

Online Appendix for “A Computational Framework for Analyzing Dynamic Auctions: The Market Impact of Information Sharing”

1 Results from models with a restricted state space

In this section, we consider a model in which firms condition their strategies on a subset of payoff or informationally relevant variables. In this version, the information set includes the time since the last information exchange, $\tau_t \in [1, \dots, T]$, and the last revealed ω vectors, $\omega_{t-\tau_t}$. We compute a baseline model with the restricted state space (B^r) in which information is shared every $T = 4$ periods and an information-exchange model with the restricted state space (IE^r) with $T = 1$. In addition, we compute each of the models with the restricted state space (B^r and IE^r) with three firms.

The computation uses the same following procedure used in computing the B and VIE models.

1. First, the computational algorithm is run for 50 million iterations, resetting the counters for the states every 10,000 iterations as follows:

$$h(J_i | \text{iteration } 10,001) = \begin{cases} 10 & \text{if } h(J_i | \text{iteration } 10,000) \geq 10 \\ h(J_i | \text{iteration } 10,000) & \text{otherwise.} \end{cases}$$

2. Then, the algorithm is run for 5 million iterations without resetting the counter.
3. Next, a run of 5 million iterations is used to form the test for the REBE (recall that the test requires an R^2 statistic to be greater than .999).
4. If the test is passed, we stop the algorithm. Otherwise we repeat steps 1 to 3.

All four specifications with the restricted state space (B^r , IE^r , B^r with three firms, IE^r with three firms) converge after the first round of steps 1 through 3. A summary of computation times is available in table 1.

Table 2 reports a summary of outcomes from B , B^r , B^r with three firms, IE , IE^r , and IE^r with three firms. Restricting the information set leads to increased competition. The average bid increases from 1.09 to 1.29 for the baseline case (going from B to B^r), and from 0.94 to 1.16 for the information-exchange case (going from IE to IE^r). The winning bids and winning bids conditional on the number of bidders also increase. By contrast, the participation rate falls from 0.76 to 0.72 for the baseline model and from 0.81 to 0.71 for the information-exchange model. The results show that restricting the information set by conditioning on fewer variables (moving from B to B^r or from IE to IE^r) has qualitatively the same effects on bids and participation as restricting the information set by revealing information less frequently (moving from IE to B). The profit, total revenue, and social surplus are also lower in B^r than in B (and in IE^r than in IE).

Bids are lower in IE^r than in B^r . Participation is almost the same in B^r and IE^r in contrast to the unrestricted case where participation is higher in IE than in B . As in

Table 1: Computational details

Size of recurrent class:

B^r	$B^r, N = 3$	IE^r	$IE^r, N = 3$
5776	7576	1467	1642

Number of all states visited during computation:

B^r	$B^r, N = 3$	IE^r	$IE^r, N = 3$
9514	17282	1784	3405

Computation times per 5 million iterations (in hours):

B^r	$B^r, N = 3$	IE^r	$IE^r, N = 3$
0:22	0:37	0:21	0:37

Computation times for testing for a REBE (5 million iterations, in hours):

B^r	$B^r, N = 3$	IE^r	$IE^r, N = 3$
0:21	0:37	0:20	0:36

Notes: Computation was conducted in MATLAB version R2017b using (an iMac Pro desktop with) a 3.2 GHz Intel Xeon processor and 32GB Ram on macOS Mojave.

the unrestricted case, the profit is higher and social surplus is almost unchanged (slightly smaller) when there are more frequent information revelations. However, the total revenue is smaller in IE^r than in B^r (3.28 vs 3.32) unlike in the unrestricted case.

Competition intensifies with an additional firm. The average bid increases from 1.29 to 1.98 for the baseline case with the restricted state space, and from 1.16 to 1.97 for the information-exchange case with the restricted state space. The average number of participants increases from 1.44 for B^r with two firms to 1.71 in B^r with three firms, although the participation rate decreases from 0.72 to 0.57. There is a large decrease in the firm profit from 0.66 to 0.18. The revenue and social surplus decrease as well.

Table 2: Summary statistics, in per-period terms, by model

	B	B^r	$B^r, N = 3$	IE	IE^r	$IE^r, N = 3$
Avg. bid	1.09	1.29	1.98	0.94	1.16	1.97
Avg. winning bid (revenue for the auctioneer)	1.11	1.27	1.85	0.98	1.14	1.85
Avg. winning bid with ≥ 1 firm participating	1.16	1.34	1.99	0.98	1.22	1.99
Avg. winning bid with 1 firm participating	1.06	1.28	1.98	0.67	1.11	1.98
Avg. winning bid with 2 firms participating	1.23	1.40	2.00	1.16	1.31	2.00
Avg. # of participants	1.52	1.44	1.71	1.63	1.43	1.71
Avg. # of participants with ≥ 1 firm participating	1.59	1.52	1.84	1.63	1.52	1.84
Avg. participation rate	0.76	0.72	0.57	0.81	0.71	0.57
% of periods with no participation	4.39	5.06	7.27	0.15	6.33	7.23
Avg. total revenue	3.35	3.32	3.25	3.49	3.28	3.25
Avg. profit	0.81	0.66	0.18	0.87	0.71	0.18
% of periods; lowest omega wins	66.37	76.51	76.00	60.80	67.39	76.56
Avg. total social surplus	2.73	2.60	2.39	2.72	2.57	2.39

Table 3: Probability Distribution by ω -tuple for B , B^r , IE and IE^r

(ω_i, ω_{-i})	Prob. Dist. (%)				Profit			
	B	B^r	IE	IE^r	B	B^r	IE	IE^r
$(\leq 4, \leq 4)$	65.51	87.38	32.59	73.76	0.68	0.68	0.52	0.68
$(\leq 4, 5 - 7)$	12.61	5.80	19.09	10.67	0.57	0.41	0.58	0.61
$(\leq 4, \geq 8)$	4.05	0.45	10.55	2.21	0.60	0.42	0.59	0.70
$(5 - 7, \leq 4)$	12.61	5.80	19.09	10.67	1.51	1.73	1.26	1.59
$(5 - 7, 5 - 7)$	0.88	0.10	5.72	0.40	1.49	1.58	1.46	1.60
$(5 - 7, \geq 8)$	0.14	0.01	1.12	0.05	1.49	1.47	1.13	1.59
$(\geq 8, \leq 4)$	4.05	0.45	10.55	2.21	1.62	1.85	1.58	1.77
$(\geq 8, 5 - 7)$	0.14	0.01	1.12	0.05	1.66	1.54	1.87	1.73
$(\geq 8, \geq 8)$	0.01	0.00	0.17	0.00	1.72	1.67	1.56	1.61

Table 4: Probability Distribution by ω -tuple for B^r and IE^r with three firms

$\omega_i, (\omega_{-i})$	Prob. Dist. (%)		Profit	
	$B^r, N = 3$	$IE^r, N = 3$	$B^r, N = 3$	$IE^r, N = 3$
$\leq 4, (\leq 4, \leq 4)$	96.32	96.67	0.28	0.29
$\leq 4, (\leq 4, 5 - 7)$	1.21	1.10	-0.13	-0.15
$\leq 4, (\leq 4, \geq 8)$	0.02	0.01	-0.16	-0.23
$\leq 4, (5 - 7, 5 - 7)$	0.00	0.00	-0.29	-0.26
$\leq 4, (5 - 7, \geq 8)$	0.00	0.00	-0.02	0.56
$5 - 7, (\leq 4, \leq 4)$	1.21	1.10	1.93	1.96
$5 - 7, (\leq 4, 5 - 7)$	0.00	0.00	1.50	1.34
$5 - 7, (\leq 4, \geq 8)$	0.00	0.00	1.75	0.76
$\geq 8, (\leq 4, \leq 4)$	0.02	0.01	1.62	1.87
$\geq 8, (\leq 4, 5 - 7)$	0.00	0.00	1.91	2.07

Table 5: Bids by ω -tuple for B and B^r

(ω_i, ω_{-i})	Bids										Profit	
	B					B^r					B	B^r
	\emptyset	0.5	1	1.5	2	\emptyset	0.5	1	1.5	2		
$(\leq 4, \leq 4)$	0.22	0.13	0.27	0.31	0.07	0.26	0.09	0.10	0.54	0.00	0.68	0.68
$(\leq 4, 5-7)$	0.11	0.32	0.45	0.11	0.02	0.10	0.16	0.22	0.52	0.00	0.57	0.41
$(\leq 4, \geq 8)$	0.08	0.58	0.29	0.04	0.02	0.06	0.15	0.42	0.37	0.00	0.60	0.42
$(5-7, \leq 4)$	0.43	0.18	0.34	0.04	0.01	0.67	0.05	0.15	0.13	0.00	1.51	1.73
$(5-7, 5-7)$	0.37	0.50	0.09	0.02	0.01	0.52	0.23	0.22	0.02	0.00	1.49	1.58
$(5-7, \geq 8)$	0.39	0.53	0.06	0.01	0.01	0.40	0.41	0.18	0.01	0.00	1.49	1.47
$(\geq 8, \leq 4)$	0.51	0.25	0.22	0.02	0.00	0.76	0.08	0.13	0.03	0.00	1.62	1.85
$(\geq 8, 5-7)$	0.53	0.39	0.06	0.01	0.00	0.47	0.31	0.21	0.00	0.00	1.66	1.54
$(\geq 8, \geq 8)$	0.61	0.36	0.03	0.00	0.00	0.50	0.35	0.15	0.00	0.00	1.72	1.67

Notes: This table shows the probability of bids by intervals of ω -tuples for B and B^r . \emptyset indicates non-participation.

Table 6: Bids by ω -tuple for IE and IE^r

(ω_i, ω_{-i})	Bids										Profit	
	IE					IE^r					IE	IE^r
	\emptyset	0.5	1	1.5	2	\emptyset	0.5	1	1.5	2		
$(\leq 4, \leq 4)$	0.07	0.13	0.28	0.47	0.06	0.27	0.12	0.16	0.41	0.04	0.52	0.68
$(\leq 4, 5-7)$	0.02	0.53	0.37	0.08	0.00	0.12	0.29	0.50	0.09	0.00	0.58	0.61
$(\leq 4, \geq 8)$	0.00	0.88	0.12	0.00	0.00	0.07	0.48	0.45	0.00	0.00	0.59	0.70
$(5-7, \leq 4)$	0.33	0.10	0.52	0.05	0.00	0.54	0.06	0.37	0.03	0.00	1.26	1.59
$(5-7, 5-7)$	0.40	0.59	0.01	0.00	0.00	0.41	0.58	0.02	0.00	0.00	1.46	1.60
$(5-7, \geq 8)$	0.11	0.89	0.00	0.00	0.00	0.43	0.57	0.00	0.00	0.00	1.13	1.59
$(\geq 8, \leq 4)$	0.60	0.14	0.26	0.00	0.00	0.65	0.17	0.18	0.00	0.00	1.58	1.77
$(\geq 8, 5-7)$	0.84	0.16	0.00	0.00	0.00	0.54	0.46	0.00	0.00	0.00	1.87	1.73
$(\geq 8, \geq 8)$	0.47	0.53	0.00	0.00	0.00	0.47	0.52	0.01	0.00	0.00	1.56	1.61

Notes: This table shows the probability of bids by intervals of ω -tuples for IE and IE^r . \emptyset indicates non-participation.

Table 7: Bids by ω -tuple for B^r and IE^r with three firms

(ω_i, ω_{-i})	Bids										Profit	
	B^r					IE^r					B^r	IE^r
	\emptyset	0.5	1	1.5	2	\emptyset	0.5	1	1.5	2		
$\leq 4, (\leq 4, \leq 4)$	0.43	0.01	0.00	0.00	0.57	0.43	0.01	0.01	0.00	0.56	0.28	0.29
$\leq 4, (\leq 4, 5-7)$	0.34	0.00	0.00	0.00	0.66	0.31	0.01	0.01	0.00	0.67	-0.13	-0.15
$\leq 4, (\leq 4, \geq 8)$	0.28	0.02	0.01	0.00	0.68	0.19	0.07	0.02	0.03	0.69	-0.16	-0.23
$\leq 4, (5-7, 5-7)$	0.23	0.05	0.00	0.02	0.70	0.09	0.14	0.09	0.18	0.50	-0.29	-0.26
$\leq 4, (5-7, \geq 8)$	0.00	0.25	0.00	0.00	0.75	0.00	0.00	0.67	0.33	0.00	-0.02	0.56
$5-7, (\leq 4, \leq 4)$	0.78	0.19	0.01	0.00	0.03	0.80	0.14	0.06	0.00	0.00	1.93	1.96
$5-7, (\leq 4, 5-7)$	0.52	0.04	0.04	0.02	0.38	0.50	0.04	0.04	0.17	0.26	1.50	1.34
$5-7, (\leq 4, \geq 8)$	0.40	0.00	0.00	0.00	0.60	0.50	0.00	0.00	0.00	0.50	1.75	0.76
$\geq 8, (\leq 4, \leq 4)$	0.61	0.05	0.03	0.01	0.31	0.75	0.01	0.12	0.04	0.09	1.62	1.87
$\geq 8, (\leq 4, 5-7)$	0.67	0.00	0.00	0.00	0.33	0.60	0.00	0.20	0.00	0.20	1.91	2.07

Notes: This table shows the probability of bids by intervals of ω -tuples for B^r and IE^r with three firms. \emptyset indicates non-participation.

2 Probability Distribution and Actions by ω -tuple for B and IE^*

Table 8: Probability Distributions and Actions by ω -tuple for B and IE

(ω_i, ω_{-i})	Prob. Dist. (%)		Bids										Profit		Joint Profit	
	B	IE	0	0.5	B			IE			B	IE	B	IE		
(0, 0)	3.17	0.50	0.12	0.07	0.12	0.41	0.28	0.01	0.00	0.09	0.12	0.78	-0.22	-0.48	-0.43	-0.97
(0, 1)	3.70	0.88	0.12	0.08	0.13	0.46	0.20	0.04	0.00	0.09	0.44	0.43	-0.17	-0.44	0.24	-0.53
(0, 2)	4.91	1.48	0.11	0.09	0.17	0.49	0.15	0.05	0.08	0.05	0.60	0.23	-0.09	-0.31	0.92	0.35
(0, 3)	4.83	1.94	0.10	0.10	0.25	0.47	0.08	0.03	0.17	0.02	0.71	0.07	-0.02	-0.26	1.32	0.52
(0, 4)	3.83	2.27	0.08	0.13	0.39	0.38	0.03	0.02	0.19	0.17	0.62	0.00	0.04	-0.24	1.40	0.49
(0, 5)	3.02	2.47	0.07	0.15	0.53	0.24	0.02	0.02	0.20	0.38	0.40	0.00	0.14	-0.09	1.63	0.94
(0, 6)	2.19	2.48	0.06	0.18	0.61	0.13	0.02	0.02	0.19	0.61	0.18	0.00	0.19	-0.00	1.72	1.15
(0, 7)	1.49	2.36	0.05	0.23	0.61	0.09	0.03	0.01	0.33	0.62	0.03	0.00	0.22	0.02	1.76	1.19
(0, 8)	0.97	2.14	0.04	0.36	0.53	0.05	0.02	0.00	0.58	0.42	0.00	0.00	0.24	-0.01	1.78	1.13
(0, 9)	0.64	1.80	0.04	0.51	0.41	0.04	0.01	0.00	0.82	0.18	0.00	0.00	0.31	0.17	1.86	1.47
(0, 10)	0.41	1.29	0.04	0.64	0.27	0.04	0.01	0.00	1.00	0.00	0.00	0.00	0.38	0.69	1.99	2.44
(0, 11+)	0.53	1.21	0.03	0.78	0.12	0.05	0.02	0.00	1.00	0.00	0.00	0.00	0.50	0.81	2.22	2.67
(1, 0)	3.70	0.88	0.18	0.06	0.13	0.49	0.15	0.01	0.04	0.00	0.29	0.66	0.41	-0.08	0.24	-0.53
(1, 1)	2.36	0.80	0.18	0.12	0.23	0.40	0.07	0.03	0.09	0.00	0.74	0.15	0.46	0.20	0.93	0.39
(1, 2)	2.54	1.07	0.17	0.14	0.32	0.32	0.05	0.03	0.10	0.07	0.81	0.00	0.49	0.32	1.52	0.96
(1, 3)	2.09	1.16	0.15	0.16	0.43	0.23	0.02	0.02	0.13	0.33	0.53	0.00	0.55	0.32	1.89	1.19
(1, 4)	1.42	1.13	0.13	0.22	0.52	0.13	0.01	0.01	0.16	0.59	0.24	0.00	0.59	0.40	1.97	1.45
(1, 5)	0.98	1.08	0.11	0.29	0.51	0.08	0.01	0.00	0.28	0.72	0.00	0.00	0.62	0.52	2.10	1.65
(1, 6)	0.64	0.97	0.08	0.43	0.43	0.05	0.01	0.00	0.52	0.48	0.00	0.00	0.66	0.55	2.20	1.74
(1, 7)	0.40	0.83	0.08	0.50	0.38	0.03	0.01	0.00	0.79	0.21	0.00	0.00	0.69	0.64	2.26	1.85
(1, 8)	0.24	0.63	0.07	0.62	0.27	0.03	0.01	0.00	1.00	0.00	0.00	0.00	0.74	0.82	2.37	2.34
(1, 9)	0.14	0.40	0.08	0.68	0.19	0.03	0.02	0.00	1.00	0.00	0.00	0.00	0.75	0.89	2.40	2.63
(1, 10)	0.08	0.19	0.08	0.70	0.16	0.03	0.03	0.00	1.00	0.00	0.00	0.00	0.76	0.95	2.47	2.84
(1, 11+)	0.09	0.10	0.10	0.69	0.15	0.04	0.03	0.00	1.00	0.00	0.00	0.00	0.79	0.97	2.56	2.92
(2, 0)	4.91	1.48	0.28	0.07	0.19	0.41	0.05	0.05	0.10	0.00	0.86	0.00	1.01	0.66	0.92	0.35
(2, 1)	2.54	1.07	0.28	0.14	0.27	0.29	0.02	0.06	0.09	0.00	0.85	0.00	1.03	0.64	1.52	0.96
(2, 2)	2.57	1.32	0.26	0.17	0.34	0.22	0.01	0.04	0.18	0.11	0.66	0.00	1.01	0.62	2.02	1.24
(2, 3)	2.02	1.36	0.24	0.23	0.39	0.13	0.01	0.03	0.21	0.39	0.37	0.00	1.02	0.72	2.36	1.66
(2, 4)	1.33	1.26	0.21	0.32	0.40	0.06	0.01	0.01	0.29	0.65	0.05	0.00	1.04	0.86	2.40	1.94
(2, 5)	0.91	1.20	0.18	0.45	0.32	0.04	0.01	0.01	0.43	0.56	0.00	0.00	1.06	0.92	2.51	2.02
(2, 6)	0.58	1.06	0.15	0.57	0.24	0.03	0.01	0.01	0.73	0.27	0.00	0.00	1.06	0.97	2.59	2.27
(2, 7)	0.35	0.89	0.14	0.64	0.18	0.02	0.01	0.00	1.00	0.00	0.00	0.00	1.06	1.07	2.63	2.46
(2, 8)	0.22	0.62	0.14	0.69	0.14	0.02	0.01	0.00	1.00	0.00	0.00	0.00	1.08	1.03	2.71	2.75
(2, 9)	0.13	0.37	0.14	0.70	0.12	0.03	0.01	0.00	1.00	0.00	0.00	0.00	1.08	1.01	2.73	2.87
(2, 10)	0.07	0.17	0.13	0.72	0.11	0.03	0.01	0.00	1.00	0.00	0.00	0.00	1.06	1.01	2.78	2.92
(2, 11+)	0.07	0.09	0.17	0.68	0.11	0.03	0.01	0.00	1.00	0.00	0.00	0.00	1.08	1.00	2.87	2.97
(3, 0)	4.83	1.94	0.35	0.07	0.26	0.30	0.02	0.06	0.03	0.07	0.83	0.00	1.34	0.78	1.32	0.52
(3, 1)	2.09	1.16	0.34	0.16	0.31	0.16	0.02	0.12	0.10	0.23	0.55	0.00	1.34	0.87	1.89	1.19
(3, 2)	2.02	1.36	0.33	0.22	0.33	0.11	0.02	0.12	0.13	0.39	0.35	0.00	1.33	0.93	2.36	1.66
(3, 3)	1.54	1.34	0.31	0.30	0.32	0.06	0.01	0.11	0.20	0.57	0.12	0.00	1.34	1.03	2.68	2.06
(3, 4)	0.97	1.22	0.28	0.40	0.28	0.04	0.00	0.07	0.43	0.50	0.00	0.00	1.35	1.20	2.72	2.20
(3, 5)	0.65	1.17	0.25	0.48	0.23	0.03	0.00	0.04	0.67	0.29	0.00	0.00	1.33	1.19	2.79	2.35
(3, 6)	0.41	1.03	0.22	0.57	0.19	0.03	0.00	0.01	0.93	0.05	0.00	0.00	1.30	1.16	2.84	2.65
(3, 7)	0.25	0.80	0.20	0.61	0.16	0.03	0.01	0.00	1.00	0.00	0.00	0.00	1.27	1.13	2.88	2.78
(3, 8)	0.15	0.51	0.21	0.67	0.09	0.02	0.01	0.00	1.00	0.00	0.00	0.00	1.31	1.06	2.94	2.91
(3, 9)	0.08	0.27	0.21	0.67	0.09	0.02	0.01	0.00	1.00	0.00	0.00	0.00	1.28	1.05	2.97	2.95
(3, 10)	0.05	0.11	0.22	0.65	0.11	0.01	0.01	0.00	1.00	0.00	0.00	0.00	1.28	1.02	3.00	2.98
(3, 11+)	0.05	0.06	0.27	0.63	0.09	0.01	0.00	0.00	1.00	0.00	0.00	0.00	1.33	1.02	3.10	2.98
(4, 0)	3.83	2.27	0.37	0.06	0.34	0.22	0.01	0.09	0.00	0.13	0.78	0.00	1.36	0.73	1.40	0.49
(4, 1)	1.42	1.13	0.35	0.14	0.40	0.10	0.01	0.17	0.00	0.61	0.23	0.00	1.38	1.06	1.97	1.45
(4, 2)	1.33	1.26	0.35	0.19	0.38	0.07	0.01	0.17	0.00	0.78	0.05	0.00	1.36	1.08	2.40	1.94
(4, 3)	0.97	1.22	0.34	0.29	0.33	0.03	0.01	0.16	0.07	0.77	0.00	0.00	1.36	1.00	2.72	2.20
(4, 4)	0.58	1.11	0.31	0.44	0.22	0.02	0.01	0.13	0.32	0.56	0.00	0.00	1.40	1.13	2.80	2.25
(4, 5)	0.38	1.11	0.29	0.51	0.17	0.02	0.01	0.10	0.61	0.30	0.00	0.00	1.38	1.13	2.87	2.47
(4, 6)	0.23	0.95	0.26	0.57	0.13	0.02	0.02	0.08	0.85	0.08	0.00	0.00	1.35	1.13	2.88	2.81
(4, 7)	0.13	0.69	0.26	0.61	0.10	0.02	0.02	0.04	0.96	0.00	0.00	0.00	1.36	1.09	2.95	2.94
(4, 8)	0.07	0.38	0.27	0.62	0.08	0.02	0.01	0.02	0.98	0.00	0.00	0.00	1.35	1.04	2.97	2.98
(4, 9)	0.04	0.17	0.25	0.63	0.09	0.03	0.01	0.00	1.00	0.00	0.00	0.00	1.33	1.02	2.99	3.00
(4, 10)	0.02	0.06	0.32	0.56	0.09	0.02	0.01	0.00	1.00	0.00	0.00	0.00	1.38	0.98	3.08	2.98
(4, 11+)	0.02	0.02	0.40	0.52	0.07	0.01	0.00	0.05	0.95	0.00	0.00	0.00	1.46	1.06	3.24	2.96
(5, 0)	3.02	2.47	0.41	0.09	0.42	0.08	0.01	0.16	0.00	0.51	0.34	0.00	1.49	1.02	1.63	0.94
(5, 1)	0.98	1.08	0.41	0.19	0.35	0.05	0.01	0.24	0.00	0.71	0.05	0.00	1.48	1.13	2.10	1.65
(5, 2)	0.91	1.20	0.40	0.25	0.30	0.04	0.00	0.25	0.00	0.75	0.00	0.00	1.45	1.10	2.51	2.02
(5, 3)	0.65	1.17	0.39	0.34	0.24	0.03	0.00	0.28	0.16	0.56	0.00	0.00	1.46	1.16	2.79	2.35
(5, 4)	0.38	1.11	0.37	0.44	0.16	0.02	0.01	0.29	0.41	0.30	0.00	0.00	1.49	1.33	2.87	2.47
(5, 5)	0.24	1.07	0.35	0.49	0.12	0.03	0.01	0.28	0.65	0.07	0.00	0.00	1.46	1.39	2.93	2.77
(5, 6)	0.15	0.86	0.34	0.52	0.10	0.03	0.01	0.21	0.79	0.00	0.00	0.00	1.44	1.29	2.94	2.93
(5, 7)	0.08	0.59	0.33	0.55	0.07	0.03	0.01	0.12	0.88	0.00	0.00	0.00	1.42	1.16	3.03	2.98
(5, 8)	0.04	0.32	0.33	0.57	0.06	0.01	0.02	0.05	0.95	0.00	0.00	0.00	1.42	1.07	3.06	3.00
(5, 9)	0.02	0.14	0.35	0.58	0.05	0.01	0.01	0.00	1.00	0.00	0.00	0.00	1.44	1.01	3.10	2.98
(5, 10)	0.01	0.05	0.38	0.56	0.05	0.01	0.00	0.01	0.99	0.00	0.00	0.00	1.46	1.01	3.13	2.98
(5, 11+)	0.01	0.02	0.47	0.49	0.03	0.01	0.00	0.10	0.90	0.00	0.00	0.00	1.57	1.17	3.36	2.98
(6, 0)	2.19	2.48	0.44	0.10	0.43	0.03	0.01	0.20	0.00	0.74	0.06	0.00	1.54	1.15	1.72	1.15

(6, 1)	0.64	0.97	0.46	0.20	0.31	0.02	0.01	0.34	0.00	0.66	0.00	0.00	1.54	1.18	2.20	1.74
(6, 2)	0.58	1.06	0.46	0.26	0.26	0.02	0.01	0.40	0.12	0.48	0.00	0.00	1.53	1.30	2.59	2.27
(6, 3)	0.41	1.03	0.44	0.34	0.19	0.02	0.01	0.49	0.26	0.25	0.00	0.00	1.54	1.49	2.84	2.65
(6, 4)	0.23	0.95	0.42	0.42	0.12	0.02	0.02	0.54	0.42	0.04	0.00	0.00	1.53	1.67	2.88	2.81
(6, 5)	0.15	0.86	0.40	0.47	0.09	0.02	0.02	0.54	0.46	0.00	0.00	0.00	1.51	1.64	2.94	2.93
(6, 6)	0.08	0.65	0.40	0.50	0.08	0.02	0.00	0.44	0.56	0.00	0.00	0.00	1.53	1.49	3.06	2.97
(6, 7)	0.04	0.42	0.40	0.52	0.07	0.01	0.00	0.31	0.69	0.00	0.00	0.00	1.52	1.33	3.07	2.98
(6, 8)	0.02	0.22	0.39	0.51	0.08	0.01	0.00	0.15	0.85	0.00	0.00	0.00	1.48	1.17	3.10	2.99
(6, 9)	0.01	0.10	0.45	0.48	0.05	0.02	0.00	0.06	0.94	0.00	0.00	0.00	1.57	1.10	3.25	3.00
(6, 10)	0.00	0.03	0.45	0.47	0.07	0.02	0.00	0.07	0.93	0.00	0.00	0.00	1.57	1.10	3.17	3.00
(6, 11+)	0.00	0.02	0.59	0.34	0.03	0.03	0.00	0.23	0.77	0.00	0.00	0.00	1.71	1.31	3.46	3.05
(7, 0)	1.49	2.36	0.46	0.10	0.41	0.03	0.01	0.26	0.00	0.74	0.00	0.00	1.55	1.17	1.76	1.19
(7, 1)	0.40	0.83	0.48	0.23	0.26	0.02	0.00	0.40	0.03	0.57	0.00	0.00	1.57	1.21	2.26	1.85
(7, 2)	0.35	0.89	0.48	0.29	0.21	0.02	0.00	0.50	0.11	0.39	0.00	0.00	1.57	1.39	2.63	2.46
(7, 3)	0.25	0.80	0.47	0.36	0.14	0.02	0.00	0.65	0.17	0.18	0.00	0.00	1.61	1.65	2.88	2.78
(7, 4)	0.13	0.69	0.46	0.43	0.09	0.02	0.01	0.76	0.24	0.00	0.00	0.00	1.59	1.84	2.95	2.94
(7, 5)	0.08	0.59	0.43	0.47	0.09	0.01	0.00	0.76	0.24	0.00	0.00	0.00	1.61	1.82	3.03	2.98
(7, 6)	0.04	0.42	0.42	0.49	0.08	0.02	0.01	0.63	0.37	0.00	0.00	0.00	1.55	1.65	3.07	2.98
(7, 7)	0.02	0.26	0.45	0.47	0.06	0.01	0.00	0.47	0.53	0.00	0.00	0.00	1.61	1.49	3.22	2.99
(7, 8)	0.01	0.13	0.49	0.46	0.05	0.01	0.00	0.25	0.75	0.00	0.00	0.00	1.58	1.28	3.16	2.98
(7, 9)	0.00	0.06	0.52	0.40	0.06	0.01	0.00	0.19	0.81	0.00	0.00	0.00	1.57	1.24	3.37	3.02
(7, 10)	0.00	0.02	0.56	0.34	0.09	0.01	0.00	0.24	0.76	0.00	0.00	0.00	1.57	1.35	3.28	3.18
(7, 11+)	0.00	0.01	0.69	0.27	0.03	0.00	0.00	0.34	0.66	0.00	0.00	0.00	1.80	1.44	3.50	3.16
(8, 0)	0.97	2.14	0.47	0.12	0.38	0.03	0.00	0.32	0.02	0.66	0.00	0.00	1.54	1.14	1.78	1.13
(8, 1)	0.24	0.63	0.48	0.32	0.18	0.01	0.00	0.56	0.17	0.27	0.00	0.00	1.63	1.52	2.37	2.34
(8, 2)	0.22	0.62	0.49	0.38	0.12	0.01	0.00	0.67	0.21	0.12	0.00	0.00	1.63	1.71	2.71	2.75
(8, 3)	0.15	0.51	0.48	0.42	0.10	0.01	0.00	0.78	0.20	0.03	0.00	0.00	1.64	1.85	2.94	2.91
(8, 4)	0.07	0.38	0.48	0.43	0.08	0.01	0.00	0.90	0.10	0.00	0.00	0.00	1.62	1.94	2.97	2.98
(8, 5)	0.04	0.32	0.49	0.43	0.07	0.01	0.00	0.91	0.09	0.00	0.00	0.00	1.64	1.93	3.06	3.00
(8, 6)	0.02	0.22	0.49	0.44	0.05	0.02	0.00	0.80	0.20	0.00	0.00	0.00	1.63	1.82	3.10	2.99
(8, 7)	0.01	0.13	0.49	0.44	0.04	0.02	0.00	0.66	0.34	0.00	0.00	0.00	1.58	1.69	3.16	2.98
(8, 8)	0.00	0.07	0.53	0.42	0.04	0.01	0.00	0.45	0.55	0.00	0.00	0.00	1.69	1.52	3.38	3.04
(8, 9)	0.00	0.03	0.56	0.43	0.01	0.00	0.00	0.50	0.50	0.00	0.00	0.00	1.66	1.60	3.45	3.18
(8, 10)	0.00	0.01	0.66	0.28	0.07	0.00	0.00	0.42	0.58	0.00	0.00	0.00	1.72	1.56	3.52	3.19
(8, 11+)	0.00	0.00	0.83	0.17	0.00	0.00	0.00	0.44	0.56	0.00	0.00	0.00	1.89	1.49	3.51	3.08
(9, 0)	0.64	1.80	0.50	0.16	0.32	0.02	0.00	0.47	0.03	0.51	0.00	0.00	1.56	1.29	1.86	1.47
(9, 1)	0.14	0.40	0.51	0.34	0.12	0.03	0.01	0.69	0.21	0.11	0.00	0.00	1.65	1.74	2.40	2.63
(9, 2)	0.13	0.37	0.51	0.39	0.08	0.02	0.00	0.76	0.24	0.01	0.00	0.00	1.65	1.86	2.73	2.87
(9, 3)	0.08	0.27	0.52	0.38	0.08	0.01	0.00	0.84	0.16	0.00	0.00	0.00	1.68	1.90	2.97	2.95
(9, 4)	0.04	0.17	0.54	0.37	0.08	0.01	0.01	0.97	0.03	0.00	0.00	0.00	1.65	1.98	2.99	3.00
(9, 5)	0.02	0.14	0.55	0.37	0.07	0.01	0.00	0.97	0.03	0.00	0.00	0.00	1.66	1.97	3.10	2.98
(9, 6)	0.01	0.10	0.54	0.39	0.06	0.01	0.00	0.88	0.12	0.00	0.00	0.00	1.68	1.90	3.25	3.00
(9, 7)	0.00	0.06	0.61	0.35	0.04	0.00	0.00	0.73	0.27	0.00	0.00	0.00	1.80	1.78	3.37	3.02
(9, 8)	0.00	0.03	0.64	0.33	0.02	0.00	0.00	0.50	0.50	0.00	0.00	0.00	1.79	1.58	3.45	3.18
(9, 9)	0.00	0.01	0.67	0.31	0.02	0.00	0.00	0.49	0.51	0.00	0.00	0.00	1.77	1.57	3.54	3.14
(9, 10)	0.00	0.00	0.58	0.42	0.00	0.00	0.00	0.42	0.58	0.00	0.00	0.00	1.48	1.56	3.24	3.13
(9, 11+)	0.00	0.00	0.83	0.17	0.00	0.00	0.00	0.38	0.62	0.00	0.00	0.00	2.04	1.52	3.73	3.24
(10, 0)	0.41	1.29	0.51	0.23	0.24	0.01	0.01	0.63	0.32	0.05	0.00	0.00	1.61	1.75	1.99	2.44
(10, 1)	0.08	0.19	0.54	0.35	0.09	0.02	0.00	0.79	0.21	0.00	0.00	0.00	1.70	1.88	2.47	2.84
(10, 2)	0.07	0.17	0.54	0.37	0.08	0.01	0.00	0.85	0.15	0.00	0.00	0.00	1.72	1.91	2.78	2.92
(10, 3)	0.05	0.11	0.56	0.37	0.06	0.01	0.00	0.90	0.10	0.00	0.00	0.00	1.72	1.96	3.00	2.98
(10, 4)	0.02	0.06	0.58	0.33	0.08	0.01	0.00	0.99	0.01	0.00	0.00	0.00	1.70	2.00	3.08	2.98
(10, 5)	0.01	0.05	0.62	0.30	0.07	0.01	0.00	0.96	0.04	0.00	0.00	0.00	1.66	1.97	3.13	2.98
(10, 6)	0.00	0.03	0.58	0.32	0.09	0.02	0.00	0.86	0.14	0.00	0.00	0.00	1.60	1.89	3.17	3.00
(10, 7)	0.00	0.02	0.66	0.31	0.02	0.01	0.00	0.75	0.25	0.00	0.00	0.00	1.71	1.83	3.28	3.18
(10, 8)	0.00	0.01	0.74	0.26	0.00	0.00	0.00	0.51	0.49	0.00	0.00	0.00	1.81	1.63	3.52	3.19
(10, 9)	0.00	0.00	0.79	0.21	0.00	0.00	0.00	0.47	0.53	0.00	0.00	0.00	1.75	1.56	3.24	3.13
(10, 10)	0.00	0.00	0.83	0.17	0.00	0.00	0.00	0.30	0.70	0.00	0.00	0.00	1.57	1.49	3.13	2.98
(10, 11+)	0.00	0.00	NaN	NaN	NaN	NaN	NaN	0.36	0.64	0.00	0.00	0.00	NaN	1.48	NaN	3.12
(11+, 0)	0.53	1.21	0.57	0.30	0.12	0.01	0.00	0.75	0.25	0.00	0.00	0.00	1.72	1.86	2.22	2.67
(11+, 1)	0.09	0.10	0.62	0.32	0.06	0.01	0.00	0.91	0.09	0.00	0.00	0.00	1.78	1.94	2.56	2.92
(11+, 2)	0.07	0.09	0.63	0.32	0.05	0.00	0.00	0.94	0.06	0.00	0.00	0.00	1.79	1.97	2.87	2.97
(11+, 3)	0.05	0.06	0.64	0.31	0.05	0.00	0.00	0.94	0.06	0.00	0.00	0.00	1.77	1.96	3.10	2.98
(11+, 4)	0.02	0.02	0.67	0.29	0.04	0.00	0.00	0.89	0.11	0.00	0.00	0.00	1.78	1.91	3.24	2.96
(11+, 5)	0.01	0.02	0.65	0.31	0.04	0.00	0.00	0.80	0.20	0.00	0.00	0.00	1.79	1.81	3.36	2.98
(11+, 6)	0.00	0.02	0.66	0.29	0.05	0.00	0.00	0.65	0.35	0.00	0.00	0.00	1.76	1.74	3.46	3.05
(11+, 7)	0.00	0.01	0.69	0.27	0.03	0.00	0.00	0.57	0.43	0.00	0.00	0.00	1.70	1.73	3.50	3.16
(11+, 8)	0.00	0.00	0.75	0.25	0.00	0.00	0.00	0.50	0.50	0.00	0.00	0.00	1.62	1.59	3.51	3.08
(11+, 9)	0.00	0.00	0.83	0.17	0.00	0.00	0.00	0.47	0.53	0.00	0.00	0.00	1.69	1.71	3.73	3.24
(11+, 10)	0.00	0.00	NaN	NaN	NaN	NaN	NaN	0.58	0.42	0.00	0.00	0.00	NaN	1.63	NaN	3.12
(11+, 11+)	0.00	0.00	NaN	NaN	NaN	NaN	NaN	0.62	0.38	0.00	0.00	0.00	NaN	1.73	NaN	3.46