Two-Period (Static) Entry/Exit Models^{*}

1 Introduction: Why do we study entry models?

When we examine questions such as the determination of prices, we need to that market structure itself is a decision of firms, and hence is itself endogenous. Firms will enter or exit a market based on their perceptions of current and future market conditions, which may induce selection on the types of firms that are present in an industry.

Some reasons to think carefully about entry and exit include understanding:

- Market Structure and Prices. A interesting case of accounting for the endogeneity of market structure is the analysis of the Office Depot and Staples merger. When you regress prices on the presence of either Office Depot, Staples or both firms being present in the market, you find that prices are lower in markets in which both of these firms are present. However, this might just be because marginal costs are lower in some markets than others, and thus firms will enter into these markets with lower marginal costs.
- Is there too much or too little entry? Sometimes factors such as barriers to entry limit the number of firms in a market. One the other hand, the number of entrants might be above what is socially optimal, as discussed by Mankiw and Whinston RAND 1986 (later). When I enter, I don't take into account the fact that by entering I lower the profits of my rival, hence there is a business-stealing externality to entry.
- Innovation: Which types of products do firms choose to develop. For instance, will product differentiation induce Microsoft and Nintendo to produce similar or different gaming platforms?
- Endogeneity of Product Characteristics: When BLP look at demand for automobiles, they take product characteristics as exogenous. What happens if we try to figure out which type of cars lead to the highest profits for manufacturers? (Take the example of the minivan: it filled a gap between getting a van and a large sedan and yielded huge profits for Chrysler)
- Auctions: how much competition will there be for a good? In the German spectrum auction, the most important factor behind the very high prices was the fact that there were more large wireless firms interested in getting a license than licenses available.

Sutton Example (a digression): Often you will see people assume that something like the Herfindahl appropriately captures the concept of a more competitive market. John Sutton has an interesting example of why more concentrated markets are not necessarily more competitive.

^{*}Parts of these notes borrow generously from notes shared by Allan Collard-Wexler and Robin Lee.

Suppose that demand is characterized by a Cournot model of competition, where demand is:

$$P = a - bQ \tag{1}$$

and assume for that marginal costs are $c(q) = c \cdot q$. The fixed cost of entering a market is F. A firm's profits as a function of the number of competitors is:

$$\pi = \left(\frac{(a-c)}{(N+1)}\right)^2 \frac{1}{b} - F \tag{2}$$

Taking logs of this expression (for variable profits) we get:

$$\log(\pi) = -2\log(N+1) + 2\log(a-c) - \log(b)$$
(3)

We will use this expression to justify some of the functional forms used later on as being additively separable in the number of firms and other parameters, and the log form often used in these models.

So the number of firms in the market will be determined by the free-entry condition, i.e.: $\max Ns.t. \left(\frac{(a-c)}{(N+1)}\right)^2 \frac{1}{b} - F > 0.$

Now think of the Bertrand model of competition, since the price is set to marginal cost for any number of firms (p=c), then firms will never be able to cover their fixed costs of entry if they have a competitor. Thus the Bertrand model predicts either 0 or 1 firms in a market. This means that a Bertrand competitive market always has fewer firms than a Cournot market, while the Herfindahl would say that the Bertrand market has little competition while the Cournot market is more competitive. However, this result is only because of the entry process, not the toughness of product market competition.

2 On Two-Period Models

We will begin with static entry/exit models. Generally, these models abstract from many dynamic considerations (e.g., a past history of play or continuation values following the second period), but are useful in their own right; they provide insights and ways of organizing the data, as well as providing a foundation for more complicated dynamic analysis.

In general, the firm's entry problem is inherently dynamic: I enter if the continuation value V(x) (a function of some set of "state variables" x):

$$V(x^{0}) = E \sum_{t=0}^{\infty} \beta^{t} \pi(x^{t}) \Pr[x^{t} | x^{0}]$$
(4)

is greater than the entry cost, i.e. $V(x) \ge \phi$ (Entry Cost).

However these problems are quite difficult, so let's look at the case where an industry is in "equilibrium", i.e. the state at which you entered and today are roughly the same. This could be very misleading for non-stationary environments: e.g., for internet businesses, a large portion of the value of entry is not current profits but the fact that market size is assumed to be increasing.

Thus, we use a simple two-stage extensive form to start where typically there is a first period in which potential competitors determine actions such as whether or not to enter (or in some cases, investment levels); in the second period, competition takes place (typically in prices or quantities).



Figure 1: Relationship between observed entry patterns and parameter configuration.

2.1 Another Digression: Multiple Equilibria

Note that there are *multiple equilibria* in these games. For instance, suppose firms payoffs are the following:

Thus the only Nash Equilibria in this game involve firm 1 entering and firm 2 staying out, or firm 2 entering and firm 1 staying out. Thus the equilibrium is not pinned down. This is a problem for many estimation techniques since the same outcome of the game could have been generated by two sets of parameters if two different equilibria of the game were being played.

Look at the following example:

Firm 1
 Enter

 Firm 2
 Out

$$0,0$$
 X_1 ,0

 Enter
 $0,X_2$
 $X_1 - 10, X_2 - 10$

The goal is to estimate X_1 and X_2 , so which combinations of $\{X_1, X_2\}$ lead to different patterns of entry by firm 1 and 2? The following diagram shows how different entry patterns could have been caused by different parameter configurations.

The problem with this setup is that if $\{X_1, X_2\} \in [0, 10] \times [0, 10]$ then it is impossible to predict which entry pattern would be observed. This is a problem for most estimation techniques which try to relate an observed outcome Y to covariate data X and parameters θ . For instance, suppose we are estimating the parameters of the entry game via Maximum Likelihood. There are unobservables ϵ_1 and ϵ_2 which are added to the payoffs X_1 and X_2 . Suppose that ϵ_1 and ϵ_2 are distributed as



Figure 2: Relationship between observed entry patterns and parameter configuration given the assumption that the most profitable firm moves first.

independent normally distributed variables N(0, 1). Then the probability of observing the following outcomes is:

$$\begin{aligned} \Pr[\text{firm 1 enters, firm 2 enters}] &= \int_{\epsilon_1} \int_{\epsilon_2} 1(X_1 + \epsilon_1 > 10, X_2 + \epsilon_2 > 10) dF(\epsilon_1, \epsilon_2) \\ \Pr[\text{firm 1 out, firm 2 out}] &= \int_{\epsilon_1} \int_{\epsilon_2} 1(X_1 + \epsilon_1 < 0, X_2 + \epsilon_2 < 0) dF(\epsilon_1, \epsilon_2) \\ \Pr[\text{firm 1 enters, firm 2 out}] &= ? \\ \Pr[\text{firm 1 out, firm 2 enters}] &= ? \end{aligned}$$

The problem is that it is impossible to compute the probability of observing either firm 1 or firm 2 entering by itself. All we know is that this probability is greater than zero and smaller than $1 - \Pr[\text{firm 1 enters}, \text{firm 2 enters}] - \Pr[\text{firm 1 out}, \text{firm 2 out}].$

There are a number of approaches to fixing this problem of multiplicity:

1. Refining the set of equilibria, i.e. picking out an equilibrium which seems more "plausible". In particular, suppose it is the case that the most profitable firm is always the first firm to enter. This leads to the following relationship between parameters and entry: This leads to firm 1 entering alone if and only if $(X_1 \in [0, 10] \text{ and } X_2 < 0)$ or $(X_1 > 10 \text{ and } X_2 < 10)$ or $(X_2 \in [0, 10] \text{ and } X_1 > X_2 \text{ and } X_1 \in [0, 10])$.

Likewise firm 2 enters alone if and only if $(X_2 \in [0, 10] \text{ and } X_1 < 0)$ or $(X_2 > 10 \text{ and } X_1 < 10)$ or $(X_1 \in [0, 10] \text{ and } X_2 > X_1 \text{ and } X_2 \in [0, 10])$. Both firms enter if and only if $X_1 > 10$ and $X_2 > 10$, while neither firm enters if and only if $X_1 < 0$ and $X_2 < 0$.

Thus the equations used for Maximum likelihood are:

$$\begin{aligned} \Pr[\text{firm 1 enters, firm 2 enters}] &= \int_{\epsilon_1} \int_{\epsilon_2} 1(X_1 + \epsilon_1 > 10, X_2 + \epsilon_2 > 10) dF(\epsilon_1, \epsilon_2) \\ \Pr[\text{firm 1 out, firm 2 out}] &= \int_{\epsilon_1} \int_{\epsilon_2} 1(X_1 + \epsilon_1 < 0, X_2 + \epsilon_2 < 0) dF(\epsilon_1, \epsilon_2) \\ \Pr[\text{firm 1 enters, firm 2 out}] &= \int_{\epsilon_1} \int_{\epsilon_2} 1(X_1 + \epsilon_1 > 0, X_1 + \epsilon_1 > X_2 + \epsilon_2, \\ X_2 + \epsilon_2 < 10) dF(\epsilon_1, \epsilon_2) \\ \Pr[\text{firm 1 out, firm 2 enters}] &= \int_{\epsilon_1} \int_{\epsilon_2} 1(X_2 + \epsilon_2 > 0, X_2 + \epsilon_2 > X_1 + \epsilon_1, \\ X_1 + \epsilon_1 < 10) dF(\epsilon_1, \epsilon_2) \end{aligned}$$

2. Partial Identification: finding implications of the model that could have been generated by some of the equilibria. With enough variation in the data it is possible to identify X_1 and X_2 up to an interval without making any assumptions on equilibrium selection. For instance, I can bound the probability that firm 1 enters by making the appropriate assumptions about the order of entry. So the probability that firm 1 enters is bounded from below by: "I only observe firm 1 entering when firm 2 has already decided not to enter *or* firm 1 enters when it is always profitable for it to enter, and firm 2 also finds it profitable to enter (but not always)":

$$\begin{aligned} \Pr[\text{firm 1 enters, firm 2 out}] \geq & \int_{\epsilon_1} \int_{\epsilon_2} \left[1(X_1 + \epsilon_1 > 0, X_2 + \epsilon_2 < 0) \right. \\ & \left. + 1(X_1 + \epsilon_1 > 10, X_2 + \epsilon_2 \in (0, 10)) \right] dF(\epsilon_1, \epsilon_2) \end{aligned}$$

Likewise the probability of firm 1 entering is bounded from above by the assumption that firm 1 always moves first:

$$\Pr[\text{firm 1 enters, firm 2 out}] \le \int_{\epsilon_1} \int_{\epsilon_2} 1(X_1 + \epsilon_1 > 0, X_2 + \epsilon_2 < 10) dF(\epsilon_1, \epsilon_2)$$

I can represent these inequality constraints in Figure ??.

These inequality constraints can be used to estimate a model of entry.

- 3. Looking a the number of firms that enter, a feature which is pinned down across different equilibria. Suppose that all firms are identical, and thus look at the case where $X_1 = X_2$ and where $\epsilon_1 = \epsilon_2 = \epsilon$. There are still many different equilibria in this simplified model. However, the number of firms in the market is pinned down: there are no firms in the market if X < 0, there is one firm in the market if $X \in [0, 1]$ and there are two firms in the market if X > 10. This allows ? to estimate their model of entry.
- 4. Model of equilibrium selection: Following the ideas of Bajari, Hong and Ryan (2006) we can think of an equilibrium selection equation where the probability of equilibrium κ is determined by:

$$\Pr[\kappa_j|X] = \frac{\exp(X_j\beta)}{\sum_{\kappa_k} \exp(X_k\beta)}$$
(5)



Figure 3: Bounds for observed entry patterns and parameter configuration.

where X_j are characteristics of the equilibrium and the market which might affect which equilibrium gets played. For instance, I might believe that the fact that I played an equilibrium in the last period makes it more likely that this equilibrium gets played in the current period. Alternatively, maybe I am more likely to play a Pareto better equilibria if there are fewer firms in the market (for example because we can communicate better). In any case, whichever covariates you think are important for equilibrium selection can be included into the vector of X's. This can then be folded into a likelihood.

This means we can estimate the selection equation and the parameters of firm profits at the same time via maximum likelihood.

3 Homogeneous Firms

Many of the earliest models of entry and exit focused on homogeneous firms with identical fixed entry costs. They can be seen as providing an intuition for what can happen when accounting for endogeneous entry, and a means of understanding some of the cross-sectional differences in the number and types of firms ("market structure") within an industry across markets.

3.1 Mankiw Whinston 1986

This is a classic paper which uses a simple two-period entry model to explore conditions under which free entry may lead to a socially inefficient and excessive number of firms present in a market. "If an entrant causes incumbent firms to reduce output, entry is more desirable to the entrant than it is to society. There is therefore a tendency toward excessive entry in homogeneous product markets."

In the first stage, there is a large number of identical entrants that must pay a fixed cost K to enter. In the second stage, firms compete given identical cost functions c(q). Look at symmetric equilibria where each firm produces q_N . Free entry implies that $\pi_N > 0 > \pi_{N+1}$ where π_N are the firm profits (including fixed costs) if N firms enter the market.

Consider aggregate inverse demand P(Q), and let $\pi_N = P(Nq_N)q_N - c(q_N) - K$. Define the socially optimal number of firms N^* as:

$$N^* = \arg\max_N W(N) \equiv \arg\max_N \int_0^{Nq_N} P(s)ds - Nc(q_N) - NK$$

Ignore for now the integer constraint so that the competitive equilibrium under free entry yields N^e entrances, where $\pi_{N^e} = 0$. The socially optimal number of firms satisfies $W'(N^*) = 0$, which can be rewritten as:

$$0 = \pi_N + N[P(Nq_N) - c'(q_N)]\frac{\partial q_N}{\partial N}$$
(6)

Assume that $Nq_N > N'q_{N'} \forall N > N'$ (aggregate output rises as more firms enter), $\lim_{N\to\infty} Nq_N = M < \infty$ (total output approaches some finite bound as more firms enter), $q_N < q_{N'} \forall N > N'$ (individual output declines with entry), and $P(Nq_N) - c'(q_N) \ge 0 \forall N$ (prices do not fall below MC). Then the second term of (??) is non-positive, which implies that $W'(N) \le \pi_N$. And thus, at the socially efficient level of entry, $\pi_{N^*} \ge 0$ and there will be an incentive for another firm to enter the market. It thus can be shown that $N^e \ge N^*$, and there will be socially inefficient entry. The second term is the "business-stealing" effect.

3.2 Bresnahan-Reiss 1991

The Bresnahan-Reiss model was originally used to try to make inference on the nature of competition in settings where there is no cost or demand data. Bresnahan and Reiss (BR) look at the increase in the number of firms in a market as market size increases. The pattern of entry should tell us about how markups decrease as market size increases. BR look at entry patterns for various professionals (dentists, tire dealers, car dealers, and plumbers) across geographically differentiated "Isolated Markets," i.e. towns which are located far away from other towns in the US.

Q: Can entry data (alone!) tell us something about markups and competition?





Figure 4: Location of Automobile Repair Shops

There are two behavioral assumptions:

1. Firms that Enter make Positive Profits

$$\pi(N, X_m) + \varepsilon_m > 0 \tag{7}$$

2. If an extra firm entered it would make negative profits:

$$\pi(N+1, X_m) + \varepsilon_m < 0 \tag{8}$$

where $\pi(N, X_m)$ is the observable component of profit depending on demand factors X_m and the number of symmetric competitors in a market N, while ε_m are unobserved components of profitability common to all firms in a market.

Assume that market level shocks ε_m have a normal distribution with zero mean and unit variance. The probability of observing a market X_m with N plants is the following:

$$\Pr(N = n | X_m) = \Phi[-\pi(n+1, X_m)] - \Phi[-\pi(n, X_m)] 1(n > 0)$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal. I parameterize the profit function as $\pi(\theta, N, X_m)$. Parameters can be estimated via Maximum Likelihood, where the likelihood is the following:

$$\mathcal{L}(\theta) = \prod_{m=1}^{M} \Pr(N_m = n | X_m \theta)$$
(9)

Firms make sunk, unrecoverable investments when they enter a market. The decision of an incumbent firm to remain in a market differs from the decision of an entrant to build a new plant. The next series of models deal with this difference.

Authors parameterize $\pi(N, X_m) = S(\boldsymbol{Y}, \lambda)V_N(\boldsymbol{Z}, \boldsymbol{W}, \alpha, \beta) - F_N(\boldsymbol{W}, \gamma)$ where \boldsymbol{Y} describes market size, \boldsymbol{Z} and \boldsymbol{W} shift per capita demand and costs.

	Number of Firms							
Industry	N = 0	N = 1	N = 2	N = 3	N = 4	N = 5	N = 6	$N \ge 7$
Druggists	28	62	68	23	8	6	3	4
Doctors	37	61	36	16	11	7	6	28
Dentists	32	67	39	15	12	12	4	21
Plumbers	71	47	26	21	10	4	6	17
Tire dealers	45	39	39	24	13	15	6	21
Barbers	95	66	23	9	3	6	0	0
Opticians	173	19	5	1	4	0	0	0
Beauticians	10	26	19	24	26	19	11	67
Optometrists	68	85	36	7	3	3	0	0
Electricians	60	54	32	17	10	5	7	17
Veterinarians	53	80	41	21	5	0	1	1
Movie theaters	90	72	25	10	5	0	0	0
Automobile dealers	38	44	54	35	25	2	1	3
Heating contractors	117	40	19	8	4	8	3	3
Cooling contractors	153	30	13	5	1	0	0	0
Farm equipment dealers	90	39	23	19	17	9	1	4

MARKET COUNTS BY INDUSTRY AND NUMBER OF INCUMBENTS

SOURCE.—Authors' tabulations from American Business Lists, Inc.



Figure 5: Dentists by town population. Top figure: distribution of towns (by population) than have 0, 1, or 2 dentists. Bottom: markets with 3, 4, or 5 dentists. Suggests need around 500 people for 1 dentist, and 1000-2000 for 2 dentists.

Variable	Name	Mean	Standard	Min	Max
	INAIIIC	Mean	Deviation	WIII	Max
Firm counts:					
Doctors	DOCS	3.4	5.4	.0	45.0
Dentists	DENTS	2.6	3.1	.0	17.0
Druggists	DRUG	1.9	1.5	.0	11.0
Plumbers	PLUM	2.2	3.3	.0	25.0
Tire dealers	TIRE	2.6	2.6	.0	13.0
Population variables (in					
thousands):					
Town population	TPOP	3.74	5.35	.12	45.09
Negative TPOP growth	NGRW	06	.14	-1.34	.00
Positive TPOP growth	PGRW	.49	1.05	.00	7.23
Commuters out of the					
county	OCTY	.32	.69	.00	8.39
Nearby population	OPOP	.41	.74	.01	5.84
Demographic variables:					
Birth ÷ county population	BIRTHS	.02	.01	.01	.04
65 years and older ÷					
county population	ELD	.13	.05	.03	.30
Per capita income					
(\$1,000's)	PINC	5.91	1.13	3.16	10.50
Log of heating degree					
days	LNHDD	8.59	.47	6.83	9.20
Housing units ÷ county					
population	HUNIT	.46	.11	.29	1.40
Fraction of land in farms	FFRAC	.67	.35	.00	1.27
Value per acre of farm-					
land and buildings					
(\$1,000's)	LANDV	.30	.23	.07	1.64
Median value of owner-					
occupied houses					
(\$1,000's)	HVAL	32.91	14.29	9.90	106.0
Commuters out of the county Nearby population Demographic variables: Birth ÷ county population 65 years and older ÷ county population Per capita income (\$1,000's) Log of heating degree days Housing units ÷ county population Fraction of land in farms Value per acre of farm- land and buildings (\$1,000's) Median value of owner- occupied houses (\$1,000's)	OCTY OPOP BIRTHS ELD PINC LNHDD HUNIT FFRAC LANDV HVAL	.32 .41 .02 .13 5.91 8.59 .46 .67 .30 32.91	.69 .74 .01 .05 1.13 .47 .11 .35 .23 14.29	.00 .01 .01 .03 3.16 6.83 .29 .00 .07 9.90	8.5 5.8 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0

SAMPLE MARKET DESCRIPTIVE STATISTICS

SOURCE —Firm counts¹ American Business Lists, Inc.; population variables: U.S. Bureau of the Census (1983) and *Rand McNally Commercial Atlas and Marketing Guide* (annual); demographic variables: U.S. Bureau of the Census (1983).

Figure 6: Variables conditioned on in X_m

Variable					Tire
Name	Doctors	Dentists	Druggists	Plumbers	Dealers
OPOP (λ_1)	1.15	46	.08	.27	53
	(.85)	(.32)	(.37)	(.60)	(.43)
NGRW (λ_2)	-1.89	.63	30	.68	2.25
	(1.60)	(.85)	(.97)	(1.10)	(.75)
PGRW (λ_3)	1.92	35	24	45	.34
	(1.01)	(.41)	(.41)	(.36)	(.59)
OCTY (λ_4)	.80	2.72	.16	28	.23
	(1.26)	(.98)	(.34)	(.71)	(.94)
BIRTHS (β_1)	59	9.86	11.34		
	(6.57)	(8.29)	(10.10)		
ELD (β_2)	11	.22	2.61		49
	(.55)	(.74)	(.78)		(.75)
PINC (β_3)	00	.04	.02	.05	03
	(.00)	(.03)	(.02)	(.03)	(.04)
LNHDD (β_4)	.013	.28	.08	.003	.004
	(.05)	(.07)	(.06)	(.06)	(.06)
HUNIT (β_5)				.51	
				(.46)	
HVAL (β_6)				.42	
				(.03)	
FFRAC (β_7)					02
$\mathbf{V}_{(\alpha)}$	69	1.05	10	0.0	(.08)
$V_1(\alpha_1)$.03	-1.85	13	.06	.86
$V = V (\alpha)$	(.40)	(.01)	(.58)	(.52)	(.45)
$v_1 - v_2 (\alpha_2)$.34		.29		.03
$V = V (\alpha)$	(.17)	19	(.21)	15	(.15)
$v_2 - v_3 (a_3)$.12	.19	.15	.15
$V = V (\alpha)$	07	(.14)	(.17)	(.09)	(.10)
$v_3 v_4 (u_4)$.07	.20	.25	.07	
$V_{\rm c} = V_{\rm c} (\alpha_{\rm c})$	(.05)	(.00)	(.14)	(.08)	0.0
$v_4 v_5 (a_5)$.04	.04	.08
$F_{i}(\gamma_{i})$	Q9	1 10	(.12)	(.07)	(.05)
	(30)	(95)	(90)	(26)	.00
$F_{\alpha} = F_{\alpha}(\gamma_{\alpha})$	(.50)	(.23)	(.25)	(.20)	(.25)
19 11(12)	(30)	(10)	(35)	(14)	.70
$F_{a} = F_{a}(\gamma_{a})$	(.30)	(.15)	(.33)	(.14)	(.21)
13 12(13)	(13)	(46)	(54)	(98)	.40
$F_{i} = F_{i}(\gamma_{i})$	18	(.10)	(.54)	(.28)	(.21)
1 4 1 3 (14)	(23)		(70)	(35)	.00
$F_r - F_r(\gamma_r)$	49	66	51	(.55)	(.12)
- 5 - 4 (15/	(13)	(60)	(95)	(35)	(20)
LANDV (γ_{ℓ})	-1.02	-131	- 84	(.33) -1.18	(.20)
	(.53)	(37)	(51)	(48)	74
Log likelihood	-233.49	- 183.20	- 195.16	- 228.27	-263.09

BASELINE SPECIFICATIONS

NOTE.—Asymptotic standard errors are in parentheses

Figure 7: λ are coefficients for market size variable. β are coefficients for variable costs. γ are coefficients for fixed costs. V_i and F_i represent variable and fixed costs given *i* firms active in market.

A. ENTRY THRESHOLD ESTIMATES

Profession	ENTRY THRESHOLDS (000's)					Per Firm Entry Threshold Ratios			
	<i>S</i> ₁	S ₂	S ₃	<i>S</i> ₄	S ₅	s_2/s_1	s ₃ /s ₂	s ₄ /s ₃	s5/s4
Doctors	.88	3.49	5.78	7.72	9.14	1.98	1.10	1.00	.95
Dentists	.71	2.54	4.18	5.43	6.41	1.78	.79	.97	.94
Druggists	.53	2.12	5.04	7.67	9.39	1.99	1.58	1.14	.98
Plumbers	1.43	3.02	4.53	6.20	7.47	1.06	1.00	1.02	.96
Tire dealers	.49	1.78	3.41	4.74	6.10	1.81	1.28	1.04	1.03

B. LIKELIHOOD RATIO TESTS FOR THRESHOLD PROPORTIONALITY

	Test for	Test for	Test for	Test for
Profession	$s_4 = s_5$	$s_3 = s_4 = s_5$	$s_2 = s_3 = s_4 = s_5$	$s_1 = s_2 = s_3 = s_4 = s_5$
Doctors	1.12 (1)	6.20 (3)	8.33 (4)	45.06* (6)
Dentists	1.59 (1)	12.30*(2)	19.13* (4)	36.67* (5)
Druggists	.43 (2)	7.13 (4)	65.28* (6)	113.92* (8)
Plumbers	1.99 (2)	4.01 (4)	12.07 (6)	15.62* (7)
Tire dealers	3.59 (2)	4.24 (3)	14.52* (5)	20.89* (7)

NOTE.-Estimates are based on the coefficient estimates in table 4. Numbers in parentheses in pt. B are degrees of freedom.

Figure 8: s_i is per-firm entry threshold for entry of *i*th firm. s_i/s_k is ratio of number of people needed to support *i*th firm vs. *k*th firm. E.g., for doctors, market needs to go from 1 to 4 in order to support 2 doctors ($s_2/s_1 = 1.98$), and go to 6.5 to support 3 doctors. note: $s_{n+1}/s_n = \{S_{n+1}/(N+1)\}/\{S_n/(N)\}$



Figure 9: Post entry competition increases at a rate that decreases with number of incumbents: most competition effects occur with entry of 2nd and 3rd firms. Once 3 firms are present, additional population to support additional firms remains almost the same.



Figure 10: Entry Threshold ψ and Exit Threshold ϕ based on static profits.

3.3 Bresnahan-Reiss Model of Exit (1994)

The BR94 model of exit distinguishes between two types of firms: firms which are already active and firms which are deciding to enter the market. Entrants and incumbents have the same profits, and hence the same continuation values. However, entrants always have lower values than incumbents, since they pay an entry cost that incumbents do not, as is shown by Figure ??. This implies that there cannot be simultaneous entry and exit: either firms exit, enter, or nothing happens. This is a feature of all models which do not have firm specific shocks and where firms are symmetric: they cannot rationalize the same type of plant in the same market making different choices. Thus market-years in which there is both entry and exit are dropped. With yearly data and markets with on average less than 3 incumbents there is very little simultaneous entry and exit, less than 5% of markets need to be dropped. Moreover, including these markets in the data does not significantly change estimated parameters. So the selection caused by this procedure does not seem to be of great import for this data. Three regimes need to be considered: *entry, exit and stasis*.

1. Net Entry: $N^t > N^{t-1}$

$$\begin{aligned} \pi(N^t, X^t_m) + \varepsilon^t_m &> \psi \\ \pi(N^t + 1, X^t_m) + \varepsilon^t_m &< \psi \end{aligned}$$

2. Net *Exit*: $N^t < N^{t-1}$

$$\begin{aligned} \pi(N^t, X^t_m) + \varepsilon^t_m &> \phi \\ \pi(N^t + 1, X^t_m) + \varepsilon^t_m &< \phi \end{aligned}$$

3. No Net Change: $N^t = N^{t-1}$

$$\begin{aligned} \pi(N^t, X^t_m) + \varepsilon^t_m &> \phi \\ \pi(N^t + 1, X^t_m) + \varepsilon^t_m &< \psi \end{aligned}$$

where ψ is the entry fee that an existing firm pays to enter the market and ϕ is the scrappage value of a firm. Entry fees and scrap value are not identified from fixed costs, since it is always possible to increase fixed costs and decrease entry/exit fees by the same amount without changing the likelihood of observing a particular market configuration. Yet, the *difference* between entry and exit fees is identified and can be compared to other quantities such as the effect of an extra competitor.

These equations can be combined into:

$$\pi(N^t, X_m^t) + \varepsilon_m^t > 1(N^t > N^{t-1})\psi + 1(N^t \le N^{t-1})\phi$$
(10)

$$\pi(N^t + 1, X_m^t) + \varepsilon_m^t < 1(N^t \ge N^{t-1})\psi + 1(N^t < N^{t-1})\phi$$
(11)

The probability of observing a market X_m with N^t plants today and N^{t-1} plants in the last period is:

$$\begin{aligned} \Pr(n^t = N^t, n^{t-1} = N^{t-1} | X_m^t) \\ &= \Phi[-\pi(n^t + 1, X_m^t) + 1(n^t + 1 \ge n^{t-1})\psi + 1(n^t + 1 < n^{t-1})\phi] \\ &- \Phi[-\pi(n^t, X_m^t) + 1(n^t > n^{t-1})\psi + 1(n^t \le n^{t-1})\phi]1(n^t > 0) \end{aligned}$$

which is used to form a maximum likelihood estimator.

4 Heterogeneous Firms

4.1 Berry 1992

The Berry'92 model allows for some degree of heterogeneity between firms entering the market, in particular differences in the fixed costs of entry.

The empirical point of this paper is in getting better estimates for the extent to which substantial presence at an airport makes it easier for an airline to enter routes to and from that airport. That is, does incumbency generate lower barriers to entry. When the US deregulated airline markets this was a significant issue in the extent to which the contestable markets hypothesis could be applied.

- Airline Markets: Questions about the nature of the barriers to entry for airlines: gates, slots, hub structures (frequent flyer points),
- Deregulation in the industry since the 1970s (before that routes and prices were heavily regulated), which has led to a enormous decrease in the prices and costs of airline travel.
- However, the main firms around since deregulation such as United and American Airlines are still in the market. Why have they not been displaced by Southwest Airlines (for instance), a lower cost and more profitable carrier?
- Very clear market definition: city pairs like Saint-Louis to Savannah.
- Terrific airline data: the origin and destination survey captures 10% of airline tickets from flights with a U.S. airport, both prices and quantity are available, but not the class of ticket (business or coach).
- 1. Profits: Profits in market i for firm k are given by:

$$\pi_{ik}(s) = v_i(N(s)) + \phi_{ik}$$

where s are strategies employed by each firm (no entry, entry).

Note that heterogeneity only enters into the entry cost (which is firm specific), not the profits given the type of entrants.

2. Entry:

Firms enter simultaneously.

Berry shows that the number of firms is unique, but not which firms will enter!

• order entry costs (so most profitable firm enters first):

$$\phi_{i1} > \phi_{i2} > \dots > \phi_{ik}$$

- Suppose firms with lowest entry costs 'enter first': The number of firms N_i is: $N_i = \max_n \text{ s.t. } v_i(n) + \phi_{in} \ge 0$, i.e. $v_i(n) + \phi_{in} \ge 0$ and $v_i(n+1) + \phi_{in+1} \le 0$
- The number is invariant across equilibria (a simple proof by contradiction argument).
- 3. Parametrize entry costs: Fixed costs are parametrized as:

$$\phi_{ik} = \alpha Z_{ik} (\text{covariates}) + \sigma u_{ik} (\text{shocks})$$

why could there be differences in entry cost: previous presence in the airport to negotiate better slot assignments, not the presence of a hub which affects competition between firms.

4. Parametrize firm profits:

$$\underbrace{v_i(N)}_{\text{common to all firms in the market}} = X_i\beta + \underbrace{h(\delta, N)}_{-\delta\ln(N)} + \rho \underbrace{u_{i0}}_{\text{market level unobservable}}$$

Note that all firm level idiosyncratic stuff gets captured in the entry costs. The total value of entering is just:

$$X_i\beta - \delta \ln(N) + \alpha Z_{ik} + \rho u_{i0} + \sigma u_{ik}$$

So we have two shocks here, and we can define the error term as:

$$\epsilon_{ik} = \rho u_{i0} + \sigma u_{ik}$$

Since we have a discrete choice problem, we need to normalize both the mean of the error term and it's variance. So set

$$\sigma = \sqrt{1 - \rho^2}$$

and let u_{i0} and u_{ik} be iid standard normal, so that ϵ_{ik} has unit variance.

5. Probability of N firms in the market? This is quite difficult since the probability of observing firm 1 enter which I will call $a_{i1} = 1$ depends on the entire vector of shocks $\vec{\epsilon} = \{\epsilon_{i1}, \epsilon_{i2}, \cdots, \epsilon_{ik}\}$ and the parameters θ :

$$\Pr[a_{i1}|\theta] = \int_{\epsilon_{i1}} \int_{\epsilon_{i2}} \cdots \int_{\epsilon_{ik}} 1(a_{i1} = 1|\theta, \vec{\epsilon}) df(\vec{\epsilon})$$

which is the area of the $\vec{\epsilon}$ space where firm 1 decides to enter.

To see the difficulty, assume that there are just 2 firms. Denote the set B_{kj} as the set of ϵ 's where 1 firm enters:

$$B_{kj} = \{\epsilon : \epsilon_k \ge -v(1)\&\epsilon_j < -v(2)\}$$

Thus the probability of observing only 1 firm enter is:

$$\Pr[N^* = 1] = \Pr(\epsilon \in B_{12}) + \Pr(\epsilon \in B_{21}) - \Pr(\epsilon \in B_{12} \int B_{21})$$

As the number of potential entrants grows, it will be very difficult to compute this integral analytically (because of the odd shape of the domain of integration, which can be quite difficult to describe), so we will do what we did with the random coefficient logit model: Integrate by SIMULATION.

Var	Est Parm (Std. Error)	Mean Value of Var. (Std. Dev.)
N		1.629 (1.393)
Const	-0.727	
	(0.097)	
Рор	2.729	0.558
	(0.255)	(0.114)
Dist	- 1.591	1.149
	(0.827)	(0.093)
Dist ²	0.337	0.022
	(1.850)	(0.039)
Tourist	0.134	0.116
	(0.089)	(0.320)
Citv N2	0.338	4.574
	(0.011)	(2.684)
City $N +$	0.084	10.377
• - · ·	(0.009)	(3.656)
	R-squared is: 0.612	2

TABLE IV Regression Results for Number of Firms

Figure 11: Reduced form relationship between number of firms on a given city pair and observables.

- 6. Algorithm
 - (a) Pick θ .
 - (b) Draw a vector of ϵ 's: $\{\epsilon_{ik}\}_{k=1}^{N}$ (Remember to keep these the same over the algorithm, just like in BLP.
 - (c) Find the fixed costs $\{\phi_{ik}\}_{k=1}^N$ and order them using an order of entry assumption.
 - (d) Add firms from $n = 1, \dots, N$ until:

$$v(N^*|\theta) + \phi_{iN} \ge 0$$
$$v(N^* + 1|\theta) + \phi_{iN+1} < 0$$

- (e) The predicted number of firms is $N^*(\theta, \bar{\epsilon}^d)$.
- (f) Compute the criterion function

$$\xi = \sum_{d} N^*(\theta, \vec{\epsilon}^d) - \hat{N}$$

(g) Finally you can estimate the model via GMM as usual, with the criterion function (and which ever instruments you want, OLS works fine as well):

$$Q(\theta) = (\xi \mathbf{Z})(\mathbf{Z}'\mathbf{Z})^{-1}(\xi \mathbf{Z})'$$

• <u>Note</u>: If we had tried to estimate this model using Maximum Likelihood, we often could encounter cases where the model has difficulty rationalizing certain outcomes such as 10 firms entering. This leads us to get zero probability events (at least as far as the computer can tell), which lead to the computer stalling out when it encounters the log of 0. In practice, you should include what I like to think of as probability dust, i.e. that the probability of any event is the max of a "dust" constant (like 10e-15) and what is predicted by the model.

- In contrast, GMM is both more stable computationally than Maximum Likelihood, and is weaker than ML since we are only assuming mean zero error term ξ , not a parametric distribution on ξ . Furthermore, a simulation estimator will be biased if the simulation error enters non-linearly.
- In practice you can choose the moments that you use in GMM so that your model fits any particular moment that you really care about.

Variable:	(1)	(2)	(3)
Const	- 3.53	-3.23	- 3.36
	(0.069)	(0.101)	(0.064)
Pop	1.21	0.18	1.21
	(0.082)	(0.100)	(0.082)
Dist	0.03	0.010	1.18
	(0.004)	(0.005)	(0.169)
Dist ²	-0.001	-0.001	
	(0.0001)	(0.001)	
Tourist	0.12	-0.071	
	(0.044)	(0.050)	
City2	2.05	1.83	2.06
	(0.054)	(0.054)	(0.054)
City share	5.58		5.47
	(0.164)		(0.162)
# Routes	_	0.732	
	<u> </u>	(0.018)	
– 2 log-likelihood:	6799	6220	6834

^aObservations are 18218 firm/market combinations. Standard errors are in parentheses.

Figure 12: Simple analysis of entry into city-pair markets, assuming profits are unaffected by number of other entrants.

Variable	Most Profitable Move First	Incumbents Move First
Constant	-5.32	- 3.20
	(0.354)	(0.258)
Population	1.36	5.28
	(0.239)	(0.343)
Dist	1.72	- 1.45
	(0.265)	(0.401)
City2	4.89	5.91
	(0.295)	(0.149)
City Share	4.73	5.41
	(0.449)	(0.206)
δ	0.527	4.90
	(0.119)	(0.206)
ρ	0.802	0.050
	(0.105)	(0.048)
Value of the objective fn:	33.3	26.2

SIMULATION ESTIMATES^a

^aObservations are 1219 markets. Standard errors are in parentheses.

Figure 13: Simulation Estimates

4.2 Mazzeo

The Mazzeo model extends the original BR model by allowing firms to chose which type of firms they enters as. In doing so, it sheds light on why the number of firms in a market affect entry thresholds. In many empirical applications, firms can differentiate between each other by choosing to enter in different areas of the product space, so for instance I might build a Chinese restaurant if you decide to build an Italian restaurant. In the specific application Mazzeo considers, firms can enter either as high or low quality motels, where we denote the firm's type as $\theta_i \in \{h, l\}$.

- We want to understand how firms decide the "location" of the products that they produce. This is driven in large part by the incentive to differentiate my product from those of my competitor.
- Allow for different types of producers to compete with each other: not just a homogenous good.
- Mazzeo chooses the Motel Industry: High and Low quality motels, and a clearly defined market: exit on a highway.

A firm's profit function depends on total demand in the market denoted as X and the number of firms that choose to enter as either high or low quality hotels:

$$\pi_{\theta_i}(N_l, N_h, X) = X\beta_{\theta_i} - g_{\theta_i}(N_l, N_h) + \epsilon_i \tag{12}$$

with the addition of a market/type unobservable to profits denoted ϵ_{θ_i} which is common to all firms in a market which are of the same type. Note that this ϵ_{θ_i} should be correlated within the market across firms of the same type.

<u>Note</u>: Note that firms are identical except for their type. So they get the same shocks (which means we don't have to specify the number of potential entrants in a market).

Mazzeo assumes the game is Stackelberg: i.e., firms play sequentially and make irrevocable decisions about entry and product type. The equilibrium conditions in this market are:

1. Firms that are in the market make positive profits:

$$\pi_{\theta_h}(N_l, N_h, X) + \epsilon_{\theta_h} > 0 \tag{13}$$

$$\pi_{\theta_l}(N_l, N_h, X) + \epsilon_{\theta_l} > 0 \tag{14}$$

(15)

2. If an additional firm entered, it would make negative profits:

$$\pi_{\theta_h}(N_l, N_h + 1, X) + \epsilon_{\theta_h} < 0 \tag{16}$$

- $\pi_{\theta_l}(N_l+1, N_h, X) + \epsilon_{\theta_l} < 0 \tag{17}$
 - (18)

3. Firms do not want to switch the decision that they made in terms of product type:

$$\pi_{\theta_h}(N_l, N_h - 1, X) > \pi_{\theta_l}(N_l, N_h - 1, X)$$
(19)

$$\pi_{\theta_l}(N_l - 1, N_h, X) > \pi_{\theta_h}(N_l - 1, N_h, X)$$
(20)

(21)

4. Mazzeo also assumes that additional market participant always decreases profits, and it is larger if the market participant is of the same product type; this guarantees existence of equilibrium.

Estimation: The model is estimated via simulated maximum likelihood:

$$\mathcal{L}(\theta) = \prod_{m} \Pr[N_l, N_h | X_m, \theta] = \int_{\epsilon_l} \int_{\epsilon_h} 1(\text{all equations satisfied}) d\epsilon_h d\epsilon_l$$

where we assume that the error term:

$$(\epsilon_h, \epsilon_l) \longrightarrow \mathcal{N}(0, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix})$$

Simulated Maximum Likelihood:

1. Draw:

$$(u_l^k, u_h^k) \longrightarrow \mathcal{N}(0, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix})$$

2. Transform these draws via the Cholesky decomposition:

$$(\epsilon_h, \epsilon_l) = chol(\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix})(u_l^k, u_h^k)$$

3. Get the number of times that the model gets the right answer:

$$\mathcal{L}^{S}(\theta) = \prod_{m} \frac{1}{\#K} \sum_{k} 1(\pi_{\theta_{h}}(N_{l}, N_{h}, X) + \epsilon_{\theta_{h}} > 0, \cdots, \\ \pi_{\theta_{l}}(N_{l} - 1, N_{h}, X) > \pi_{\theta_{h}}(N_{l} - 1, N_{h}, X))$$

4. Maximize $\mathcal{L}^{S}(\theta)$ over θ .

One of the issues with Mazzeo's estimator is that it is an accept-reject simulator, which is not smooth. Thus, if I change θ , my objective function is a step function which can be tough to maximize over. In contrast, BLP was set up to have a smooth simulator, i.e. very small changes in θ always lead to changes in the criterion function.

FIGURE 1 PARTITIONING FOR EQUILIBRIUM OUTCOMES



TABLE 4 Observed Market Configurations for the Two-Product-Type Models

Market Configuration	Number of Markets	Percent of Total (%)
(1, 0)	61	12.4
(0, 1)	67	13.6
(2, 0)	26	5.3
(1, 1)	40	8.1
(0, 2)	30	6.1
(3, 0)	10	2.0
(2, 1)	22	4.5
(1, 2)	30	6.1
(0, 3)	33	6.7
(3, 1)	13	2.6
(2, 2)	17	3.5
(1, 3)	35	7.1
(3, 2)	20	4.1
(2, 3)	30	6.1
(3, 3)	58	11.8
Total	492	

Parameter		Estimate	Standard Error	Estimate	Standard Error
Effect on low-type payoffs					
Constant	C_L	1.6254	.9450	1.5420	.9192
Low competitor #1	θ_{LL1}	-1.7744	.9229	-1.6954	.8931
Low competitor #2	θ_{LL2}	6497	.0927	6460	.0922
High competitor #1 (0 lows)	θ_{L0H1}	8552	.9449	7975	.9258
Additional high competitors (0 lows)	θ_{L0HA}	1247	.0982	1023	.0857
Number of high competitors (1 low)	θ_{L1H}	0122	.1407	0154	.0444
Number of high competitors (2 lows)	θ_{L2H}	0000	.0000	-1.12E-6	.0001
PLACEPOP	β_{L-P}	.2711	.0550	.2688	.0554
TRAFFIC	β_{L-T}	0616	.1070	0621	.1069
SPACING	β_{L-S}	.3724	.1271	.3700	.1271
WEST	β_{L-W}	.5281	.1515	.5246	.1511
Effect on high-type payoffs					
Constant	C_H	2.5252	.9395	2.5303	.8925
High competitor #1	θ_{HH1}	-2.0270	.9280	-2.0346	.8810
High competitor #2	θ_{HH2}	6841	.0627	6841	.0627
Low competitor #1 (0 highs)	θ_{H0L1}	-1.2261	.9314	-1.2176	.8841
Additional low competitors (0 highs)	θ_{H0LA}	-5.25E-6	.0006	0000	.0000
Number of low competitors (1 high)	θ_{H1L}	-2.82E-7	.0001	.0000	.0001
Number of low competitors (2 high)	θ_{H2L}	0000	.0000	-5.34E-6	.0003
PLACEPOP	β_{H-P}	.6768	.0551	.6801	.0570
TRAFFIC	β_{H-T}	.2419	.1137	.2419	.1142

 β_{H-S}

 β_{H-W}

Two-Substage Version

Stackelberg Version

.5159

.2588

.1328

.1592

-1143.12

TABLE 6 Estimated Parameters: Two-Product-Type Models

SPACING

Log-likelihood

WEST

.5157

.2562

.1332

.1585

-1143.01

4.3 Seim 2006

An issue with the Mazzeo model is that it is difficult to estimate the model when there are multiple "types" of firms, since the number of inequalities which need to be satisfied rises exponentially. We can use a moment inequality model, or change the structure of the game in such a way that makes the estimation of models with many types of entrants feasible.

The way that Seim does this is by introducing private information into the model, i.e. firms have information about the payoffs of entering a market that other firms do not see. This is by now a very common strategy in empirical I.O.: assume that the unobservables about other firms are also unobserved by other firms. To make this point as ridiculous as possible, this implies that you know as much about Wal-Mart's probability of entry as K-Mart does!

Second, private information shocks will lead to ex-post regret in the sense that two firms might enter when only one firm would make positive profits. This could also happen in the perfect information case if firms are playing a mixed strategy equilibria.

Questions:

- How do firms choose their geographic locations: tradeoff between density of demand (lots of consumers) versus competition.
- Choice of different type of modems technologies (Greenstein, Augereau and Rysman have a paper on this).

The model that Seim uses has:

- Simultaneous Moves.
- Asymmetric Information: I don't know my competitors idiosyncratic payoff shocks.
- The profits of firm f entering into location l:

$$\Pi_{fl} = X_l^m \beta + \underbrace{\xi^m}_{\text{market shocks}} + h(\theta_l^m, n^m) + \underbrace{\eta_{fl}^m}_{\text{private information idiosyncratic shocks}}$$

• Key idea: pure strategy equilibria might be difficult to find (or admit multiplicity); "smoothing" out opponent's strategies (similar to mixing/purification) makes computation much easier, and may aid with uniqueness (though latter is a bit hand-wavy).

However, using asymmetric information means some firms may wish to exit ex-post, leading to an unstable configuration in the market. If this is the case, why do we then observe these unstable configurations in the market?

• Parametrize competitive effect to be function of number of firms in distance band b from location l

$$\Pi_{fl} = X_l^m \beta + \xi^m + \sum_b \gamma_b n_{bl} + \eta_{fl}^m$$

• What are the expected payoffs of entering into location *i*?

$$E_{\eta_{-f}}[\Pi_{fl}] = \int_{\eta^{-f}} \left(X_{fl}^m \beta + \xi^m + \sum_b \gamma_b n_{bf} + \eta_{fl}^m \right) df(\eta^m)$$

Because this a linear function, we just care about the probability of other firms entering in location b:

$$E_{\eta_{-f}}[\Pi_{fl}^{m}] = X_{fl}^{m}\beta + \xi^{m} + \sum_{b} \gamma_{b}(\hat{N} - 1)p_{b}^{*} + \eta_{fl}^{m}$$

where \hat{N} are number of actual entrants and p_b^* is probability a firm chooses to locate in distance band b.

• Assume that the private information shocks η_{fl}^m are distributed as i.i.d. logit draws. To get this probability of entry in location *l* conditional on entry:

$$p_l^* = \frac{\exp(\xi^m)\exp(X_l^m\beta + \gamma_0 + (\hat{N} - 1)\sum_b \gamma_b \sum_j 1_{ij}^b p_j^*)}{\exp(\xi^m)\sum_k \exp(X_k^m\beta + \gamma_0 + (\hat{N} - 1)\sum_b \gamma_b \sum_j 1_{kj}^b p_j^*)}$$

Key: Note the unobservable market shocks drop out.

We need to solve for the fixed point of this equation. We can do this just by iterating on this equation since it is a contraction mapping.

• Then the probability of entry is simply:

$$Pr(entry) = \frac{\exp(\xi^m) \sum_l \exp(\Pi_{fl})}{1 + \exp(\xi^m) \sum_l \exp(\Pi_{fl})}$$

Assume that the observed number of entrants exactly equals the expected number of entrants: $\hat{N} = F \times Pr(entry)$ where F is number of *potential entrants* into market.

We can recover the market level shock ξ^m in exactly the same way that we did using BLP (with no non-linear parameters). Notice that:

$$\xi^{m} = \ln(\hat{N}) - \ln(F - \hat{N}) - \ln(\sum_{k} \exp(X_{k}^{m}\beta + \theta_{0} + (\hat{N} - 1)\sum_{b} \gamma_{b} \sum_{j} 1_{kj}^{b} p_{j}^{*}))$$

• Suppose $\xi \sim \mathcal{N}(\mu, \sigma)$. How do you estimate this model? Do it by maximum likelihood:

$$\mathcal{L}(\theta) = \prod_{m} \left[\phi(\frac{\xi^m(\theta) - \mu}{\sigma}) \prod_{l} (p_{lm}^*(\theta))^{n_l^m} \right]$$

(where the first part, by computing the probability of a given realization of ξ , controls for \hat{N}_m firms entering)

	151 Sample Markets			
	Mean	Minimum	Maximum	
Market level				
Population, market	74,367	$41,\!352$	142,303	
Population, main city	$59,\!428$	$40,\!495$	140,949	
Population, all tracts in market	$92,\!563$	$41,\!614$	193,322	
Largest Incorporated Place within 10 mi	$2,\!618$	-	$9,\!972$	
Largest Incorporated Place within 20 mi	7,916	-	24,725	
Tract level				
Number of tracts	21.13	8	49	
Number of store locations	18.72	7	44	
Tract population	$4,\!380$	247	$32,\!468$	
Area (sqmi)	10.10	0.10	181.50	
Average distance (mi) to				
other locations in market	3.49	1.08	8.05	
Notor				

Table 1	: Descrip	ptive Sta	atistics.	Markets	and	Locations
TODIC I	• DODUII			TVI GI KC US	and	Docarions

Notes:

The largest incorporated place within 10 and 20 miles is relative to the centroid of the market's main city. The distance between locations within a market is computed as the distance between the tracts' population-weighted centroids. Demographic data is as of 1999.

Figure 2: Sample Market - Great Falls, MT



	Potential Entrant Pool =					
	2 x Total H	Entrants	$50~{ m Fi}$	$50 { m Firms}$		
	Coefficient	Marginal	Coefficient	Marginal		
Variable	(Std. Error)	Effect	(Std. Error)	Effect		
Population ₀ (000)	1.8191	0.0333	2.1258	0.0393		
· · · · · · · · · · · · · · · · · · ·	(0.1534)		(0.1764)			
Population ₁ (000)	1.3109	0.0236	1.7349	0.0314		
	(0.1200)		(0.1498)			
Population ₂ (000)	0.6070	0.0121	1.1348	0.0227		
_ , , ,	(0.1192)		(0.1486)			
Business density	-0.8077	-0.0155	-0.8889	-0.0173		
·	(0.1458)		(0.1477)			
Avg. Per-Cap. Income ₀ (0000)	0.9309	0.0180	1.0380	0.0204		
/	(0.1136)		(0.1233)			
Avg. Per-Cap. Income ₁ (0000)	1.0081	0.0193	0.9188	0.0178		
<u> </u>	(0.2081)		(0.2043)			
Avg. Per-Cap. Income ₂ (0000)	0.4851	0.0092	0.4884	0.0094		
, , ,	(0.2512)		(0.2601)			
γ_0	-3.4520		-3.3853			
	(0.3111)		(0.3266)			
γ_1	-1.0103		-1.0087			
/1	(0.0745)		(0.0923)			
γ_2	-0.3448		-0.4870			
12	(0.0738)		(0.0934)			
σ	3.5829		4.6760			
	(0.3110)		(0.4316)			
μ	-2.8764		-7.0364			
	(1.3425)		(1.5801)			

 Table 4: Parameter Estimates, Entry and Location Choice Model

Notes:

Results based on 1999 demographic and firm data. Subscript 0 denotes the immediately adjacent locations to the chosen tract, within 0.5 miles in distance; subscript 1 denotes tracts at 0.5 to 3 miles in distance from the chosen tract; and subscript 2 denotes tracts at more than 3 miles distance from the chosen tract. Tract-level business density is defined as the number of establishments (0000) per square mile. γ denotes competitive effects, and σ and μ the estimates of the parameters of the distribution of ξ .

4.4 Jia 2008

- Question: What is the impact of chain stores on other retailers and communities?
- Extends entry literature to allow for: flexible (spatial) competition patterns and scale economics for chains operating stores in similar areas. Thus relaxes independence of entry decisions across markets.
- Store location problem becomes complicated with interdependence with N locations (2000 markets), 2^N potential strategies. Instead, will focus on Wal-Mart v. Kmart (2 players) and analyze a supermodular entry game.

Year	Number of Discount Stores	Total Sales (2004 \$, billions)	Average Store Size (thousand ft ²)	Number of Firms
1960	1329	12.8	38.4	1016
1980	8311	119.4	66.8	584
1989	9406	123.4	66.5	427
1997	9741	198.7	79.2	230

 TABLE I

 The Discount Industry From 1960 to 1997^a

^aSource: Various issues of *Discount Merchandiser*. The numbers include only traditional discount stores. Wholesale clubs, supercenters, and special retailing stores are excluded.

- Data (1988-1997)
 - Data on discount chains from Chain Store Guide (directory of >10K sq ft stores): size, address, store format, name.
 - Market: county w/ 5000-64,000 population (2065 out of 3140 in US). Treats chain store location in other (larger) markets as exogeneous. County Business Patterns data and US Census county population, demographic, and retail sales data.
 - Ignores Target (340 in 1988, 800 stores in 1997).

Three stage model

1. First stage: small retailers make entry decisions w/o anticipating Kmart or Walmart (prechain period).

$$\Pi^{0}_{s,m} = X^{o}_{m}\beta_{s} + \delta_{ss}\ln(N^{o}_{s,m}) + \sqrt{q - \rho^{2}}\epsilon^{o}_{m} + \rho\eta^{0}_{s,m} - SC$$
(22)

where s is small store; $X_m^o \beta_s$ is market size parameterized by demand shifers (e.g., population); $\delta_{ss} \ln(N_{s,m}^o)$ is competition effect; and unobserved profit shocks and sunk cost of entry. Correlation across firms within market captured by ϵ_m^o ; both ϵ_m^0 and $\eta_{s,m}$ distributed iid standard normal. No anticipation of K or W.

2. Second stage: Kmart and Walmart emerge and optimally locate stores across entire set of markets, accounting for potential spillovers. Not an independent decision across markets.

Let $i \in \{k, w\}$; let $D_{i,m} \in \{0, 1\}$ denote decision by *i* to enter/stay out of market *m*. Chain profits are given by:

$$\Pi_{i,m}(D_i, D_{j,m}, N_{s,m}; X_m, Z_m, \epsilon_m, \eta_{i,m}) = D_{i,m} \Big[X_m \beta_i + \delta_{ij} D_{j,m} + \delta_{i,s} \ln(N_{sm} + 1) + \delta_{ii} \sum_{l \neq m} \frac{D_{i,l}}{Z_{ml}} + \sqrt{1 - \rho^2} \epsilon_m + \rho \eta_{im} \Big]$$

where Z_{lm} is distance between markets l and m.

3. Existing small firms decide to exit, and potential entrants decide whether or not to enter. (Homogeneous small firms, no exit/entry for Kmart and Walmart).

Small firm profits here are similar to first stage, except for addition of $\sum_{i \in \{k,w\}} \delta_{si} D_{lm}$ to capture chain effect on profits.

Estimation and Solution Algorithm:

• Start with single agent problem. Let X_m represent $X_m\beta_i + \sqrt{1-\rho^2}\epsilon_m + \rho\eta_{im}$ and suppress firm subscripts for now.

$$\max_{D_1,\dots,D_M} \sum_{m=1}^M \left[D_m \left(X_m + \delta \sum_{l \neq m} \frac{D_l}{Z_{ml}} \right) \right]$$
(23)

• Large dimensionality (2^{2065}) problem. Transform into search over fixed points of the necessary conditions; can show that these are:

$$D_m^* = 1 \left[X_m + 2\delta \sum_{l \neq m} \frac{D_l^*}{Z_{ml}} \ge 0 \right] \forall m$$
(24)

This is market m's marginal contribution to total profit.

- Define $V_m(D) = 1[X_m + 2\delta \sum_{l \neq m} (D_l/Z_{ml}) \ge 0]$ and let $V(D) = \{V_1(D), \ldots, V_M(D)\}$. If $\delta_{ii} > 0$ (by assumption), and $V(D') \ge V(D'')$ whenever $D' \ge D''$ (which is true), then D^* is one of V's fixed points. Paper shows that $V(\cdot)$ is an increasing function on a complete lattice D, and thus has a greatest and least fixed point of this set (where all $D \in D$ can be ordered).
- Algorithm: start with $D^0 = \sup(\mathbf{D}) = \{1, \ldots, 1\}$ and apply $V(\cdot)$ until converge to largest element in set D^U . Repeat with $D^o = \inf(\mathbf{D}) = \{0, \ldots, 0\}$ to find smallest element D^L . Then can finally grid search for the optimal profit maximizing vector D^* in between these points.
- To account for competition from rival, the paper shows that a chain's profit function is supermodular in own strategy D_i and has "decreasing differences" in opponent strategy (gain to additional store is lower if rival has store). The game can be reframed as a supermodular game, which will have a greatest and least element in the set of Nash equilibria.
- Algorithm: start with Walmart $D_w^0 = \inf(\mathbf{D}) = \{0, \dots, 0\}$. Find Kmart's best response $D_k^1 = K(D_w^0)$ using method described before. Then find Walmarts best response $W(D_k^1)$. Repeat. By supermodularity, Kmarts best response will be decreasing and Walmarts increasing. Algorithm converges to Kmarts highest profit equilibrium. Walmart's highest profit equilibrium can be found similarly by starting with Kmart at $\{0, \dots, 0\}$.

• Estimation: estimates model with (i) Kmart preferred equilibrium (first-mover advantage); (ii) Walmart preferred eq; (iii) Walmart in south, and Kmart for rest of the country. Results relatively robust.

Main Results:

- 1. Kmart had greater negative impact on Walmart in 1988, while opposite true in 1997 (Walmart negative on Kmart).
- 2. Presence of a chain store leads 50% of (local) discount stores unprofitable.
- 3. Walmart expansion can explain 37-55% net change in small discount stores and 34-41% of all other discount stores.
- 4. Scale economies were important to Walmart.

TABLE XII

COMPETITION EFFECT AND CHAIN EFFECT FOR KMART (KM) AND WAL-MART (WM)^a

	1988		1997	
Number of	Percent	Total	Percent	Total
Kmart stores				
Base case	100.0	437	100.0	393
Wm in each market	85.1	371	82.2	323
Wm exits each market	108.6	474	141.9	558
Not compete with small stores	101.3	442	104.3	410
No chain effect	94.7	414	93.5	368
Wal-Mart stores				
Base case	100.0	651	100.0	985
Km in each market	61.4	400	82.2	809
Km exits each market	119.5	778	105.7	1042
Not compete with small stores	101.7	662	105.1	1035
No chain effect	84.4	550	92.9	915

^aBase case is the number of stores observed in the data. For each exercise, I resolve the full model under the specified assumptions. For the last two rows of both panels where the counterfactual exercise involves multiple equilibria, I solve the model using the equilibrium that is most profitable for Kmart.

	Profit Positive		Profit Recove	Profit Recovers Sunk Cost	
	Percent	Total	Percent	Total	
		198	38		
No Kmart or Wal-Mart	100.0	9261			
Only Kmart in each Market	76.2	7057	47.9	4440	
Only Wal-Mart in each Market	77.5	7173	49.1	4542	
Both Kmart and Wal-Mart	56.1	5195	31.6	2925	
		199	97		
No Kmart or Wal-Mart	100.0	8053			
Only Kmart in each Market	89.8	7228	54.1	4357	
Only Wal-Mart in each Market	82.4	6634	47.9	3854	
Both Kmart and Wal-Mart	72.9	5868	40.3	3244	

TABLE XIII
NUMBER OF SMALL STORES WITH DIFFERENT MARKET STRUCTURE

^aI fix the number of Kmart and Wal-Mart stores as specified and solve for the equilibrium number of small stores. If stores have perfect foresight, the columns labeled Profit Recovers Sunk Cost would have been the number of stores that we observe, as they would not have entered in the pre-chain period if their profit after entry could not recover the sunk cost.

TABLE XIV

NUMBER OF ALL DISCOUNT STORES (EXCEPT FOR KMART AND WAL-MART STORES) WITH DIFFERENT MARKET STRUCTURE^a

	Profit Positive		Profit Recove	Profit Recovers Sunk Cost	
	Percent	Total	Percent	Total	
	1988				
No Kmart or Wal-Mart	100.0	10,752			
Only Kmart in each Market	82.7	8890	47.1	5064	
Only Wal-Mart in each Market	78.5	8443	43.6	4692	
Both Kmart and Wal-Mart	62.7	6741	31.5	3383	
		1997	7		
No Kmart or Wal-Mart	100.0	9623			
Only Kmart in each Market	91.9	8842	51.7	4976	
Only Wal-Mart in each Market	79.8	7683	42.0	4043	
Both Kmart and Wal-Mart	72.4	6964	36.5	3508	

^aI fix the number of Kmart and Wal-Mart stores as specified and solve for the number of all other discount stores. See the additional comments in the footnote to Table XIII.

TABLE XV

THE IMPACT OF WAL-MART'S EXPANSION^a

	1988	1997
Observed decrease in the number of small stores between 1988 and 1997	693	693
Predicted decrease from the full model	380	259
Percentage explained	55%	37%
Observed decrease in the number of all discount stores		
(except for Kmart and Wal-Mart stores) between 1988 and 1997	1021	1021
Predicted decrease from the full model	416	351
Percentage explained	41%	34%

^aIn the top panel, the predicted 380 store exits in 1988 are obtained by simulating the change in the number of small stores using Kmart's and the small stores' profit in 1988, but Wal-Mart's profit in 1997. The column of 1997 uses Kmart's and small stores' profit in 1997, but Wal-Mart's profit in 1988. Similarly for the second panel.

5 References on Entry Games

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