibrium bidding
- Let

$$u_i = \Gamma\left(z_t\right) + a_{it}$$

where a_{it} is the bidders private information. (note multiplicative separability has also been explored)

– Let a normalization exist such that

$$\Gamma\left(z_0\right) = 0$$

– so now (this is shown in HHS):

$$\begin{aligned} \beta_i\left(u_i;\mathcal{N},\mathbf{z}\right) &= \Gamma\left(\mathbf{z}\right) + \beta_i\left(u_i;\mathcal{N},\mathbf{z}_0\right) \\ &= \beta\left(\Gamma\left(\mathbf{z}\right) + a_{it};\mathcal{N}\right) \end{aligned} \tag{6}$$

– Now we can write the inverse bid function as

$$a_{it} + \Gamma\left(\mathbf{z}_t\right) = \beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right) + \frac{G_{M_i|B_i}\left(\beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right) \mid \beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right);\mathcal{N}\right)}{g_{M_i|B_i}\left(\beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right) \mid \beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right);\mathcal{N}\right)} \tag{7}$$

– Now we have to note a few things which are due to the additive separability of the last few equations

  * the events $\{\beta_i\left(\Gamma\left(\mathbf{z}\right) + A_i;\mathcal{N}\right) = \beta_i\left(\Gamma\left(\mathbf{z}\right) + a_i;\mathcal{N}\right)\}$ and $\{\beta_i\left(\Gamma\left(\mathbf{z}_0\right) + A_i;\mathcal{N}\right) = \beta_i\left(\Gamma\left(\mathbf{z}_0\right) + a_i;\mathcal{N}\right)\}$ are equivalent for any $\mathbf{z}$.
  * the events $\{\beta_j\left(\Gamma\left(\mathbf{z}\right) + A_j;\mathcal{N}\right) = \beta_i\left(\Gamma\left(\mathbf{z}\right) + a_i;\mathcal{N}\right)\}$ and $\{\beta_j\left(\Gamma\left(\mathbf{z}_0\right) + A_j;\mathcal{N}\right) = \beta_i\left(\Gamma\left(\mathbf{z}_0\right) + a_i;\mathcal{N}\right)\}$ are also equivalent for any $\mathbf{z}$ and $j \neq i$.
  * it follows that the expression

$$\frac{G_{M_i|B_i}\left(\beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right) \mid \beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right);\mathcal{N}\right)}{g_{M_i|B_i}\left(\beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right) \mid \beta_i\left(\Gamma\left(\mathbf{z}_t\right) + a_{it};\mathcal{N}\right);\mathcal{N}\right)}$$

    is invariant to $\mathbf{z}_t$.

– The upshot is that (6) implies (7) holds for all $\mathbf{z}_t$ whenever it is for $\mathbf{z}_t = \mathbf{z}_0$

– The whole point of this is that auction heterogeneity can now be controlled for by what amounts to a hedonic regression

  * Let

$$b_{it} = \alpha\left(\mathcal{N}\right) + \Gamma\left(\mathbf{z}_t\right) + \varepsilon_{it}$$

  * This is estimated using standard regression techniques.
  * From this little regression we get a homogenized bid

$$b_{it}^h = b_{it} - \widehat{\Gamma}\left(\mathbf{z}_t\right)$$

  * Then we run through the by now usual approach.

– The only technical point to note is that equilibrium bidding implies that the distribution of the sampling error, ε_{it}, should vary with $\mathcal{N}$. Hence the final stage, where the distribution of the private information is estimated, should be done separately for each $\mathcal{N}$

9.2 Unobserved Auction Heterogeneity

- The problems here are that things which are observed by all the bidders but not by the econometrician that affects valuations, and these things vary across auctions.
- There are at least three issues here:

1. (a) whether this auction heterogeneity is empirically distinguishable from other assumptions about the private information (i.e. is and IPV auction with unobserved heterogeneity distinguishable from an APV auction?)
   (b) is the distribution of the private information identified (assuming you know the dgp)?
   (c) is the identification adequate? That is, can we use the inference to answer useful questions if we don't see the unobserved stuff. This will depend on the project you have in mind.

- This is an area where more applied econometric work would be useful.

9.2.1 Dealing with it in a FPSB Auction

- currently the state-of-the-art for dealing with this unobserved auction heterogeneity is Krasnokutskaya (2011) which is an application of Li and Vuong (1998).
- She does something similar to a random effects estimator familiar in panel data, but in a non-parametric context. When we discuss the Asker 2010 I will show you the mechanics of this.
- I leave you to go through it if you are interested. However, the key thing to note is that identification of $F_{\mathbf{U}}(\cdot|\mathbf{Z})$ is now possible only up to a locational normalization. So you can work out the shape of the distribution but not where it sits.
- So in dealing with unobserved heterogeneity we put a constraint on the applicability of the methods. There have been other recent approaches, some which use a random effects estimator as a parametric version of the non-parametric deconvolution estimator (e.g. Bajari Houghton Tadelis forthcoming in the AER). Also there are some recent new identification results that may be helpful, but I am not aware if they have been implemented (see work by David McAdams and co-authors).

9.3 Bidder Heterogeneity

- As far as I can tell people have very little idea of how to handle bidder heterogeneity that is observed by bidders but not the econometrician. If we always knew what we were doing, it wouldn't called research...

9.4 Endogenizing entry

- This is the focus of Krasnokutskaya and Seim (AER, 2011“Bid Preference Programs and Participation in Highway Procurement Auctions”) and Roberts and Sweeting (AER, 2013 “When Should Sellers Use Auctions”)
- This is first order important when considering the effect of policy changes
- Also has seemed important to me in thinking about collusion (more later)