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Bidding up, buying out and cooling-off: an examination of auctions with withdrawal rights*

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Summary. This paper considers a model in which bidders in an auction are faced with uncertainty as to their final valuation of the auctioned object. This uncertainty is resolved after the auction has taken place. It is argued that the inclusion of a cooling-off right raises the expected revenue to the seller when bidders face a risk of the object being a strict 'bad', in that owning the object incurs negative utility to the winner of the auction. The model is then tested in a laboratory setting. The evidence from this experiment supports the predictions of the theory.

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1 Auctions and cooling-off rights

A fairly common assumption in economics is that contracts are binding. In many markets, however, one party to a contract has a right to withdraw from the contract after it has been entered into. This right, when it applies to buyers, is often called a cooling-off right. Such a right may arise from the terms of the contract or may be imposed by statute.

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When a cooling-off right was introduced to the residential housing market in Victoria, Australia, the Hon. W.A. Landeryou, MLC, commented in the second reading speech of the Sale of Land (Amendment) Bill [11/24/1982] that

> "The cooling-off period will provide purchasers of land with some protection against impetuous buying or persuasive sales techniques and will enable a purchaser to obtain further advice in respect of the transaction."

In other words, if an object has different valuations in different states of the world, cooling-off rights exist to enable the buyers to ascertain which state of the world they are in.

There are many examples of markets where cooling-off rights exist. Examples of markets in Australia where statutory cooling-off rights exist are: residential housing markets; motor vehicle markets; and door-to-door sales (also in the USA). The details of the rights differ from jurisdiction to jurisdiction. However, there are several trends: the cooling-off right tends to last for 3 to 7 business days after the contract has been entered into; a nominal fee tends to be attached to the exercise of the right (often 1% of the buying price); and the right does not apply to sales by auction.

Von Ungern-Sternberg [13] reports instances of non-statutory cooling-off rights. He notes that it is relatively common for the winner of a tendering process in the Swiss and German construction industries to be given a similar option to withdraw from a contract. The idea of the right in this industry is to allow winning firms to evaluate their position with regard to other commitments before being bound to a new job. It is difficult to measure how prevalent such non-statutory cooling-off rights are as they depend on the express and implied terms of individual contracts. However, it is noteworthy that instances have arisen when parties have litigated over such terms, for instance in *Darley v John Valentine Health Group Pty Ltd (In Liq) (1987) 21 IR 441*, an Australian case where a cooling-off right was contained in a health club franchise agreement.

There seems sufficient evidence to suppose that cooling-off rights operate in several different types of markets around the world. This raises the interesting question of how such rights affect behavior in markets. This paper attempts to address this question in the context of a first price sealed bid auction, a stylized representation of markets that operate by tendering or bidding processes. It takes the independent private valuations (IPV) model developed by Vickrey [12] and incorporates the notion of a cooling-off right. The model is then tested empirically. A laboratory experiment was conducted in which undergraduate economics students at the Australian National University were faced with the exact problem faced by the bidders in the theoretical model. The data from the experiment is used to verify the equilibrium strategies predicted by the theory. Thus the contribution of this paper is two-fold: it examines the effect of cooling-off rights within the most comprehensively understood auction framework; and it offers an empirical evaluation of the resulting model.

A key innovation in the paper is the modeling of valuations. In the model values, while privately known, are state dependent. Bidders know their values in each state but are uncertain about which state they will find themselves in after the auction. This state is revealed during the cooling-off period. To give this structure some intuition, consider the following example.

A residential home is put up for sale. Prospective buyers may have the opportunity to inspect the home once or twice before they have to enter a bid. These inspections give prospective buyers an accurate impression of their value of the house, if what they see is what they get. However, in the back of their minds, they may acknowledge the possibility of other states of the world; perhaps they fear the house is infected with termites (a ’disaster’ state of the world) or perhaps they suspect the council plans to build a school nearby (a possibly good ‘surprise’ state of the world). On the day of the house auction it is hard to verify either of these suspicions; however after the auction both may be easily verified in a few days (by a building inspection or a call to the local council, respectively).

In both of these scenarios, termites or school, the prospective buyers of the house have different valuations depending on the state that they find themselves in. As compared with the state where what you see is what you get, the presence of termites would dramatically lower the value of the house (possibly making it negative if the problem was severe enough and the purchasers attach negative utility to the inconvenience it creates). In the second case, having a new school nearby may raise the value of the house.

Conceivably, the presence of the cooling-off right will dramatically affect bidding behavior in each example. In the case of suspected termites, bidders can ignore this possibility because they can withdraw from the contract if their fears are realized. This would make bidders more willing to bid aggressively for the house. Similarly, the possibility of having a new school nearby is more likely to induce higher bids if bidders have the option of withdrawing if they find they have bid too high and a school is not planned. Without a cooling-off right a bidder would want to hedge their bids in both instances, protecting themselves against the risk of being in the least attractive state of the world.

The question of how a cooling-off right affects behavior in auctions has not been widely explored in the literature. One paper that examines cooling-off rights and auctions explicitly is von Ungern-Sternberg [13]. This paper examines a multi-object auction where the bidder can withdraw their bid after winning the auction. The model considers a procurement auction where capacity-constrained firms bid for two jobs simultaneously. The firm will withdraw from a contract if both contracts are won and the firm is too constrained to perform both profitably. Von Ungern-Sternberg demonstrates that, in his setting, given sufficiently capacity-constrained firms, the cooling-off right is desirable for the employer of the tendering firms. This is due to the ability of firms to bid more aggressively without having to hedge against the possibility of taking on more than they can handle.

In von Ungern-Sternberg’s framework types are distributed over a known distribution with an unknown mean, with each bidder knowing their precise valuation. This allows considerable simplification of the strategic elements in the

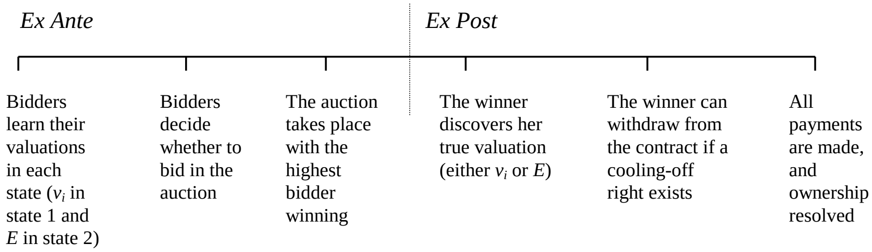

Figure 1. The timing of the model

auction he considers; specifically, bidders are unable to infer their probability of winning from their valuations. This makes it desirable to reconsider the effect of cooling-off rights in the standard IPV structure adopted in this paper.

As in von Ungern-Sternberg's paper, it is found that the hedging behavior of bidders lies at the heart of the problem. When faced with the chance of disaster, the expected revenue of the seller in the auction rises with the introduction of a cooling-off right. This is because bidders with a cooling-off right are able to ignore the disaster case and bid aggressively, while bidders with no such right have to hedge against it. The negative effect this hedging has on revenue is greater than the loss the seller experiences when the cooling-off right is exercised. The experimental data lends support to these theoretical conclusions.

This paper is organized as follows: Part 2 presents a model of an auction with a cooling-off right; Part 3 presents the empirical research; while Part 4 offers some closing remarks.

2 A theoretical model of an auction with a cooling-off right

2.1 The structure of the model

Figure 1 shows the timing of the model used to analyze the effect of a cooling-off period on a first price sealed bid auction.

When a potential bidder first inspects the object that is to be auctioned she forms a set of valuations. Each valuation corresponds to a state of the world. There are two potential states of the world. The value corresponding to state 1 is denoted v_i and is an independent draw from a commonly known distribution, F. However, the value of v_i is private information to bidder i.

In state 2 the value is denoted E. E is common to all bidders and known to be common. It corresponds to the value of the object in the "disaster" or "surprise" cases that were used earlier in discussing cooling-off rights.

The bidder also knows the probability of each state arising. The probability of state 2 arising is γ, with the probability of state 1 being $(1 - \gamma)$, where $\gamma \in (0, 1)$. After observing v_i, E and γ the bidder decides whether to participate in the auction. The highest bid wins the auction.

The seller is assumed to value the object at zero in both states of the world and have the same information as bidders about the relative likelihood of each state occurring.

If the bidder wins, the bidder is now in the *ex post* region of Figure 1. The state of the world (either state 1 or state 2) is realised and the bidder observes their true (or *ex post*) valuation (that is, either E or v_i). At this stage, if the cooling-off right exists, the bidder may choose to withdraw from the contract.

Once these steps are complete the payoffs to the parties are realized.

As in Vickrey [12], I assume risk neutral bidders, with bidder's state one valuations drawn from a uniform distribution on [0,1]. This has the advantage of making the exposition simple and easily applicable to an experimental setting.

In the following discussion the pure symmetric bidding strategies are derived for the benchmark case, where no cooling-off right exists. Then the equilibrium strategies are considered for the case where bidders have a costless cooling-off right. These two cases are compared on the basis of the bidding strategies and the expected revenue for the seller.

2.2 The benchmark case: no cooling-off right

In the benchmark case the bidder does not have the right to cool-off after the contract to buy has been entered into. This means the outcome of the auction is binding. In this setting the expected value of the object to the bidder is $(1-\gamma)\, v_i + \gamma E$. Thus, each bidder faces an individual rationality (or participation) constraint that must be satisfied if the bidder is to participate in the auction. Given that the only constraint on bids is that they be non-negative, this constraint is

$$(1-\gamma)\, v_i + \gamma E \geq 0\,. \tag{1}$$

When E is non-negative this constraint is satisfied for all types of bidders. When E is strictly negative this constraint will not be satisfied for some types of bidder and, hence, some types of bidder will not participate in the auction. Thus when E is negative the shape of the auction resembles an auction with a reserve price, where the reserve price is equal to $v^* = -\frac{\gamma E}{1-\gamma}$. When E is non-negative $v^* = 0$. That is, no bidder with an expected valuation less than v^* will enter a bid.

Assume that the equilibrium strategy is strictly monotonic and symmetric. Let $V(b_i)$ be the inverse of the bidding function to be followed by all bidders. It follows that the profit of bidder i is

$$\pi = [(1-\gamma)\, v_i + \gamma E - b_i]\ [V\,(b_i)]^{n-1} \tag{2}$$

where the last part of the expression, $[V\,(b_i)]^{n-1}$, is the probability that bidder i wins the auction (or has the highest valuation, v_i). The first order condition, with respect to b_i, is

$$0 = (n-1)\,[v_i]^{n-2}\, v_i'\,(b_i)\,[(1-\gamma)\, v_i + \gamma E - b_i] - [v_i]^{n-1}\,.$$

The assumption of symmetry allows $V(b_i)$ to be replaced by v_i. We can now proceed by either deriving $b_i(v_i)$ directly from this differential equation, or, leaving it defined implicitly, use the envelope theorem to derive $b_i(v_i)$ indirectly. This derivation follows the later route. From Equation 2, making b_i the maximization variable and v_i the parameter being varied, the envelope theorem yields $(1-\gamma)[V(b_i)]^{n-1} = \pi'(v_i)$. This means that

$$\pi(v_i) = \pi\left(v^*\right) + \int_{v^*}^{v_i} (1-\gamma)\, x^{n-1} dx \,. \tag{3}$$

Noting that $\pi(v^*) = 0$, as this 'marginal' bidder is indifferent between winning and losing the auction, we can use (2) and (3) to obtain

$$b_i = (1-\gamma)\, v_i + \gamma E - \frac{1}{v_i^{n-1}} \int_{v^*}^{v_i} (1-\gamma)\, x^{n-1} dx \,. \tag{4}$$

From the participation constraint (Equation (1)), we have the initial conditions that when $E \geq 0$, $\pi_i = 0$ if $v_i = 0$ and when $E < 0$, $\pi_i = 0$ if $v_i = -\frac{\gamma E}{1-\gamma}$. Equation (4) then gives the following symmetric equilibrium strategy for a risk neutral bidder in an auction with no cooling-off right:

$$b_i(v_i) = \begin{cases} (1-\gamma)\, \frac{n-1}{n} v_i + \gamma \mathrm{E} & \text{if } E \geq 0 \\ (1-\gamma)\left[\frac{(n-1)}{n} v_i + \frac{1}{n v_i^{n-1}} \left(\frac{-\gamma E}{1-\gamma}\right)^n\right] + \gamma E & \text{if } E < 0 \text{ and } v_i \geq -\frac{\gamma E}{1-\gamma} \\ no & \text{if } E < 0 \text{ and } v_i < -\frac{\gamma E}{1-\gamma} \end{cases} . \tag{5}$$

This bid function has an intuitive explanation. When $E \geq 0$ the equilibrium bid is the weighted average of the optimal bid in each possible state of the world. If the first state of the world were certain to occur the optimal strategy is to bid $\frac{(n-1)}{n} v_i$. However the chance of this happening is $(1-\gamma)$, so the bid $\frac{(n-1)}{n} v_i$ has a weight $(1-\gamma)$. In the second state of the world (where the true valuation is E) the best thing to do is bid E. Since E is common to all bidders, a form of (inverted) Bertrand competition emerges in this state of the world. Thus bids must converge on E. The bidder weights this strategy by γ, the chance of state 2 eventuating.

When $E < 0$ the bid function is again a weighted sum of the bid functions in the two states of the world. In state 1, the bid function is that of an IPV auction with reserve price $-\frac{\gamma E}{1-\gamma}$, since that reserve price is the value of v_i at which a risk neutral bidder is indifferent between winning and losing the auction. In state 2, each bidder wishes to bid the negative amount E, which would not be accepted by the auctioneer on its own, but can enter the overall bid function when combined with the positive bid function from state 1.

This case, where no cooling-right exists, is the benchmark against which to compare an auction in which bidders have the option to withdraw from the contract after they discover the state of the world.

2.3 Bidding in an auction with a costless cooling-off right

The introduction of a costless cooing-off right changes the payoff function, conditional on winning, from

$$\pi\Big|_{\text{win}} = \begin{cases} v_i - b_i & \text{in state 1} \\ E - b_i & \text{in state 2} \end{cases}$$

to

$$\pi\Big|_{\text{win}} = \begin{cases} v_i - b_i & \text{in state 1 and } \; v_i - b_i \geq 0 \\ 0 & \text{in state 1 and } \; v_i - b_i < 0 \\ E - b_i & \text{in state 2 and } \; E - b_i \geq 0 \\ 0 & \text{in state 2 and } \; E - b_i < 0 \end{cases} .$$

Now the winner can observe her profit after the state of the world is realized and decide whether, on the basis of this new information, she wishes to purchase the item. The new payoff function is made complicated by the fact that the bidder can now withdraw from the contract without cost. This means that whenever the winner of the auction will realize a loss, this loss can be reduced to zero.

To make the problem tractable it is helpful to divide bidders into two groups, those with $v_i \leq E$ and those with $v_i > E$.

Consider the problem facing bidders with $v_i \leq E$. If this type places a bid such that $b_i < E$ the best response from any competing bidder is to bid $b_i + \varepsilon$; that is, to bid marginally higher than b_i. By doing so the competing bidder gets, at least, the profit $E - (b_i + \varepsilon)$ should state two arise. In state one the competing bidder earns a profit of at least zero, due to the cooling-off right. Thus all bidders have an incentive to bid higher in response to a bid of less than E, regardless of the value of v_i or γ. Thus a form of Bertrand competition arises which drives all bids up to at least the level of E. Any bid less than E may be bettered by any bidder, and all bidders have an incentive to better such a bid.

This establishes the fact that in a pure strategy equilibrium E must be the absolute lower bound of any bidding strategy. However when faced with a bid of E the best response of a bidder in the group characterised by $v_i \leq E$ is to bid anything in their action set as, regardless of whether they win or lose the auction, their profit will be zero. Any bid greater than E will be withdrawn if it wins, while any bid less than E will not win. A bid of E will be withdrawn if it wins and state one arises, and earns zero profit in state two.

Any bidder with $v_i \leq E$ is indifferent between any possible bid in equilibrium. This creates multiple equilibria in the game. For example, $b_i = E + 10$ and $b_i = E + v_i$ are both equilibrium bidding strategies for bidders in this group. The symmetric pure strategy equilibrium bid function for $v_i \leq E$ can be thought of as $b_i = A(v_i)$ where $A(v_i)$ maps from the type space to $[E, \infty)$.

All but one of these possible equilibria has bidders entering spurious bids. Bids may be considered spurious if the cooling-off right is certain to be exercised. For example, if bidders are bidding $b_i = E + 10$ when $v_i \leq E$ and this bid wins the auction, the winning bidder is certain to exercise the cooling-off right regardless of the state of the world that eventuates. The reason that such bids are included in the set of best responses is that the cooling-off right is costless. If $v_i \leq E$ a zero payoff is incurred by bidding E or spurious bids of anything higher than E.

There are several arguments in support of rejecting the spurious equilibria in favor of equilibria where the ownership of the object to be auctioned is transferred in at least one state of the world. First, when bidders enter spurious bids they know that they will never get possession of the object. Once this observation is made it becomes difficult to see why a bidder would enter an auction if they have no intention of acquiring possession of the object. Second, non-spurious equilibria are attractive as they might form the limit of the case where a fee is attached to the exercise of the cooling-off right. While this is a conjecture, it makes intuitive sense. If we consider an epsilon (small) fee attached to the cooling-off right and then reduce this fee toward zero, the hedging behavior of the bidder against this fee will become insignificant and strategies will approach the non-spurious equilibria. However, such a fee will always remove any spurious bids from the best response set. Lastly, spurious bids require more actions by the bidder than non-spurious bids. The bidder has to make a positive action by electing to cool-off, whereas if a non-spurious bid is entered the bidder may not have to act at this stage in the game. However, the payoff for both types of bid is the same. It seems counter-intuitive to expect a bidder to choose to engage in more actions for the same payoff when doing so incurs no strategic advantage.

On this basis, the equilibrium bids of bidders with $v_i \leq E$ is restricted to $b_i = E$. This is the only non-spurious bid function that constitutes a symmetric pure strategy equilibrium.

It remains to consider those bidders with $v_i > E$.

If $E < 0$ all types will elect to cool-off if state 2 eventuates. Since this can be done costlessly E will be ignored and all types will bid according to their normal strategy for a first price seal bid auction, as first proposed by Vickrey [12], so that $b_i(v_i) = \frac{n-1}{n} v_i$.

If $E \geq 0$ then types with $v_i \leq E$ bid $b_i = E$. It was argued earlier, in the context of types with $v_i \leq E$, that E is the lower bound of any equilibrium bidding strategy. This argument is directly applicable to types with $v_i > E$. This implies that any bid less than E is pointless in equilibrium as it has a certain profit of zero. This makes the problem of bidders with $v_i > E$ isomorphic with the problem of bidders in an auction with a reserve price of E – in both cases bids in equilibrium are restricted to be above E. Thus for $v_i > E$, $b_i = \frac{n-1}{n} v_i + \frac{E^n}{n v_i^{n-1}}$ when $E \geq 0$.

Hence, refining equilibria such that spurious bidding is eliminated, the equilibrium bidding function in the presence of a costless cooling-off right is

$$b_i = \begin{cases} E & \text{if } v_i \leq E \\ \frac{n-1}{n} v_i + \frac{E^n}{n v_i^{n-1}} & \text{if } v_i > E \text{ and } E \geq 0 \\ \frac{n-1}{n} v_i & \text{if } E < 0 \end{cases} . \tag{6}$$

2.4 Comparing equilibria on the basis of bidding strategies

Equation (6) gives the non-spurious equilibrium bidding strategy in the presence of a costless cooling-off right. Equation (5) gives the equilibrium bidding strategy in the absence of a cooling-off right. Comparing the bidding strategies under the two regimes establishes the following proposition:

Proposition 1. *In the symmetric, pure strategy, non-spurious equilibria derived above, the bidding strategy under cooling-off always results in a higher bid, for a given type, than the bidding strategy used when a cooling-off right does not exist.*

Proof. See Appendix.

This is because the existence of a cooling-off right promotes more aggressive bidding as it allows bidders to limit their downside.

From Proposition 1 one can derive the following corollary:

Corollary 1. *In the symmetric, pure strategy, non-spurious equilibria derived above, the expected winning bid with a costless cooling-off right is always greater than that in an auction without a cooling-off right.*

If every type enters a higher bid under costless cooling-off, then the expected winning bid must also be higher as the entire range of bids is shifted upwards with the introduction of the cooling-off right.

2.5 Comparing the equilibria on the basis of expected revenue

Table 1 reports the expected revenue of auctions for both types of cooling-off regime and various ranges of E. Expected revenues are calculated by taking the sum of the bids of each type, weighted by the probability that that type wins. Table 1 also compares the expected revenue of auctions with and without cooling-off; the regime with the larger expected value is indicated using an inequality. Propositions 2 through 4 establish these relative magnitudes.

Proposition 2. *When E is strictly positive, bidders play symmetric, pure strategies, and spurious equilibria are eliminated, the expected revenue from an auction without a cooling-off right is strictly greater than the expected revenue from an auction with a costless cooling-off right.*

Proof. See Appendix.

When a costless cooling-off right is present, bidders will focus their bidding on the state of the world in which they have the highest valuation, and expect to cool-off in the other state. This leads to more aggressive bidding. From the seller’s perspective this aggressive bidding is off-set by the fact that in one state of the world the winner of the auction will withdraw from the contract, resulting in zero revenue for the seller. This ability to withdraw costlessly means that bidders do not bother to hedge against their least preferred state of the world

Table 1. Expected revenue across auctions

Value of E	With cooling-off	Relative magnitude	Without cooling-off[a]
$E < 0$	$(1-\gamma)\,\frac{n-1}{n+1}$	$>$	$\gamma E\left[1-\left(\frac{-\gamma E}{1-\gamma}\right)^n\right] + (1-\gamma)\,\frac{n-1}{n+1}\left[1-\left(\frac{-\gamma E}{1-\gamma}\right)^{n+1}\right] + (1-\gamma)\left(\frac{-\gamma E}{1-\gamma}\right)^n\left(1+\frac{\gamma E}{1-\gamma}\right)$
$E = 0$	$(1-\gamma)\,\frac{n-1}{n+1}$	$=$	$(1-\gamma)\,\frac{n-1}{n+1}$
$0 < E \leq 1$	$\gamma E + (1-\gamma)\left(\frac{n-1}{n+1} + E^n\right)(1-E)$	$<$	$(1-\gamma)\,\frac{n-1}{n+1} + \gamma E$
$E > 1$	γE	$<$	$(1-\gamma)\,\frac{n-1}{n+1} + \gamma E$

[a] Note that $0 \leq \frac{-\gamma E}{1-\gamma} \leq 1$

occurring. Where E is greater than zero, those bidders who have v_i greater than E will behave as if they are in an auction with a reserve price of E. However, the amount that this pushes up bids can never be equal to the size of the reserve price itself. Hence, it does not fully compensate the seller for the revenue lost from the bidder being able to withdraw in state 2. When compared with an auction without cooling-off, the ability of bidders to disregard one state of the world results in 'under-hedging' - their bids do not incorporate the hedging needed in auctions without cooling-off. From the seller's point of view this results in a situation analogous to an auction without cooling-off, where one of v_i (if E is greater than v_i) or E is set to zero. When both states of the world result in the object for sale being a good (i.e. $E > 0$), this means that the seller is unable to extract value in both states of the world and hence loses revenue.

Proposition 3. *When E equals zero, bidders play symmetric, pure strategies, and spurious equilibria are eliminated, auctions with a costless cooling-off right and auctions with no cooling-off right are revenue equivalent.*

This follows straight from the observation that, when E equals 0, auctions under both regimes mirror the first price auction considered by Vickrey [12].

Proposition 4. *When E is strictly negative, bidders play symmetric, pure strategies, and spurious equilibria are eliminated, the expected revenue from an auction without a cooling-off right is strictly less than the expected revenue from an auction with a costless cooling-off right.*

Proof. See Appendix.

The cooling-off right creates a tendency for the bidder to under-hedge when faced with risk. In the case where E $>$ 0 this lead to a loss in revenue as it inhibited the seller's ability to extract expected value. When E $<$ 0 the opposite is observed, because the bidder does not have to hedge against the 'bad' state of the world, the seller is not made to accept bids that incorporate compensation for the possibility of state 2 arising. The seller benefits from the under-hedging, as, in this instance, it more than compensates for the revenue lost when the cooling-off right is exercised. As E tends to large negative amounts, and when no cooling-off right exists, this effect becomes starker as large numbers of bidders choose not to participate in the auction. These bidders are kept in the auction when the cooling-off right is introduced.

These propositions have been established in the context of a first price sealed bid auction. This corresponds to the empirical testing of the model in part 3. It is conjectured that the revenue equivalence result for risk neutral bidders (Milgrom and Weber [8] for its most general statement) may allow us to extend the results across other auction types.

It is also interesting to note that the expected surplus of bidders is increased by a cooling-off right when $E < 0$ and decreased when $E > 0$ (calculated using the bid functions in Equations (5) and (6)). This coincides with the effects on expected revenue set out in Propositions 2 through 4. Thus, when it is more efficient to have a cooling-off right *ex ante*, in the sense of increasing expected joint surplus, it is also revenue increasing. When $E < 0$ a cooling-off right is also *ex post* efficient in that the agent with the highest value will always get possession of the object. This is not the case without a cooling-off right as E may be so low that no bidder bids even though, *ex post*, exchange would have been welfare enhancing. These efficiency results may go some of the way toward explaining why we observe parties agreeing to include cooling-off periods in some contracts.

3 Testing the model in the laboratory

This section seeks to verify the bidding strategies that were derived for auctions with, and without, a cooling-off right (contained in Equations (5) and (6)). Those aspects of the predicted strategies tested experimentally are:

1. The point predictions of the bid functions: Do bids by participants in the experiment deviate significantly from those predicted by theory?
2. The proposed refinement excluding spurious bids: How often do spurious bids occur?
3. The explanatory power of the theory: What proportion of the observed variation in the bids does the theory explain?
4. Coefficients in linear bid functions: When bid functions are linear in the variables, do the estimated coefficients differ significantly from those predicted?
5. Changes in the bid functions: As E changes, do the bids exhibit structural breaks when, and only when, the form of the predicted bid function changes?

If these strategies are supported by the data, then Propositions 1 through 4 are similarly supported.

3.1 The design of the experiment

This experiment was conducted on six groups of six subjects. 10 first price sealed bid auctions were conducted on each group. In each auction the subject was given two values corresponding to v and E in the preceding discussion. The subject was then invited to enter a bid of either *no*, indicating that they did not wish to participate in the auction, or any non-negative amount expressed as dollars and cents. The *ex post* valuation was decided by a dice roll. There was a one-in-six chance that E was the *ex post* valuation, and a five-in-six chance that v was the *ex post* valuation. Once the *ex post* valuation was resolved, the profit of the winning bidder was calculated by subtracting the winning bid from the *ex post* valuation.

The values of E were common to all bidders. This was common knowledge. The values of v that were assigned to each subject were independent draws from a uniform distribution over the integers in the set $\{0,1, 2, ..., 199, 200\}$, expressed in cents. Thus v could range from \$0.00 to \$2.00 Australian.

In the first five auctions there was no cooling-off right. In the last five auctions there was a costless cooling-off right. An automatic decision rule was imposed on the auctions with a cooling-off right so that if the winning bidder was going to realize a loss this loss was reduced to zero. Thus the auctions with cooling-off rights were explained in the instructions as “you cannot make a loss in an auction if you win”.

The subjects were given \$5.00 for arriving at the experiment on time. This \$5.00 then served as a float, so that, if a winning bid turned out to be higher than the *ex post* valuation in an auction without cooling-off, a negative profit could be realised. The final payoff to a subject was calculated as \$5.00 plus the net profit of that subject over the ten auctions. The highest payoff paid was \$6.93 and the lowest was \$3.38. The mean payoff was \$5.21.

This up-front lump-sum payment of \$5 may have affected behavior in the auction by dampening the anticipation of negative profits. One of the main channels through which cooling-off affects bidding is via the possibility of making a negative profit. If participants perceived this up-front payment as substantially removing this possibility, then the payment would distort bidding behavior away from the predicted bids. This raises a problem, common to experimental work, in distinguishing rejection due to the model not capturing behavior, from rejection due to a design problem in the experiment. On the other hand, if the participants had incorporated the payment into the reference point from which they judge gains and losses, then the problem will not exist. The potential impact on the interpretation of results needs to be weighed against the practical difficulty of inviting participants to engage in an experiment that costs them money. Since, as argued later, the data from this experiment tend to support the theory, it seems unlikely that bids were significantly distorted by the up-front payment of \$5.

The progressive profit of each subject was not reported during the course of the experiment. The primary reason for this was that keeping 36 subjects up to date as to their profit would have taken too long with the resources at hand. It is unclear whether not reporting profit would have changed the common problem of wealth effects influencing behavior as profit changes during the experiment. Generally, there is little that can be done to address this problem in experiments of this kind.

The 36 subjects were gathered together in one room and split into groups of six. The subjects in each group bid against each other in the subsequent auctions. It was decided to have six subjects in each group as experiments in the past have indicated that this was a sufficient size to effectively deter any collusive behavior (Cox, Roberson and Smith [2]).

After being divided into groups the subjects were given a set of written instructions. The same booklet also contained the forms on which participants entered their bids in each auction. It was made clear to the subjects, both verbally and in the instructions, that any form of communication between subjects would render the experiment a failure and was prohibited.

Before the experiment began, subjects were given two practice runs. In these practice runs, and in the ten real auctions, subjects had two minutes to make their bid. This was felt to be more than adequate time.

It was useful to hold all the auctions for all the groups at the same time in the same room. This meant that any information given to one group in response to a question was given to all groups. It also meant that the timing of the experiment was the same for all groups. It follows that any other environmental factors that may affect results are constant across the groups.

Six laboratory assistants were assigned to oversee the auction process, one for each group. These assistants were familiar with the experiment structure, having been the subjects of a test run of the experiment. The assistants were instructed to submit any substantive questions from the participants to the author, who oversaw the experiment.

The subjects were drawn from undergraduate economics students at the Australian National University, in August 1998, with the author presenting a short address to each of several economics classes. From the 50 students who responded, the 36 subjects were randomly selected.

3.2 A description of the data

The experiment yielded 360 separate data points (bids), taken from 60 different auctions. The sixty auctions were divided into 10 sets of 6, with each set characterised by different values of E. These parameter values are shown in Table 2. The values of E, in Australian dollars, ranged from -0.4 to 3. These values were chosen to test all of the variations in the bid functions predicted by the theory. The theoretical predictions for each value of E were tested using 36 observations. Each auction type is referred to using the following mnemonic: the letter, N or C, refers to no cooling-off and cooling-off, respectively; the number, 1–5, refers to the level of E, in ascending order. Thus, N5 is the auction with no cooling-off right and the highest level of E, \$3.

Table 2. Parameter values for each auction

Auction type	Value of E	Costless cooling-off?
N1	−0.40	N
N2	0.00	N
N3	0.50	N
N4	1.20	N
N5	3.00	N
C1	−0.40	Y
C2	0.00	Y
C3	0.50	Y
C4	1.20	Y
C5	3.00	Y

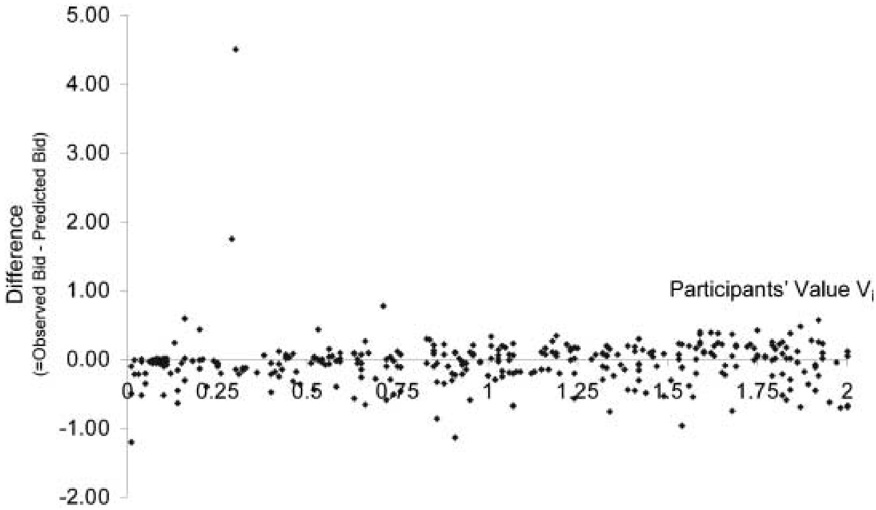

Figure 2. All auctions excluding C5 (mean difference: −0.05)

Each observed bid was compared with the predicted bid generated from the theory. The difference between the prediction and the observed bid was calculated, with a negative difference indicating that the observed bid was lower than the predicted bid. These differences are plotted for all auctions except C5 in Figure 2. Auction C5, which is somewhat unique, is shown separately in Figure 3.

In most of the auction types there is a very slight upward trend in the data points. This is evident in Figure 2. As the value of v_i increases, the difference between the predicted value of the bid and the actual bid tends to increase. For small values of v_i there is a slight tendency for the predictions to overestimate bids, and for high values of v_i there is a slight tendency for the predictions to underestimate bids. Generally, however, the differences tend to be scattered around zero.

Auction C5, however, is an exception to this generalization. All the bids in auction C5 were at or below the predicted bid. This suggests that something

Table 3. Spurious bidding

Participant no.	Value of v	Value of E	Observed bid
c2	0.29	−0.40	1.99
c2	0.30	0.50	5.00
d1	0.71	0.50	1.38
d1	1.16	1.20	1.27
e2	1.16	1.20	1.36

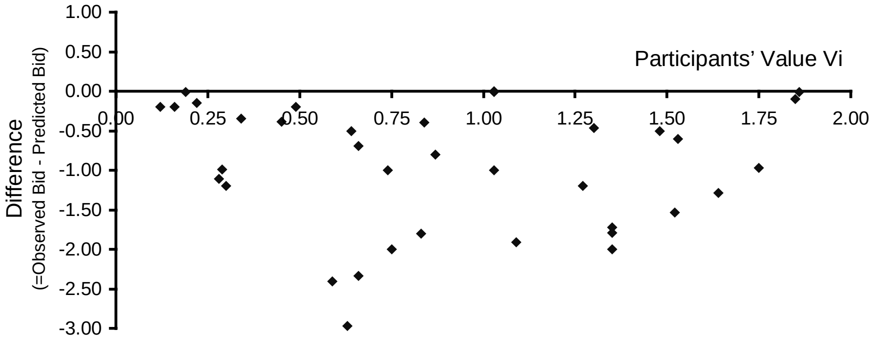

Figure 3. Auction 5 (mean difference: −0.967)

about auction C5 makes it significantly different from the rest of the auctions. The possible reasons for this will be explored later.

Before engaging in econometric analysis it is useful to explore some of the patterns that emerge from a cursory examination of the data.

Spurious bidding behavior was observed in some of the auctions with cooling-off. Table 3 records the parameter values for these bids and the bids themselves. Table 3 shows that, out of 180 bids collected from auctions with cooling-off, only five were spurious in the sense of inviting automatic cooling-off regardless of the state of the world that arose. All of the 180 observations from auctions with cooling-off rights invited spurious bids – to see this note that in any of these auctions it is a Nash equilibrium for all types to bid, say, \$100, which is much more than either v_i or E.

These spurious bids can be seen as positive signals that people were aware of the structure of the bidding problem and had a feel for the equilibrium strategies and consequent payoffs. It is also comforting to note that the theory acknowledges the possibility of this behavior, while the fact only 5 spurious bids were observed suggests that the proposed equilibrium refinement was perhaps justified as a first approximation. These spurious bids raise the interesting question of how to generalize the theory to accommodate occasional deviations.

A significant proportion of the subjects (at least 7) exhibited a reluctance to bid higher than their value of v in the auctions with no cooling-off right. The theory predicted that bids should have been higher than v in 105 of the

Table 4. Loss averse bids (in auctions with no cooling-off)

Participant no.	Value of E	Value of v	Observed bid	Predicted bid
d3	1.20	0.51	0.50	0.55
d4	1.20	0.33	0.30	0.43
d6	1.20	0.32	0.30	0.42
f5	1.20	0.14	0.14	0.30
d1	3.00	0.10	0.05	0.57
e1	3.00	0.48	0.47	0.83
f4	3.00	0.91	0.90	1.13
f5	3.00	1.17	1.17	1.31

observations. In 36 of these observations this failed to occur. Table 4 shows examples of this behavior.

This behavior seems to be due to something more than 'normal' errors in bidding. The mean differences between observed and predicted bids for these 36 observations is -0.36 compared with -0.14 for the entire experiment. A simple t-test rejects the null hypothesis that these means are the same (p= 0.005). The behavior may be explained by fairly strong aversion to the chance of making a loss. In each of the instances shown in Table 4 the subject's predicted bid was above their value of v. However, the subjects placed their bid at, or just below, their value of v. This suggests that subjects had no wish to incur the possibility of a making a loss should they win in the auction and v turned out to be the true valuation. Interestingly, there were no clear cases of people being averse to bidding above E, even from those in the above sample. This suggests that the relatively high likelihood of having v_i as the true valuation was an important factor in the decision of when to remove any chance of making a loss.

Overall the auction procedure was a fairly efficient allocation mechanism. 74% of all the auctions conducted (not counting the auctions of type C5) resulted in the predicted winner winning the auction. Each of the two cooling-off regimes incurred seven inefficient auctions. Of the 14 auctions that did not have the predicted winner winning, 10 were won by the bidder with the second highest expected value. It is not useful to consider auction C5 in respect to efficiency as the parameter values were such that, in equilibrium, it was expected that all subjects had an equal chance of winning.

Since the key results of the theoretical model presented in this paper concern expected revenue it is of some interest to examine the actual revenue received in this experiment. Table 5 reports the average revenue of the six auctions of each auction type. It also reports the paired difference of auctions with, and without, cooling-off for common values of E. The null hypothesis that each difference is zero is tested in each case (the test statistic is reported).

The results in this table must be treated with considerable caution. The number of observations is small because there are only six auctions to pool data from in each case. The sensitivity of the comparisons to differences in v_i and whether cooling-off occurs must also be kept in mind. Lastly, with such a low number

Table 5. Comparison of average revenue

Value of E	No cooling-off	Cooling-off	Paired difference	t-statistic[a]
−0.40	1.34	1.19	−0.152	−2.206*
0	1.47	0.93	−0.537	−4.765***
0.5	1.47	1.19	−0.272	−3.347***
1.2	1.41	1.26	−0.147	−1.762
3	1.74	0.42	−1.325	−7.450***

[a] The null hypothesis is that the paired difference is equal to zero. The test statistic is $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ which has a t distribution with $n_1 + n_2 - 2$ degrees of freedom. The pooled estimate of the population variance is s^2.

* indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.
** indicates that the null hypothesis can be rejected with only a 0.05 chance of committing a type one error.
*** indicates that the null hypothesis can be rejected with only a 0.01 chance of committing a type one error.

of observations it is necessary to assume that the population variances are equal when performing the t-test. It is hard to justify this assumption from the theory. That said, it is nice to observe that as E increases there is an increased tendency for no cooling-off to dominate cooling-off in terms of revenue.

3.3 Analysis of the data

The mean difference between the observed bids and the theoretical predictions was calculated for various combinations of auctions. A hypothesis test was then conducted with a null hypothesis that the mean difference was zero. The results of these tests are reported in Table 6.

For the overall sample of 60 auctions and 360 bids the mean difference was −0.138. The corresponding test statistic was −5.020, strongly rejecting the null hypothesis that the mean difference is zero. This suggests that the theoretical model is not a good predictor of the bidding in these auctions. However, much of this error in prediction can be attributed to certain parameter settings. In particular, the bidding in auction type C5 was poorly predicted. The influence of auction C5 on the results is shown by comparing the test statistic for all the auctions (auctions N1–5 and C1–5) with that for auctions N1–5 and C1–4; the removal of auction C5 more than halves the value of the test statistic. Although making this adjustment still means that the null hypothesis can be rejected at any significance level greater than 0.025, it illustrates the degree to which auction C5 skews the hypothesis test.

Examining the tests closely reveals that the theory performs well as a predictor of bids when the bulk of the probability mass of the random variable v lies above the value of E. That is, for cases with, and without, cooling-off the theory

Table 6. Tests of mean differences

Auction nos.	All	N1–5, C1–4	N1–5	N1–4	C1–5	C1–4
Mean difference	−0.138	−0.046	−0.062	−0.021	−0.214	−0.026
Standard deviation	0.522	0.386	0.294	0.274	0.670	0.478
Test statistic[a]	−5.020***	−2.150**	−2.822***	−0.929	−4.300***	−0.666
Auction nos.	N1	N2	N3	N4	N5	C1
Mean difference	0.001	−0.009	−0.007	−0.070	−0.225	0.023
Standard deviation	0.352	0.260	0.277	0.184	0.321	0.357
Test statistic[a]	0.030	−0.212	−0.162	−2.273***	−4.208***	0.382
Auction nos.	C2	C3	C4	C5	N1–2	C1–2
Mean difference	−0.068	0.089	−0.150	−0.967	−0.004	−0.023
Standard deviation	0.222	0.795	0.299	0.799	0.307	0.299
Test statistic[a]	−1.830*	0.671	−3.007***	−7.265***	−0.102	−0.641

[a] The null hypothesis is that the mean difference is equal to zero. The test statistic is $t = \frac{\bar{x}\cdot\sqrt{n}}{s}$ which has a t distribution with $n - 1$ degrees of freedom.

* indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.
** indicates that the null hypothesis can be rejected with only a 0.05 chance of committing a type one error.
*** indicates that the null hypothesis can be rejected with only a 0.01 chance of committing a type one error.

performs well when $E = 0.50, 0.00$ or -0.40. In auctions N1, N2, N3, C1, and C3 the null hypothesis cannot be rejected at any meaningful level of significance. Auction C2 is somewhat borderline in that it has a p-value of 0.06. This can be compared with the results of auctions N4, N5, C4 and C5, all of which reject the null hypothesis at a significance level of 0.01 or more. These results illustrate a stark division in the subjects approach to evaluating choice over risk. When the subjects were faced with a small chance of making a relatively significant loss, subjects tended to develop strategies more or less in line with those predicted. However, when subjects were faced with the prospect of a small chance of making a relatively significant gain they did not adopt strategies similar to those predicted. Instead, they tended to entered bids consistently lower than predicted. This suggests that the subjects were more risk averse when faced with a small chance of a gain, as opposed to a loss.

This is an interesting result as it bears some similarity to the predictions of the prospect and cumulative prospect theories of choice under risk and uncertainty (Kahneman and Tversky [5] and Tversky and Kahneman [11], respectively). In both theories the value function is concave for gains and convex for losses, implying that we should frequently observe risk aversion in gambles over gains and risk seeking in gambles over losses. This is consistent with the behavior presented here. However, in both theories the decision makers true risk preferences are determined jointly by their value function and their weighting function.

When probabilities over gains and losses are sufficiently low the shape of the weighting function may reverse the above characterization, so that risk seeking is observed in gambles over gains and risk aversion in gambles over losses. So a key issue in deciding how these experimental results relate to the various versions of prospect theory is whether the probability over gains and losses, $0.1\overline{6}$, is 'low'. In an experiment designed to explore this aspect of the theory Tversky and Kahneman [11] considered a probability of 0.1 as low and a probability of 0.5 as high (in the sense of inviting risk aversion over gains). $0.1\overline{6}$ lies between these ranges. Hence, the behavior observed in this experiment, while perhaps explained by prospect theory, cannot be easily characterized as confirming or contradicting it.

Comparing the predictions of the model on the basis of the different cooling-off regimes does not lead to any conclusion that the model performs better under one regime than the other. If the result for auctions N1–5 is compared to that for auctions C1–5, the respective test statistics are -2.8 and -4.3. However, both these test statistics are sufficient to reject the null hypothesis at the 0.01 significance level, thus comparing predictive power is, in a sense, pointless. If we exclude the auctions where $E = 3.00$ (i.e. auctions N5 and C5), the predictions for the case with cooling-off do slightly better, on the basis of the test statistics, than the case without cooling-off. However this is a very rough yardstick with which to compare predictive power. The most that can be said is that, in both cases, the predictions seem to perform quite strongly as in neither case is the null hypothesis rejected.

Auction C5 deserves special mention. It stands out as being the auction in which the theory did not perform well. The theory has posited that the symmetric pure strategy equilibrium in this auction is to bid $b_i = E$. However, there also exist asymmetric pure strategy Nash equilibria, of the form

$$\begin{aligned} b_i &\in [E, \infty) \\ b_j &\in [E, \infty) \text{ and} \\ b_k &\in no \cup [0, \infty) \end{aligned}$$

where b_k is the bid entered by those individuals in the set of bidders not including bidders i and j. This result is apparent once we observe that, if two bidders (i and j) enter bids greater than E, in equilibrium the payoffs of all other bidders (k) are zero regardless of what they do. Hence, these other bidders (k) can choose any bid. In the experiment this translates into a prediction that at least two bidders in auction C5 would bid \$2.99 or more (the predictions have to adjusted for the fact that the bidding space is discrete in cents). Once these predictions are adjusted in this manner, and compared to the observed bids, we see that one of the auctions of type C5 conforms exactly to an asymmetric Bayesian Nash equilibrium in pure strategies. Two of the others come close. It is some comfort that this tendency for strategies in Bertrand games to diverge from the standard Nash equilibrium has been observed in other experimental studies (see Plott [9], for instance). Baye and Morgan [1] investigate possible reasons for this behavior

Table 7. Percentage of the variation in bids explained by the theory

Model	% of variation in bids explained by model
All Auction Types	
Theory	0.41
$bid = \beta_0 + \beta_1 v_i + \beta_2 E$	0.43
Auction Types N1–5, C1–4	
Theory	0.55
$bid = \beta_0 + \beta_1 v_i + \beta_2 E$	0.54
Auctions N1–2, C1–2	
Theory	0.67
$bid = \beta_0 + \beta_1 v_i + \beta_2 E$	0.66

in the data. They find that two equilibrium concepts that incorporate notions of bounded rationality, Radner's [10] epsilon equilibrium and McKelvey and Palfrey's [6] quantal response equilibrium, are significantly more successful in organizing their data than the Nash equilibrium concept. This suggests that some notion of bounded rationality may explain the unique bidding behavior in auction C5.

The preceding theoretical discussion brought out some important normative prescriptions for when a cooling-off regime might be desirable for a seller. It was argued that a cooling-off regime benefits the seller when the value of E is less than zero. With this in mind particular attention was paid to the quality of the predictions in auctions N1, N2, C1 and C2. When auctions N1 and N2 were analyzed together the test statistic had a p-value of 0.94, while for auctions 6 and 7 the combined test statistic had a p-value of 0.54. Both these results suggest that that the null hypothesis was fairly robust for both cooling-off regimes when E was less or equal to zero.

To get a more precise impression of the quality of the predictions the percentage of the variation in the observed data explained by the theory was calculated. This is a calibration exercise where the calibration parameters are given by the equilibrium bid functions (Equations (5) and (6)). The r-squared statistic from an OLS linear regression was calculated to provide a point of reference. Table 7 reports the results.

When all the auctions are considered together the theory is able to explain 41% of the observed variation in the bids. This compares to 43% for the estimated linear model. When auction C5 is omitted from the data set, the theory outperforms the linear models, explaining 55% of the variation in bids. The non-linearity of the theory allows it to outperform the linear model, despite the fact that the linear model has more degrees of freedom. The linear model explains 54% of the variation. Auctions N1, N2, C1 and C2 are particularly important from a theoretical viewpoint, as in these auctions it is advantageous for a seller

Table 8. Coefficient estimates for linear strategies

Regression model:	$bid = \beta_0 + \beta_1 v_i + \beta_2 E$		
Parameter	Prediction	Estimate[a]	t-statistic[b]
Auctions N2–5			
β_0	0	−0.065 (0.045)	−1.446
β_1	0.694	0.779 (0.035)	2.383**
β_2	0.166	0.085 (0.019)	−4.258***
Auctions C1–2			
β_0	0	0.021 (0.073)	0.291
β_1	0.833	0.739 (0.057)	−1.642
β_2	0	−0.220 (0.173)	−1.272

[a] The standard errors are in parentheses.
[b] The null hypothesis is that the estimate is equal to the prediction. The test statistic is $t = \frac{\beta - B}{s(\beta)}$ which has a t distribution with $n - (k + 1)$ degrees of freedom.

* indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.
** indicates that the null hypothesis can be rejected with only a 0.05 chance of committing a type one error.
*** indicates that the null hypothesis can be rejected with only a 0.01 chance of committing a type one error.

to have a cooling-off period. Overall, the theory is successful in explaining 67% of the variation in bids in auctions N1, N2, C1 and C2. This compares to 66% by the linear model. This suggests that the theoretical model is a good explanator of bidding behavior when $E \leq 0$.

Where bidding strategies are linear it is possible to test the coefficients predicted by the theory using linear OLS regressions. Auctions N2–5 and C1–2 have linear bid functions. Table 8 compares the estimated parameters from the regression to those predicted by the theory. In both sets of auctions it is not possible to reject the null hypothesis that the intercept is zero. In fact it is not possible to reject any of the theoretical predictions in auctions C1 and C2. In auctions N2–5 the predictions of the coefficients of v_i and E are both rejected. This suggests that the coefficients predicted by the theory are most accurate in auctions with cooling-off right where E is less than or equal to zero.

The last testable hypothesis from the theory is that as E changes, so do the strategies played by bidders. Equations (5) and (6) predict that auctions N1, C3, C4 and C5 have unique biding strategies, while auctions N2, N3, N4, and N5 should have the same strategies. Auctions C1 and C2 should also share the same strategies. This should manifest itself in coefficient estimates being structurally

Table 9. Chow test statistics[a] for instances where structural breaks were expected

Auction nos.	N1	N2–5	C1–2	C3	C4	C5
N1	–					
N2–5	1.498	–				
C1–2	5.096***	3.414**	–			
C3	na	8.398***	5.550***	–		
C4	na	14.451***	9.573***	na	–	
C5	na	82.242***	18.278***	na	na	–

[a] The null hypothesis is that the coefficient estimates are the same. The structure of the linear regression model used to test this was $Bid = \beta_1 v_i + \beta_2 E$. The test statistic is $\mu = \frac{(T-2k)(SSE_R - \Sigma SSE_U)}{k\,\Sigma SSE_U}$ which is distributed over $F(k, T-2k)$.

* indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.
** indicates that the null hypothesis can be rejected with only a 0.05 chance of committing a type one error.
*** indicates that the null hypothesis can be rejected with only a 0.01 chance of committing a type one error.

stable in those auctions in which strategies are the same. It is possible to test for this using structural break tests. This was done using the Chow test.

The Chow test uses linear regression modeling to test for significant changes in coefficient estimates, which indicate an underlying change in the modeled behavior. Because the Chow test uses linear regression techniques it is not appropriate to use it to test for structural changes between auctions N1, C3, C4 and C5. This is because the theory predicts that, in auctions N1, C3, and C4, the coefficient of v_i is a step function (see Equations (5) and (6)). Since the appropriate equilibrium model for auction C5 is uncertain (after looking at the data) it is also included in this group. However, it is appropriate to use the Chow test where at least one of the equilibrium strategies is linear. Table 9 reports the results of structural break tests done where a structural break was expected. Table 10 reports the results of structural break tests done where a structural break was not expected.

Table 9 shows that, with the exception of auction N1 versus auctions N2–5, structural breaks exist where they are expected. This is strong evidence in support of the proposition that strategies change as the value of E changes. In particular, it is good to see that in all instances that were amenable to comparison, strategies under cooling-off were significantly different from those used in auctions where no cooling-off right existed.

Table 10 shows that in no instance, where strategies were expected to be the same, was there a structural change in coefficient estimates. This supports the proposition that the strategies being played in auctions N2, N3, N4 and N5 were the same. The data supports the same conclusion for auctions C1 and C2.

In totality, these structural break tests provided strong evidence in support of the prediction that as parameter values change so do the bidding strategies.

Table 10. Chow test statistics[a] for instances where structural breaks were not expected

Auctions	Test statistic
N2 vs. N3-5	0.192
N3 vs. N2, N4 andN 5	0.100
N4 vs. N2, N3 and N5	0.285
N5 vs. N2-4	0.337
C1 vs. C2	0.070

[a] The null hypothesis is that the coefficient estimates are the same. The structure of the linear regression model used to test this was $Bid = \beta_1 v_i + \beta_2 E$. The test statistic is $\mu = \frac{(T-2k)(SSE_R - \Sigma SSE_U)}{k\,\Sigma SSE_U}$ which is distributed over $F(k, T-2k)$.

* indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.

Moreover, the tests also support the proposition that these changes occur as predicted by the theory.

3.4 Commentary on the empirical results

The experimental data lends support to the theoretical predictions in part 2, however in auction C5 the bids were consistently below the predicted level. In this instance $E = 3$ and a cooling-off right exists. Such a systematic deviation from predicted behavior seems to suggest some other strategy was being played. The pattern may also be consistent with the notions of bounded rationality explored by Baye and Morgan [1]. Although this behavior contradicts the theoretical predictions, it tends to strengthen the conclusions regarding the comparison of expected revenues. When a cooling-off right exists, if bidders exhibit a tendency to bid below E when E is greater than the maximum value of v_i, the expected winning bid will be lower than predicted. This implies that the revenue to the seller will be even less than predicted. This supports the conclusion that when $E > 0$ the sellers expected revenue drops when a cooling-off right is added.

On the whole, the experimental evidence lends support to the theoretical conclusions. Changes in strategies occur as predicted, and for most values of E the point predictions of the strategies are fairly close to those observed in the data. However, as in most experimental work, it may be overly ambitious to expect the data to fit the specialized parameters and assumptions of the theory perfectly. It is also encouraging to see the data support the refinement of the equilibrium under cooling-off that ruled out spurious bidding (Equation (6)). Where the predictions can be rejected, the rejection is in favor of behavior that makes the revenue conclusions in part 2 stronger.

Hence the data supports the conclusion that cooling-off is a desirable element when $E < 0$ and undesirable when $E > 0$.

4 Concluding remarks

This paper has addressed the question of when a cooling-off right is a desirable element in an auction from the seller's point of view. It has been argued that a seller will be made better off, from an expected revenue point of view, by introducing a cooling-off right when there exists a state of the world in which the object being auctioned gives dis-utility to the bidders. However, when the object gives positive utility in all states of the world it is not in the seller's best interests to introduce a cooling-off right.

From experimental testing we learn that, when a state of the world exists where the good may yield negative utility to a buyer ($E < 0$), the theory predicts behavior particularly well. The point predictions of the theory are unable to be rejected, over 65% of the variation in bidding is explained and the predicted coefficients of the linear bid functions in auctions with cooling-off are not rejected. This is encouraging as the implications of the model are strongest for this case – when $E < 0$ a cooling-off right raises revenue.

More generally, the experiment confirms that strategies change with parameters and cooling-off rights. What is more, the Chow tests suggest that the theory accurately predicts when these changes take place. The experiment also confirmed that the model is justified in ruling out spurious bidding in auctions with cooling-off. Only 5 out of a possible 180 spurious bids were observed.

The experiment illustrated behavior that was not predicted by the theory. In particular, the pervasive under-bidding in auction C5 was unforeseen. This behavior seems consistent with other, similar, Bertrand game experiments. It suggests that the point predictions of the model may be poor when E lies above the support of the privately held values v_i. However, the flavor of the revenue conclusions is strengthened by this experimental observation.

The experiment also suggested some tendency for subjects to be more risk averse when faced with the small chance of a gain, as opposed to a loss. This was discussed with reference to the prospect theories of choice under risk and uncertainty. While no firm conclusions could be drawn, the tendency was suggestive of behavior predicted by prospect theory.

Overall, the experiment seems to support the theoretical model presented in part 2. However, it also suggests at least two areas where the model could be improved; in capturing bidding behavior when E is very high and allowing for different responses to different types of risks. It has also been noted that, in practice, a nominal fee is often attached to the exercise of the cooling-off right. Bidding behavior in the presence of such a fee remains an avenue for further research.

Appendix

Proof of Proposition 1

When $E \geq 1$ this proposition is established by inspection. When $1 > E \geq 0$ the bidding strategy with cooling-off is

$$b_i = \begin{cases} E & \text{if } v_i \leq E \\ \dfrac{n-1}{n} v_i + \dfrac{E^n}{n v_i^{n-1}} & \text{if } v_i > E \end{cases}$$

and without cooling-off is $b_i(v_i) = (1-\gamma)\frac{n-1}{n} v_i + \gamma E$. Clearly $(1-\gamma)\frac{n-1}{n} v_i + \gamma E$ $< E$ and by contradiction $\frac{n-1}{n} v_i + \frac{E^n}{n v_i^{n-1}} > (1-\gamma)\frac{n-1}{n} v_i + \gamma$E.

That is, if $\frac{n-1}{n} v_i + \frac{E^n}{n v_i^{n-1}} \leq (1-\gamma)\frac{n-1}{n} v_i + \gamma$E then $\frac{E^n}{n v_i^{n-1}} \leq -\gamma \frac{n-1}{n} v_i$ which is a contradiction as it implies that a negative number is greater than a positive number.

When $E < 0$ it has to be established that

$$(1-\gamma)\frac{(n-1)}{n} v_i + \gamma E + \frac{1-\gamma}{n v_i^{n-1}} \left(\frac{-\gamma E}{1-\gamma}\right)^n < \frac{n-1}{n} v_i \qquad \text{(A1)}$$

in the region $\frac{-v_i(1-\gamma)}{\gamma} \leq E < 0$. It is sufficient to establish that in this region $\theta = \gamma E + \frac{1-\gamma}{n v_i^{n-1}} \left(\frac{-\gamma E}{1-\gamma}\right)^n \leq 0$. Now, noting that

$$\begin{aligned} \frac{d\theta}{dE} &= \gamma \left(1 - \left(\frac{1}{v_i^{n-1}}\right)\left(\frac{-\gamma E}{1-\gamma}\right)^{n-1}\right) \\ &= 0 \quad \text{when} \quad E = -\frac{(1-\gamma) v_i}{\gamma} \end{aligned}$$

and that when $E = 0$, $\frac{d\theta}{dE} = \gamma > 0$, it must be the case that Equation [A1] holds as we have a function that is monotonic in the relevant region with a positive slope, so that as E decreases that function also decreases in value, with the condition that at the upper bound of the relevant region the function equals zero. Hence we can support Proposition 1. □

Proof of Proposition 2

By observation, from Table 1, when $E > 1$ the expected revenue from an auction without a cooling-off right will exceed that from an auction with costless cooling-off. When $0 < E \leq 1$ the entries in the third row of Table 1 must be compared. Consider the case where $(1-\gamma)(1-E)\left(\frac{n-1}{n+1} + E^n\right) + \gamma E \geq$ $(1-\gamma)\frac{n-1}{n+1} + \gamma E \quad \forall E \in (0,1]$. This is equivalent to

$$E^{n-1} - E^n \geq \frac{n-1}{n+1}. \qquad \text{(A2)}$$

Now when $E = 0$, $E^{n-1} - E^n = 0$ and when $E = 1$, $E^{n-1} - E^n = 0$. If we let $f(E) = E^{n-1} - E^n$ then, $\frac{df(E)}{dE} = E^{n-2}[(n-1) - nE] = 0$.

Hence stationary points exist at $E = 0$ and $E = \frac{n-1}{n}$. From the second order sufficient condition, when $E = \frac{n-1}{n}$, $f(E)$ is at a maximum. Hence for a given number of bidders the maximum value of $f(E)$ is $\left(\frac{n-1}{n}\right)^{n-1}\left(\frac{1}{n}\right)$. Since $\left(\frac{n-1}{n}\right)^{n-1}$ has to be less than one, it must be true that $\left(\frac{n-1}{n}\right)^{n-1}\left(\frac{1}{n}\right) < \frac{1}{n}$. Thus it follows that $\left(\frac{n-1}{n}\right)^{n-1}\left(\frac{1}{n}\right) < \frac{1}{2}$. Since the minimum value of $\frac{n-1}{n+1}$ is one half, it follows that $E^{n-1} - E^n < \frac{n-1}{n+1} \quad \forall E \in (0, 1]$.

Hence Equation [A2] is contradicted. Thus it is established by contradiction that

$$(1-\gamma)(1-E)\left(\frac{n-1}{n+1} + E^n\right) + \gamma E < (1-\gamma)\frac{n-1}{n+1} + \gamma E \qquad \forall E \in (0, 1]\ .$$

Thus for all strictly positive values of E the expected revenue from an auction with no cooling-off rights exceeds the expected revenue from an auction with a costless cooling-off right, establishing Proposition 2. □

Proof of Proposition 4

When E is strictly negative, it is necessary to compare the entries in the first row of Table 1. If we let

$$\begin{aligned} g(\cdot) &= \gamma E\left[1 - \left(\frac{-\gamma E}{1-\gamma}\right)^n\right] + (1-\gamma)\frac{n-1}{n+1}\left[1 - \left(\frac{-\gamma E}{1-\gamma}\right)^{n+1}\right] \\ &\quad + (1-\gamma)\left(\frac{-\gamma E}{1-\gamma}\right)^n\left(1 + \frac{\gamma E}{1-\gamma}\right) \end{aligned}$$

where $E \in \left[\frac{-(1-\gamma)}{\gamma}, 0\right)$, then

$$\begin{aligned} \frac{\partial g(\cdot)}{\partial E} &= \gamma\left[1 - \left(\frac{-\gamma E}{1-\gamma}\right)^n\right] + \gamma E\left[\frac{\gamma n}{1-\gamma}\left(\frac{-\gamma E}{1-\gamma}\right)^{n-1}\right] \\ &\quad + \gamma(n-1)\left(\frac{-\gamma E}{1-\gamma}\right)^n - \gamma n\left(\frac{-\gamma E}{1-\gamma}\right)^{n-1}\left(1 + \frac{\gamma E}{1-\gamma}\right) + \gamma\left(\frac{-\gamma E}{1-\gamma}\right)^n \\ &= \gamma + \gamma(n-1)\left(\frac{-\gamma E}{1-\gamma}\right)^n - \gamma n\left(\frac{-\gamma E}{1-\gamma}\right)^{n-1} . \end{aligned}$$

It is difficult to sign this derivative. However, it can be seen that when $E = 0$, $\frac{\partial g(\cdot)}{\partial E} = \gamma$ and when $E = \frac{-(1-\gamma)}{\gamma}$, $\frac{\partial g(\cdot)}{\partial E} = 0$. If there are no concavity changes in the interval $\left[0, \frac{-(1-\gamma)}{\gamma}\right]$, other than $E = \frac{-(1-\gamma)}{\gamma}$ or 0, then it can be concluded that, in this interval, $\frac{\partial g(\cdot)}{\partial E} \geq 0$. Taking the second derivative of $g(\cdot)$ yields;

$$\frac{\partial^2 g(\cdot)}{\partial E^2} = \left(\frac{-\gamma E}{1-\gamma}\right)^{n-2}\left(\frac{-\gamma E}{1-\gamma} - 1\right)$$
$$= 0 \quad \text{when} \quad E = 0 \quad \text{and} \quad \frac{-(1-\gamma)}{\gamma}.$$

Hence $\frac{\partial g(\cdot)}{\partial E} \geq 0$. This means that, without cooling-off, expected revenue decreases as E decreases in this interval. As the expected revenue, with a cooling-off right, is constant as E falls, and when E = 0 the revenue under both regime is the same, it must be true that $(1-\gamma)\frac{n-1}{n+1} > g(\cdot) \quad \forall E \in \left[\frac{-(1-\gamma)}{\gamma}, 0\right)$. That is, when E is strictly negative the expected revenue from an auction held under a costless cooling-off regime will be higher than that with no cooling-off, establishing Proposition 4. □

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