

Vertical Information Restraints: Pro- and Anti-Competitive Impacts of Minimum Advertised Price Restrictions

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July 31, 2019

Abstract

We consider vertical contracts in which the retail market may involve search frictions. Minimum advertised price (MAP) restrictions act as a restraint on customers' information and can therefore increase search frictions in the retail sector. Such restraints thereby soften retail competition—an impact also generated by resale price maintenance (RPM). However, by accommodating (consumer or retailer) heterogeneity, MAP restrictions can allow for higher manufacturer profits than RPM. We show that these restrictions can do so through facilitating price discrimination among consumers, encouraging service provision, and facilitating manufacturer collusion. Thus, welfare effects may be positive or negative compared to RPM or to the absence of such restrictions.

*We gratefully acknowledge useful comments from Arthur Campbell, Allan Collard Wexler, Daniel Garcia, Soheil Ghili, Gautam Gowrisankaran, Marco Haan, Bruno Jullien, Robin Lee, Matt Mitchell, Joao Montez, Volker Nocke, Martin Peitz, Patrick Rey, John Riley, Sandro Shelegia, Rona Bar-Isaac and Jidong Zhou and from seminar participants at DG Comp, the U Toronto bag lunch, MACCI 2016, CIRANO Conference on IO 2016, EARIE 2016, CEMFI, TSE, 8th Workshop on Consumer Search and Switching Costs, Barcelona Summer Forum 2017, Yale MIO Conference 2018 and the Federal Trade Commission. We thank Rami Abou-Seido, El Hadi Caoui, Zheng Gong, and Yuta Yasui for their excellent research assistance and SSHRC (435-2014-0004) for financial support. Email: johnasker@econ.ucla.edu and heski.bar-isaac@rotman.utoronto.ca

1 Introduction

When a retailer enters into a distribution agreement with a manufacturer it is well understood that the agreement may impose ‘vertical’ restraints on the retailer that restrict price or non-price aspects of retailer activity.¹ These restrictions, in turn, raise antitrust concerns, which have been the subject of litigation and scholarly debate since the early days of the Sherman Act.² Price restraints include resale price maintenance (RPM), in which the manufacturer imposes floors and/or ceilings in retail pricing. Commonly considered non-price restraints typically include exclusivity provisions or the imposition of sales territories. This paper considers a third, and largely ignored, type of vertical restraint: information restraints, specifically, minimum advertised price (MAP) policies.³ MAP policies impose a floor on the price at which retailers can advertise a product, but crucially, not the price at which it can be sold to a consumer. That is, even if a MAP policy imposes a floor of \$10 per unit on advertised prices, nothing restricts the retailer from selling it to a consumer for \$7.

¹For a survey of common practices and related economic issues, see Rey and Vergi $\frac{1}{2}$ (2008).

²15 USC 1. For overviews and examples, see Rey and Vergi $\frac{1}{2}$ (2008), Yamey (1954) or Overstreet (1983), among many others.

³MAP policies appear to be the most common form of vertical information restraint. As an illustration of the scope of their use, the first 40 policies found in a search for “minimum advertised price pdf” using the Google search engine on April 13, 2016, covered the following product categories (companies): adjustable office and industrial chairs (Neutral Posture); antennas (Winegard Company); aquarium filtration systems (Lifegard Aquatics); baby and toddler pillows (Smuggwugg); batteries, flashlights, and solar energy equipment (NOCO); cameras and security cameras (Sony); chainstays and gripping bar tape (Lizard Skins); chairs (Allseating); clutches, flywheels, and accessories (Midway Industries); data storage electronics (Seagate); designer faucets, showers, and bath accessories (Sonoma Forge); digital display mounting (Ergotron); enzymes (Enzyme Science); exhaust systems and mufflers (Flowmaster); games (Cool Mini or Not); health bars, drinks, and powders (Powercrunch); high-pressure hydraulic tools (Enerpac); holster clips (Utiliclip); home, mobile and automobile electronics (JVC Professional Video Products); instruments for measuring and calibrating (Meriam Process Technologies); LCD and plasma TVs (Microtek); mobile cameras (GoPro); mobile telephone cases (Urban Armor Gear); outerwear, corporate wear and imprintable apparel (Charles River Apparel); pistols (Glock); rock-crawling and off-road cars (GMADE); security cameras and surveillance equipment (Samsung Techwin America); scented pencils, stickers and paper clips (Scentoco); shallow water anchors (JL Marine Systems); speakers (Eminence); stove ranges, broilers, ovens, and kitchen equipment (Montague Company); temperature-controlled food display cases (Structural Concepts); thermal imaging infrared cameras (FLR Commercial Systems); toy car track tape (InRoad Toys and PlayTape); triathlon clothing and gear (Pearl Izumi); ventilation fans (Panasonic Home & Environment Co.); wall art, picture frames, jewelry, mugs, pillows (Glory Haus); water bottles (Blender Bottle); wood flooring (Kahrs International); and work boots (Thorogood Shoes). All these policies are available at www.johnasker.com/MAP.zip.

MAP policies are a ubiquitous example of a broader set of restraints that restrict retailers' ability to communicate information to consumers. The reach and impact of these restrictions may be significant. For instance, the European Commission (2017) reports on a survey of more than one thousand retailers engaged in e-commerce across Europe and suggests that a large fraction face non-price restrictions, including limitations to sell through market places (18%), websites other than their own (11%), and the use of price comparison sites (9%). These kinds of restrictions can be understood to make it more difficult for consumers to easily find products. In this paper, we focus on MAP restrictions as a lens through which to begin to explore the economics of this class of restrictions.⁴

At first sight, such restrictions seem odd. Advertising and information play an important role in ensuring that customers know of the existence of a firm or product. Price advertising can also reassure customers who are unsure about prices and therefore might be put off from shopping. Thus, there is clearly a role for advertising prices and reasons for a manufacturer to encourage retailers to do so, including competing against other brands or mitigating consumer uncertainty. However, this intuition focuses on the role of advertising in inter-brand competition. In this paper, we switch the focus to competition between a manufacturer's retailers (intra-brand competition) in order to highlight the forces that may make restrictions on price advertising attractive to a manufacturer.⁵

Superficially, MAP and RPM policies appear similar.⁶ If an advertised price is constrained to be equal to an actual price, then a MAP policy appears to operate as *de facto* RPM. Indeed, as we discuss below, this perspective appears to underlie the thinking of antitrust agencies and commentators on policy. However, this view of MAP misses the crucial distinction between MAP and RPM, which is that the MAP price does not restrict the retailer from offering an actual (transaction) price at some level below the advertised price. We show that this distinction and the flexibility that the retailer retains therein result in MAP having an economically distinct effect

⁴Israeli et al (2016) present MAP specific data for the USA. Carleton and Chevalier (2001) present data from the early days of internet commerce suggestive of the existence of MAP-like provisions.

⁵Much of the literature on vertical restraints that we discuss below has focussed on intra-brand concerns (with the exception of our discussion on upstream collusion in Section 4). There is, of course, a rich literature that considers inter-brand effects, even in a static setting, including Shaffer (1991), Perry and Besanko (1991), Rey and Vergé (2010), and Miklos-Thal, Rey and Vergé (2010).

⁶There are models in which these policies are equivalent. For example, as described in greater detail in Asker and Bar-Isaac (2018), in the Diamond (1970) model with homogeneous consumers, customers anticipate that all firms charge this advertised ceiling of prices and therefore, this ceiling acts as a minimum price restraint.

on markets.

Two themes emerge. First, for MAP policies to have any market impact, consumers must have an informational (search) friction that advertising helps to alleviate. Hence, in every setting we consider, search frictions are a central feature. If consumers can costlessly observe all product offerings, MAP is clearly irrelevant to market outcomes.

Second and of central economic interest, MAP policies soften competition by obscuring prices. Thus, consumers allocate themselves to competitors somewhat randomly or at least by relying less on prices than they would if they were perfectly informed. By contrast, RPM softens competition by equalizing prices (and, by doing so, reducing the flexibility of retailers to respond to idiosyncratic conditions). When consumers or retailers are heterogeneous, MAP affords retailers the flexibility to exploit the heterogeneity for higher profit. Thus, MAP will impact markets when search and heterogeneity among similarly situated economic actors are important features of the environment. By contrast, in these environments, RPM tends to have more limited efficacy.

We illustrate these themes through a number of simple models. It is useful to present several models in order to engage with the vast literature on RPM and the variety of pro- and anti-competitive effects of RPM therein. These models were already sufficient for a book-length treatment published more than half a century ago; see Yamey (1954). Indeed, in its *Leegin* decision, the Supreme Court reviewed several mechanisms through which RPM might operate and ruled that RPM cases should be determined on a rule-of-reason basis since there was no reason to believe that a single mechanism should predominate.⁷ Similarly, it would be unreasonable to expect that MAP should operate through a single mechanism, and our goal in this paper is not to provide “the” theory of MAP or compare MAP to RPM for every model of RPM that has been introduced in the literature. Instead, we present three example models—one in which RPM can play no role and two that show how MAP modifies canonical examples of the classic pro- and anti-competitive roles of RPM: namely, encouraging service provision and facilitating collusion. Specifically, these models are the following:

- The *price discrimination* model shows that MAP policies allow manufacturers to imperfectly separate high- and low-search-cost consumers and better extract surplus from high-value consumers with high search costs. That is, the MAP

⁷*Leegin Creative Leather Products, Inc. v. PSKS, Inc.*, 551 U.S. 877 (2007). Elzinga (2017) provides a helpful overview. Ippolito (1991) and Ippolito and Overstreet (1997) provided much of the empirical evidence relied on by the court in reaching its decision.

policy obscures actual prices, allowing search patterns to be leveraged as a screening device.⁸ In this model heterogeneity among consumers plays a key role.

- The *service* model shows that by obscuring prices, MAP policies soften competition and protect retailer profits, while allowing retailers to optimize subject to their heterogeneous marginal costs of retailing. This situation can increase the returns to retailers from providing services that expand the market (such as informative advertising). By softening competition, while retaining retailer flexibility, MAP can dominate RPM as a means for manufacturers to profitably incentivize retailers.⁹ In this model heterogeneity among retailers plays a key role.
- The *collusion* model shows that MAP raises cartel profits and stability by allowing manufacturers to more easily monitor each other's behavior. Notably, this process is done without sacrificing the ability of cartel members to tailor actual transaction prices to local market conditions.¹⁰ That is, whereas RPM may be viewed as having features of a price-fixing scheme, MAP may be viewed as analogous to a market division scheme.

In antitrust policy, investigations and discussions of RPM and, by extension, MAP, tend to be focused on the last two models; that is, the tension between service provision and the facilitation of collusion.

In understanding the way MAP restrictions work, particularly how they differ from the price restraint embodied in an RPM restriction, it is useful to examine a typical MAP provision. As an example, consider the January 1, 2016, MAP policy of Samsung Techwin America (a manufacturer of security cameras and surveillance

⁸Throughout, we assume that retailers cannot price discriminate. Instead, discrimination occurs through the consumers' choice of retailer. Chen (1999) considers the effect of RPM when retailers can engage in price discrimination. Although we do not consider this possibility in this paper, MAP might facilitate retailer price discrimination and thereby boost industry (and manufacturer) profits in addition to retailers' incentives to provide service. Asker and Bar-Isaac (2018) provide a simple example.

⁹Telser (1960) is typically cited regarding the idea that RPM promotes service (although it is also discussed in Yamey (1954), for example). More recent formalizations and developments include Mathewson and Winter (1984); Klein and Murphy (1988); and Deneckere, Marvel and Peck (1997). We follow the literature in considering pro-consumer service, although note that, as in the model of exclusion in Asker and Bar-Isaac (2014), service need not be helpful to the consumer.

¹⁰In formalizing this argument, we adapt the framework of Jullien and Rey (2007), which explores the use of RPM to facilitate upstream collusion, to accommodate MAP.

equipment).¹¹ The MAP price is specified as a percentage of the manufacturer’s suggested retail price (in this instance, 40% of MSRP). The policy explicitly applies to all advertisements in all media, including online media. The MAP restriction applies only to advertised prices and not to the price at which products are actually sold. In physical stores, this means that the posted price in the store is not affected by the MAP restriction. Regarding online pricing, the policy states:

Pricing listed on an internet site is considered an “advertised price” and must adhere to the MAP policy. Once the pricing is associated with an actual purchase (an internet order), the price becomes the selling price and is not bound by this MAP policy. Statements such as “we will match any price” and “call for price” are acceptable.

In particular, such policies allow retailers to advise customers that they will be able to see the price when the item is in a (digital) shopping basket.

Finally, Samsung reserves the right to punish non-compliance with termination.¹²

This paper contributes to the well-established literature in policy and academic circles on the antitrust implications of vertical contracts.¹³ When MAP provisions have been considered in this literature, the point of departure (as in this paper) has been RPM restrictions. As with many vertical restraints, the approach to MAP and RPM taken in U.S. and European law differs, with the E.U. being less permissive. In the 2007 *Leegin* case, federal U.S. law reversed earlier precedent, shifting RPM from a complicated version of a *per se* offense to a rule of reason regime.¹⁴ Even prior to the *Leegin* case, U.S. law had ruled in favor of MAP provisions, acknowledging their pro-competitive potential in preserving service incentives.¹⁵ Post-*Leegin* cases in the U.S. have also failed to gain traction.

¹¹Available at www.johnasker.com/MAP.zip

¹²Interestingly, the policy also describes its purpose, which, in this instance, is to “to help ensure the legacy of STA as a top producer of high performance, high quality, professional security products and to protect the reputation of its name and products ...[and]... to ensure dealers, retailers and distributors have the incentive to invest resources into services for STA’s customers.”.

¹³See Rey and Vergé (2008) for a survey.

¹⁴*Leegin Creative Leather Products, Inc. v. PSKS, Inc.*, 551 U.S. 877 (2007)

¹⁵See *Holabird Sports Discounters v. Tennis Tutor, Inc.*, 1993 No. 92-2504, 1993 U.S. App. LEXIS 10918, (4th Cir. May 7, 1993), and at appeal 993 F.2d 228 (1993) and *Commodore Business Machines, Inc. v. Montgomery Grant, Inc.*, No. 90 Civ. 7498 (LMM), 1993 U.S. Dist. LEXIS 262 (S.D.N.Y. Jan. 13, 1993). In 2003, following an FTC enforcement action, plaintiffs obtained a settlement in *In re Compact Disc Minimum Advertised Price Antitrust Litigation*, 216 F.R.D. 197, 199-200 (D. Me.2003) (approving settlement) by alleging that MAP provisions facilitated a cartel among manufacturers (in this case, distributors of recorded music). Albert (2012) and Passo (2016) provide extensive reviews of recent U.S. policy and litigation.

By contrast, in Europe, MAP provisions have tended to be viewed with more suspicion. In particular, MAP has been found to be a *de facto* form of RPM.¹⁶ In turn, RPM has been found to impose a sufficient degree of harm to competition such that there is no need to examine its effects in determining liability (that is, making it a restriction of competition by object, somewhat similar to a *per se* offense in U.S. law).¹⁷

Hence, jurisdictions differ in their approach to regulating MAP provisions. This paper contributes to understanding this divergence by providing the precise frameworks needed to make economic arguments that support each approach. By doing so, it helps clarify the trade-offs implicit in arriving at any policy position vis-a-vis MAP.

This paper is also closely related to the nascent literature (notably Lubensky (2017), Janssen and Shelegia (2015), and Janssen and Reshidi (2017)) that examines the effects of vertical contracts when the final goods market is characterized by search frictions. In particular, in such markets, it has long been understood that the law of one price need not hold, and price dispersion may arise.¹⁸ Naturally, such frictions have implications for the contracts between manufacturer and retailer and for a manufacturer's profitability. A MAP restriction that limits the price that retailers may advertise (with no restriction on the price they may charge) can make it more difficult for consumers to find the retailers charging the lowest prices and lead to retail price dispersion (if not advertised price dispersion). As is illustrated in this paper, a manufacturer (and even consumers) may benefit.

The paper is also related to the large and growing literature on obfuscation in search markets. (See, for example, Ellison and Wolitzky (2012), Wilson (2010), Piccione and Spiegler (2012), Chioveanu and Zhou (2013), Gamp (2017) and Petrikaite (2018)).¹⁹ Much of the economic force of a MAP restriction occurs by making firms

¹⁶See Commission Decision 16 July 2003 *PO/Yamaha* (COMP/37.975), OFT Decision *Agreements between Lladro Comercial SA and UK retailers fixing the price for porcelain and stoneware figures*, CP/0809-01, 31 March 2003 and, recently, the Decision of the UK Competition and Markets Authority *Online resale price maintenance in the commercial refrigeration sector* Case CE/9856/14. All these cases apply E.U. law. Hughes (2017) provides a broad discussion and criticism of the approaches of the UK Competition and Markets Authority and the EU Commission towards MAP and RPM.

¹⁷See, for instance, the overview of relevant E.U. law in the Decision of the UK Competition and Markets Authority *Online resale price maintenance in the commercial refrigeration sector* Case CE/9856/14.

¹⁸This idea dates back to at least Stigler (1961). Baye, Morgan and Scholten (2006) provide a useful overview.

¹⁹More broadly, recent literature has explored retailers' policies which influence search and pricing behavior in search markets. Such practices include stochastic discounts off the list price in Gill and

identical in the point of view of consumers. In this way, MAP plays a role in obfuscating the actual as opposed to advertised price (indeed, this is its only role in our analysis). This paper is distinct from the rest of the obfuscation literature (to our knowledge) by explicitly considering the opportunities obfuscation gives to an upstream manufacturer, as opposed to a retailer.²⁰

To our knowledge, only a few papers in the economic literature consider MAP policies. Kali (1998) and Cetinkaya (2009) explore theoretical models that treat MAP as an RPM provision with an additional advertising subsidy. A very different approach is adopted in this paper. Charness and Chen (2002) conduct an experimental study of the determinants of MAP compliance.²¹ Israeli, Anderson and Coughlan (2016) empirically examine detected violations of MAP, using data from a firm engaged, on behalf of manufacturers, in monitoring the MAP compliance of online retailers. They also provide an informative discussion of MAP's prevalence.²² Importantly for our study, Israeli et al find that 14-22% of authorized dealers violate MAP, as compared to 46-54% of unauthorized dealers, which suggests that MAP policies are genuine restraints on retailer behavior. More recently, Asmat and Yang (2019) examine the impact of the MAP policy of a large electronics manufacturer on retail pricing and shopping behavior. Asmat and Yang present empirical evidence consistent with the role of MAP in raising search costs, and in line with the price discrimination model presented in Section 2.

The rest of this paper is organized as follows. Section 2 explores the price discrimination model and introduces a modeling structure that is used in the other models. Section 3 investigates how MAP can enhance customer service. Section 4 shows how a MAP program can help coordinate a manufacturer cartel. Finally, Section 5 offers some closing remarks.

2 MAP as price discrimination

We begin by laying out a simple model whose essential feature is the presence of search frictions, which imply that advertised prices might not reveal the actual prices to consumers. We consider variations in the sections below to illustrate different

Thanassoulis (2016), and exploding offers in Armgstrong and Zhou (2016).

²⁰An exception is Shaffer and Zettelmeyer (2004), which uses a reinterpretation of a Hotelling model to explore similar themes.

²¹More recently, Israeli (2018) uses field data to evaluate manufacturer policies for improving compliance.

²²The data used in Israeli et al comes from a firm offering MAP compliance monitoring of internet-based sellers.

effects of MAP. In this section, we assume that consumers are heterogeneous in both search costs and valuations. An advertising restriction allows the industry to segment consumers and thereby raise profits. Welfare consequences, as is the case with most price discrimination, are ambiguous.

2.1 Model

Consumers have unit demand for a good, which they can buy from one of two retailers. Retailers source the good from a single manufacturer.

A fraction λ of consumers have a low valuation of the good and derive utility $l - p$ when buying the good at price p ; the remainder have a high valuation, h (we abuse the notation slightly and also use l or h to denote the consumer's type).

Minimum advertised prices have bite only if there are limits on consumers' ability to freely compare different retailers' prices. We model these limits in a simple way. Specifically low-value consumers can search costlessly and observe actual transaction prices. Instead, high-value consumers are assumed to have a high search cost such that they visit the store offering the lowest advertised price (equivalently, the highest advertised consumer surplus).^{23,24} If these customers observe the same advertised price at both retailers, they will choose one at random. That is, they have sufficiently high search costs such that they will only visit one store, and from the point of view of these consumers, advertised prices are the only point of differentiation between retailers.

Consumers who are indifferent are equally likely to visit or buy from either retailer. Consumers who observe all actual prices will choose the lowest price. To simplify exposition here, we model consumers with high search costs as non-strategic in the sense that they mechanically, visit the retailer with the lowest (advertised) price. As we demonstrate in the Online Appendix, section B.1, even if consumers are strategic, this behavior can be supported in equilibrium but requires us to keep track

²³See the Online Appendix section B.2, which provides a characterization of the case in which some fraction of low-value consumers can only visit one store, some fraction of high-value consumers search costlessly and there is heterogeneity in the marginal costs of retailing.

²⁴Having high- and low-search-cost consumers is an extreme but convenient and often-used way to examine search frictions and consumers who are heterogeneous in their search behavior. See, for example, Varian (1980). Alternatively, an *ex ante* costly decision to engage in search in the style of Burdett and Judd (1983) would allow for the fraction of searchers to be endogenously determined (at the cost of an additional notation for these search costs and a somewhat more involved analysis). In an empirical investigation of the online book market, De Los Santos, Hortaçsu and Wildenbeest (2012) suggest that such a fixed simple search model might be more appropriate than a model of costly sequential search.

of consumers beliefs about the actual prices when they observe particular advertised prices.

The two retailers are denoted R_1 and R_2 . We begin by supposing that they incur no costs beyond payments to the manufacturer. We assume throughout that the manufacturer offers the retailers identical terms.²⁵ We suppose that these retailers are indistinguishable to consumers other than through their advertising.

Retailers simultaneously set their prices, p_1 and p_2 , and their advertised prices, p_1^a and p_2^a . We suppose truthful advertising, in the sense that retailers cannot advertise prices lower than their transaction prices, i.e., $p_j^a \geq p_j$ for $j = 1, 2$.²⁶ If the manufacturer imposes a minimum advertised pricerestriction, p^{MAP} , then $p_j^a \geq \max\{p_j, p^{MAP}\}$. Combined with the model of consumer behavior described above, this expression means that no firm has an advantage in advertising a price higher than $\max\{p_j, p^{MAP}\}$.²⁷ As a result, the discussion that follows explores the reduced form of this game and sets $p_j^a = \max\{p_j, p^{MAP}\}$.

Retailers buy the product from the manufacturer (M) who can set a two-part tariff with a linear wholesale price $w \geq 0$ and fixed fee $T \geq 0$.^{28,29} In the absence of vertical restrictions, retailers take the wholesale price w as given and set the retail transaction price and advertised price. If the manufacturer imposes RPM, retailers are bound to charge at least the RPM price, p^{RPM} ; that is, we consider minimum RPM (and it is assumed, without loss, that there is no advertising restriction). If the manufacturer imposes MAP, retailers are bound to advertise at the MAP price, p^{MAP} , or higher as described above. The manufacturer's marginal cost is equal to zero.

2.1.1 Timing

Timing is as follows.

²⁵This may be due to legal restrictions or be induced by the assumptions below on timing.

²⁶There is a literature on false advertising. See Corts (2013,14), Rhodes and Wilson (2016) and Piccolo, Tedeschi, and Ursino (2015) for recent contributions.

²⁷The reasoning mirrors that of the standard homogenous good Bertrand model.

²⁸Since we assume that the manufacturer cannot distinguish between retailers, there are no subscripts on these variables to denote different retailers.

²⁹There may be cases where negative wholesale prices may benefit the wholesaler; for example to induce greater service investment in Section 3 or to induce harsher punishments in the case of collusion in Section 4. Here, we simply assume that charging such a price is infeasible; for example, at $w < 0$, a retailer may ask for an “infinite” number of goods and dump them. (With positive marginal costs, there is perhaps some room to charge wholesale prices below the marginal costs but with sufficiently low costs, a similar non-negativity constraint arises).

1. The manufacturer sets the contract terms (T, w) and any restraints (RPM, MAP, etc., and the associated prices p^{RPM} or p^{MAP}).
2. Retailers accept or reject the offered contract.
3. Retailers each set their actual sales price, p_j . The advertised price is p_j or, if a MAP restriction is in place, $\max\{p_j, p^{MAP}\}$, where p^{MAP} is the minimum advertised price.
4. Purchases are made and profits realized.

In this model, since there is no relevant private information, the relevant notion of equilibrium is subgame perfect Nash equilibrium.³⁰

2.2 Analysis

We show that MAP can facilitate price discrimination. That is, in the face of consumer preference heterogeneity, MAP can be used to sort consumers among retailers in ways that extract greater surplus than might otherwise be the case because, by preventing all consumers from searching amongst retailers costlessly, MAP can allow for price dispersion. As noted by Salop (1977), an integrated monopolist would value such price dispersion because it both segments the market and charges higher prices to less efficient searchers. An appropriate MAP contract enables the manufacturer both to allow for such price dispersion and to extract the surplus that this creates in the industry as a whole.

It is immediate that even without the use of vertical restraints, the manufacturer can induce a retail price of h or a retailer price of l and enjoy the full surplus of doing so.³¹ By setting $w = l$, the manufacturer can induce retailers who Bertrand compete to set prices at $p = l$ and sell to all consumers. This extracts all the retailer surplus, and the manufacturer sets $T = 0$. This is optimal if $p = l$ is the single price that maximizes industry profits. Conversely, if industry profit is maximized by selling to only high-value consumers, all surplus can be extracted by the manufacturer (for example, by setting $w = h$ and $T = 0$). Thus, the manufacturer's profit is equal to $\max\{l, (1 - \lambda)h\}$.

³⁰In the Online Appendix B.1, where consumers with high search costs must make inferences about actual prices from advertised prices, the solution concept is instead Perfect Bayesian Equilibrium.

³¹As is well understood, the use of a two-part rather than linear tariff allows the manufacturer to overcome the familiar double-marginalization problem highlighted in Spengler (1950).

Note that RPM does not provide an avenue through which to raise profits above this level. Setting $p^{RPM} > w$ provides a binding price floor on the retailers' prices, since they are in homogenous Bertrand competition. However, this process does not aid in surplus extraction.

However, the addition of MAP to this environment may allow for further surplus extraction by the manufacturer. It does so by inducing the different retailers to charge different prices (it is clear that optimally, in this simple environment, one will charge a price l and the other a price h). When one retailer prices high and the other low, consumers with a high valuation will purchase from the high-price store if they are randomly allocated to that store. Consumers with zero search costs, and the luckier high-valuation consumers, will purchase from the low-price store.

We begin by characterizing the optimal MAP-based scheme that involves discrimination.

Proposition 1 *In the optimal MAP-based discriminatory price scheme, the manufacturer sets (w^*, T^*) as in Equations (1) and (2)*

$$w^* = \frac{l(1 + \lambda) - h(1 - \lambda)}{2\lambda}; \quad (1)$$

$$T^* = \frac{(h - l)(1 - \lambda^2)}{4\lambda} \quad (2)$$

and the MAP price is set at or above h . R_1 sets $p_1 = l$ and R_2 sets $p_2 = h$.³² Under this scheme, the manufacturer earns

$$\frac{h(1 - \lambda) + l(1 + \lambda)}{2}. \quad (3)$$

Proof. Trivially, in a scheme that discriminates it is optimal to have one retailer charge l and the other charge h . As we argue below, this characterizes w^* and ensures that both retailers earn the same profits. The manufacturer maximizes its profit by setting the fixed fee at the level of the gross profit.

Specifically, given that R_1 sets a price of l , R_2 must prefer setting a price of h and selling to half of the high-value non-searchers (of whom there are $(1 - \lambda)$) who randomly visit R_2 to setting a price just below l and selling to all the low-value searchers and to half of the high-value consumers who visit randomly; that is:

$$\frac{1 - \lambda}{2}(h - w^*) \geq \left[\frac{1 - \lambda}{2} + \lambda \right] (l - w^*). \quad (4)$$

³²Of course, there is another equilibrium in which R_2 sets $p_2 = l$ and R_1 sets $p_1 = h$. Allowing for heterogeneity in retailers' costs, as in Online Appendix B.2, is a simple means of extending the model to resolve a coordination issue in selecting roles in these asymmetric equilibria.

Similarly, R_1 must prefer to set a price of l rather than one just below h when R_2 sets a price of h . Equivalently,

$$\left[\frac{1-\lambda}{2} + \lambda \right] (l - w^*) \geq \frac{1-\lambda}{2} (h - w^*). \quad (5)$$

These inequalities are both satisfied when they hold with equality, yielding the expression for w^* , where both retailers earn the same profits. The manufacturer sets the fixed fee T^* to extract the gross profit.

The manufacturer profits come from collecting T^* from each of the two retailers and a margin of w^* on every unit sold. ■

It is worth noting that since the MAP in Proposition 1 is set at or above h , other restrictions such as a ban on advertising or a requirement to advertise the product without any mention of prices would have the same impact in this setting.³³

While Proposition 1 characterizes which discriminatory scheme would be optimal. It remains to show that the manufacturer would gain by choosing such a discriminatory scheme. When there are sufficiently many low value consumers (λ is high enough) this is necessarily the case, as shown in the following corollary.

Corollary 1 *If in the absence of MAP it is optimal to sell at the valuation of low value consumers (that is if $l > (1 - \lambda)h$) then MAP is strictly more profitable for the manufacturer.*

Proof. MAP is strictly more profitable when $\frac{h(1-\lambda)+l(1+\lambda)}{2} > \max\{l, (1 - \lambda)h\}$.

If in the absence of MAP it is optimal to sell to only low value consumers then $\max\{l, (1 - \lambda)h\} = l$ and so the left hand side of this inequality is equal to l . Trivially the right hand side (a weighted average of h and l) is strictly greater. ■

By continuity, MAP would remain optimal when the profit from selling to all consumers (l) is below but close to the profit from selling only to high value consumers ($(1 - \lambda)h$). Consider the following parameterization.

Example 1 *Let $h = 2$ and $l = 1$. First note that, absent MAP, the maximal profit is $\max\{1, 2(1 - \lambda)\}$ and it is optimal to set $w = 1$ if there are sufficiently many low value consumers, $\lambda \geq \frac{1}{2}$, and to set $w = 2$ otherwise. Following Proposition 1, $(w^*, T^*) = (\frac{3\lambda-1}{2\lambda}, \frac{1-\lambda^2}{4\lambda})$ and $p^{MAP} = 2$. Further, the manufacturer's profit from this pricing scheme is $\frac{3-\lambda}{2}$. Thus, in this example, the MAP policy, leads to strictly greater profit for the manufacturer whenever $\lambda > \frac{1}{3}$.*

³³This does not mean MAP provisions and these other measures are always equivalent. For instance, price advertising (in combination with MAP) would be distinct in settings where inter-brand competition is important.

We illustrate this example in Figure 1.³⁴ Of the high-value consumers, due to the identical advertised price imposed by MAP and their inability to search, half go to R_1 (who charges l) and half to R_2 (who charges h). All the low-value consumers (who are assumed to be able to search costlessly between retailers) go to R_1 . Hence, the profit (ignoring T) made by R_2 is $\frac{1}{2}(h - w)(1 - \lambda)$, the sum of rectangles A and B. Similarly, the profit made by R_1 is $(l - w)(\lambda + (1 - \lambda)\frac{1}{2})$, the diagonally hatched rectangle corresponding to the sum of C and D. By setting w such that the two profits are equal, then by setting T at this level, the manufacturer can capture all the surplus other than area E .

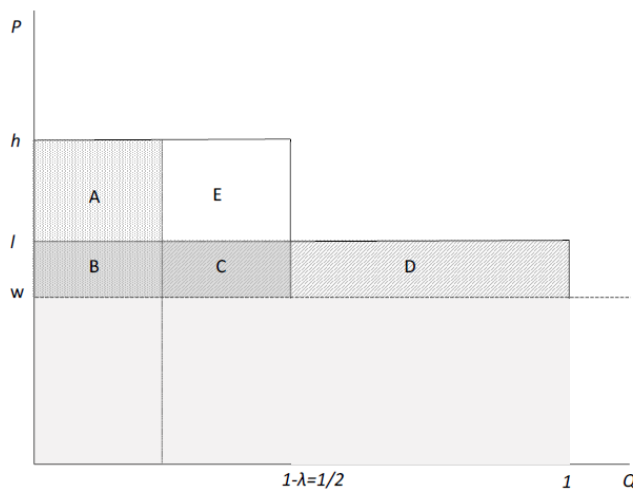


Figure 1: Outcomes with MAP, a two-part tariff and no service

All price discrimination operates by (usually imperfectly) homogenizing consumers so that targeted prices extract more surplus.³⁵ The MAP policies explored here are no exception. By introducing an information friction, the manufacturer is able to isolate at least some of the high-value consumers and extract surplus from them via the high price charged by R_2 . By contrast, RPM does little to segregate

³⁴In Figure 1, λ is assumed to be equal to one half.

³⁵Sometimes, the emphasis is tilted toward better targeting the prices (in the extreme, Type 1 price discrimination), while other times, the emphasis is on homogenizing the consumers (with bundling and Type 3 price discrimination being obvious examples).

consumers and therefore, here, would serve only to solve a possible double margin problem (which arises if T is restricted to 0). This situation illustrates that MAP has elements analogous to the much more invidious market division schemes that invite criminal sanction under antitrust laws, while RPM is similarly analogous to price fixing.³⁶ Merely agreeing on a common price does not help retailers in this setting to extract more surplus. However, if retailers can allocate high-value consumers to R_2 and low-value consumers to R_1 , as might be done in some ideal market division scheme, then they would be able to extract all available gains from trade. MAP generates an imperfect implementation along these lines, albeit ultimately to the benefit of the upstream manufacturer.³⁷

As is standard for price discrimination, welfare consequences are ambiguous and depend on quantity effects. If in the absence of MAP, the optimal scheme involves a retail price of h that excludes all low-value consumers, then MAP would raise welfare; instead, if the unrestricted case involves a price of l and sales to all customers, then the MAP scheme (in which l -types with high search costs who find themselves at a high-price retailer are not served) would reduce it.

From a consumer surplus perspective, the impact of MAP is also ambiguous. Low-value consumers are either excluded or they pay their full valuation and get no surplus. In the absence of MAP, if the optimal price is l then all high-value consumers gain $h - l$; however, if the optimal price is h , then they gain no surplus. Instead with MAP, a fraction of the high-value consumers—those fortunate enough to find themselves at a low-price retailer—gain $h - l$ and thus consumer surplus may be higher or lower with MAP.

3 MAP and service provision

In this section, we develop the idea that a service component might lead a MAP arrangement to be optimal. We further demonstrate the role of MAP in accommodating heterogeneity but focus on retailer cost heterogeneity and suppose that consumers are homogeneous, which shuts down price discrimination. To highlight the role of search costs, we set $\sigma = 0$. That is, all consumers have high costs of search and therefore visit only one store. As above, the goods sold at the different stores are homogeneous. The model is enriched such that consumers have downward-sloping,

³⁶To be clear, we do not argue that MAP and RPM should invite the same sanctions as market division and price fixing. Indeed, as much of this paper shows, both often serve to make markets more efficient and more consumer friendly.

³⁷Interestingly, contingent on IC constraints being satisfied, as more retailers are used in a market, surplus extraction via MAP will approach perfect (type 1) price discrimination in this setting.

rather than unit, demand, with each consumer’s demand given by $q(p)$.³⁸ Therefore, retailers with different marginal costs have different optimal retail prices.

Further, in order to address a classic pro-competitive rationale for vertical restraints, on the retailer side we introduce a new service provision decision, taken at stage 3 in the timing of the core model described in Section 2.1.1. Specifically, retailer j makes an investment in a service level $s_j \in [0, 1]$ at cost $I(s_j)$, which is continuously differentiable and increasing, and $I'(s)$ is sufficiently high that equilibrium investments are strictly less than 1. Services acts to increase consumer awareness of the product.³⁹ A consumer who is aware of the product can purchase from either retailer, as in Mathewson and Winter (1984) but unaware consumers do not purchase. An investment of s_j will expose a measure of consumers of size s_j to the product. The probability that a consumer is exposed to the investment of one retailer is independent of whether it is exposed to the other, so the total measure of consumers who are aware of the product is equal to $\mathcal{S} = 1 - (1 - s_1)(1 - s_2)$.

Finally, we suppose that each retailer’s marginal cost of retailing is independently drawn from an ex-ante known distribution. This marginal cost takes the value $c > 0$ with probability α , and 0 otherwise. Retailer costs remain private information; that is, retailers do not know each others’ realizations. These assumptions ensure that there is the *ex-post* price dispersion that is needed for RPM and MAP to have a meaningful role. We suppose that retailers’ costs are realized only after entering into contracts with the manufacturer; that is, after stage 2 and before stage 3 in the timing of Section 2.1.1. Hence, the two-part tariff extracts surplus by setting the fixed fee equal to a retailer’s expected net profit.⁴⁰

This model shows how MAP can lead to pro-competitive service when the price discrimination channel outlined in Section 2 is not operative. That is, the focus is shifted from MAP’s efficacy in leveraging consumer heterogeneity to its efficacy in environments dominated by retailer heterogeneity. Moreover, we highlight that MAP can play a pro-competitive role through a standard channel highlighted in the literature on vertical restraints: by encouraging pro-competitive investments in service provision.

The argument proceeds in stages. First, we characterize the benchmark case of no restrictions. Next, we turn to the case of minimum RPM and rehearse the familiar argument that minimum RPM can allow retailers higher anticipated margins on

³⁸Also, $q'(p) \in (-\infty, 0)$, $q''(p) \in [0, \infty)$.

³⁹As pointed out in Asker and Bar-Isaac (2014), service need not be pro-competitive, and (at least in their model), it can lead to exclusion when retailers are gatekeepers to a market. All the arguments made in Asker and Bar-Isaac (2014) can be transferred to environments with MAP.

⁴⁰This is implied by the timing in Section 2.1.

each unit sold, and, thereby, leads to higher service levels. Finally, we turn to the central result of this section—that MAP can raise manufacturer’s profits and be more efficient than RPM in allowing retailers to earn higher margins and, in this way, lead to higher service levels. In turn, this allows for higher consumer surplus and welfare. Thus, MAP may be pro-competitive as compared to either RPM or no restrictions.

3.1 No vertical restrictions

With no vertical restrictions, all consumers who are aware of the product can observe both prices (for example, they might consult an online price aggregator). Conditional on a realization of \mathcal{S} , the measure of aware consumers, R_1 ’s profit is given by

$$\pi(p_1, p_2) = \begin{cases} \mathcal{S}(p_1 - w - c_1)q(p_1) & \text{if } p_1 < p_2 \\ \frac{1}{2}\mathcal{S}(p_1 - w - c_1)q(p_1) & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases} \quad (6)$$

Profit for R_2 is similarly defined.

In considering the equilibrium of the pricing subgame (stage 3 in the timing in Section 2.1.1) we analyse a discrete analog to Spulber (1995), which points out that Bertrand with privately known costs mirrors the equilibrium in a first price auction with risk aversion. Equilibrium of the pricing subgame then follows Proposition 2 in Spulber (1995), which is, itself, an adaptation of Theorem 2 in Maskin and Riley (1984) and guarantees a unique equilibrium in symmetric strategies.

Lemma 1 *There exists a unique equilibrium. The equilibrium features symmetric pricing strategies, such that, for $j \in \{1, 2\}$,*

1. *if $c_j = c$, $p_H = w + c$*
2. *if $c_j = 0$, price (p_L) is drawn from the distribution $F_L(p) = \frac{(p-w)q(p) - \alpha(\bar{p}-w)q(\bar{p})}{(1-\alpha)(p-w)q(p)}$ with the support $[\underline{p}, \bar{p}]$, where $\bar{p} = \min\{p_H, p_L^M(w)\}$ and p_L^M is the monopoly price a low-cost retailer would charge; i.e., $p_L^M = \arg \max(p - w)q(p)$; and \underline{p} is implicitly defined by $(\underline{p} - w)q(\underline{p}) = \alpha(\bar{p} - w)q(\bar{p})$.*

Proof. See the appendix. ■

Next, we proceed to consider the equilibrium of the investment subgame. We denote the investment level by a retailer with high costs by s_H , and with low costs by s_L .⁴¹

Lemma 2 *There exists a symmetric equilibrium in which a retailer with high costs makes no investment ($s_H = 0$) and a retailer with low costs invests s_L where this is the solution to*

$$I'(s_L) = (1 - (1 - \alpha) s_L) \alpha (\bar{p} - w) q(\bar{p}). \quad (7)$$

The equilibrium is unique when $I''(\cdot) < \alpha(\bar{p} - w)q(\bar{p})$.

Proof. Note that given the pricing equilibrium in Lemma 1, a high-cost retailer earns no (net of investment) profits, and therefore, there is no investment at a high cost realization; that is, $s_H^1 = s_H^2 = s_H = 0$.

In choosing its investment level at a low cost realization s_L^i , retailer i , when having a low cost realization, solves the following problem (illustrated here for Retailer 1):

$$\max_{s_1} E_{s_2}(\mathcal{S}) \tilde{\pi} - I(s_L^1), \quad (8)$$

where $E(\mathcal{S}) = 1 - (1 - s_1)(1 - \alpha s_H^2 - (1 - \alpha) s_L^2) = 1 - (1 - s_1)(1 - (1 - \alpha) s_L^2)$ and $\tilde{\pi} = \alpha(\bar{p} - w)q(\bar{p})$ (note, by construction, in the mixed strategy equilibrium, profits adjusted by the probability of winning is constant over p). This yields the following best-response function in the investment subgame, which arises from the first-order condition:

$$I'(s_L^1) = (1 - (1 - \alpha) s_L^2) \alpha (\bar{p} - w) q(\bar{p}). \quad (9)$$

The best responses are symmetric and downward-sloping, since $I(\cdot)$ is convex and $\frac{ds_L^1}{ds_L^2} I''(s_1) = -(1 - \alpha) \alpha (\bar{p} - w) q(\bar{p})$, and $s_L^1(0) > 0$. Consequently, there is an equilibrium where investment the level for low-cost retailers is symmetric, i.e. $s_L^1 = s_L^2 = s_L$ and solves

$$I'(s_L) = (1 - (1 - \alpha) s_L) \alpha (\bar{p} - w) q(\bar{p}). \quad (10)$$

A sufficient condition for this equilibrium to be unique $I''(s_1) < \alpha(\bar{p} - w)q(\bar{p})$ since this ensures that $\frac{ds_L^1}{ds_L^2} < -1$. ■

⁴¹Imposing specific functional forms allows for a closed-form solution; for example, when $I(s) = \frac{s^2}{2}$ then $s_L = \frac{\alpha(\bar{p}-w)q(\bar{p})}{1+(1-\alpha)\alpha(\bar{p}-w)q(\bar{p})}$.

Finally, we can turn to the problem of the manufacturer. The manufacturer sets T equal to a retailer's expected profits so that

$$T = (1 - \alpha) [(1 - (1 - s_L(w))(1 - (1 - \alpha) s_L(w)))\alpha(\bar{p}(w) - w)q(\bar{p}(w)) - I(s_L(w))],$$

where this expression follows on noting that only the low-cost retailer earns profits, and that the expected size of the market and expected profits per customer follow from the expressions that appear below (8). The manufacturer chooses w to maximize

$$2T(w) + w \left[2\alpha(1 - \alpha)s_L(w) \int_{\underline{p}(w)}^{\bar{p}(w)} q(x)f_L(x)dx \right. \\ \left. + (1 - \alpha)^2(1 - (1 - s_L(w))^2) \int_{\underline{p}(w)}^{\bar{p}(w)} q(x)2f_L(x)(1 - F_L(x))dx \right],$$

where the term in square brackets corresponds to expected sales. To understand this term, first note that when both retailers have high costs, then there is no investment in advertising, and therefore, there are no sales; with probability $2\alpha(1 - \alpha)$, there is only one low-cost retailer, a mass of s_L consumers aware of the product and each consumer purchases $\int_{\underline{p}}^{\bar{p}} q(p)f_L(p)dp$ in expectation; finally, the second term inside the square brackets describes the event in which both retailers obtain low cost realizations. When both retailers are low cost, consumers aware of the product will purchase from the lower of two prices drawn independently from $F_L(\cdot)$; thus, the final integral reflects the expected quantity corresponding to a price distributed as the second-order static of $F_L(\cdot)$.⁴²

3.2 RPM

From the perspective of the manufacturer, resale price maintenance can strictly improve on the outcome with no restraints. The channels that allow this are standard. First, minimum RPM can soften price competition between retailers, and hence, by allowing the retailers higher expected profits, can induce greater investment. For example, consider the case in which α is close to 1, so that both retailers are almost certainly high-cost retailers. Retailer (net) profits and, hence, advertising investments would be negligible. In this case, an RPM provision that guaranteed retailers

⁴²Recall that the ultimate goal of this section is to show (by way of an example) that MAP can be the preferred method to incentivize service provision. Hence, a general characterization of the optimal w is not required. Further, inspection of the maximand suggests that such a characterization will be unhelpfully complicated.

some net profits, and induced some investment, would clearly improve manufacturer profits. Second, retail prices (for retailers with low cost realizations) may be too high due to a standard double mark-up problem; maximum RPM can help solve this problem. Here, the focus is on the case of minimum RPM, set at a level denoted by P , directed to address the need for service.

First, note that if minimum RPM is binding for both high- and low-cost retailers, the retail price (regardless of the retailers' cost realizations) is given by P . Instead, if the restriction is binding only for retailers with low cost realizations, then the pricing equilibrium may, again, involve a mixed strategy for retailers with a low-cost realization. The upper limit for the support of any such mixing distribution is \bar{p} and the mixing distribution is a truncation of $F_L(\cdot)$, where these are as defined in Lemma 1.

Lemma 3 *Given minimum RPM set at the level $P < w + c$, there exists a unique equilibrium in the pricing game. The equilibrium features symmetric pricing strategies, such that for $j \in \{1, 2\}$,*

1. if $c_j = c_H$, $p_H = w + c_H$;
2. if $c_j = 0$ then if $\underline{p} \geq P$ then p_L , is drawn from a truncation of $F_L(p)$ on $[\underline{p}, \bar{p}]$ (where these are all as described in Lemma 1); and if $\underline{p} < P$ p_L , is drawn from a truncation of $F_L(p)$ on $(\underline{p}_{\min}, \bar{p})$ and the remaining probability $F_L(\underline{p}_{\min})$ assigned to an atom at P ; where \underline{p}_{\min} satisfies

$$(\underline{p}_{\min} - w)q(\underline{p}_{\min}) = \frac{\alpha(P - w)q(P)(\bar{p} - w)q(\bar{p})}{(P - w)q(P) - 2\alpha(\bar{p} - w)q(\bar{p})}.$$

Proof. See the appendix. ■

The characterization of the pricing equilibrium suggests that if the optimal minimum RPM induces price dispersion, it cannot aid the manufacturer through inducing service provision.⁴³

Corollary 2 *Suppose that the minimum RPM price is $P < w + c_H$ (so that it does not bind, given a high cost realization) then the level of investment in service is identical to the level of investment in the absence of RPM or any other restrictions—which is the solution to (8).*

⁴³Note that, in principle, minimum RPM (not only maximum RPM) may play a role in affecting “double mark-up” concerns by changing the price distribution and therefore, from the manufacturer’s perspective, may still be preferable to no restrictions, even if it has no impact on service provision. However, we do not consider such an effect any further, and instead focus on the comparison with MAP for inducing service.

Proof. A retailer with a high cost realization makes no net profits and the low cost retailer, its profit per aware consumer is identical to the case of no restrictions. Thus, the low-cost retailer faces the identical maximization problem (8) in its choice of investment in service. ■

It follows that here RPM can play a role in inducing service provision only if it binds for both low and high cost realizations; that is, if $P > w + c_H$. We provide a complete statement of the manufacturer's problem with RPM, which is analogous to our analysis of the case with no restrictions shown above, in Appendix B.3. Here, it is worth noting that in the former case, where both high- and low-cost firms set a price equal to $P > w + c_H$, demand for a retailer irrespective of its costs is given by $\frac{q(P)}{2}$. Consequently, the following is immediate.

Lemma 4 *When RPM binds for both cost realizations, service for low cost, s_L , and high cost realizations, s_H , are implicitly characterized as follows:*

$$I'(s_L(P)) = (1 - \alpha s_H - (1 - \alpha) s_L)(P - w) \frac{q(P)}{2}, \text{ and} \quad (11)$$

$$I'(s_H(P)) = (1 - \alpha s_H - (1 - \alpha) s_L)(P - w - c_H) \frac{q(P)}{2}. \quad (12)$$

Proof. These are simply the first-order conditions that determine the choice of service level for each cost realization. ■

Note that these equations must be satisfied simultaneously; in comparison, in the case of no restrictions solving for a high-cost retailer's investment is trivial, which allows for a simple characterization of a low-cost retailer's investment in (9).⁴⁴

It is straightforward to find instances in which the manufacturer might benefit from imposing such a price restriction; in particular, this will be the case for α close to 0 or 1.

Summarizing the discussion above, we obtain Proposition 2.

Proposition 2 *Minimum RPM set at P can strictly improve the manufacturer's profits relative to the case of no restrictions in the case of smooth service investments. It can do so through raising retailers' advertising investments only when it leads retailers to choose pure pricing strategies.*

Proof. The second sentence is simply a restatement of Corollary 2. The first statement can be demonstrated by example. We do so in Example 2. ■

⁴⁴For specific parameterizations, unique closed-form solutions are ensured. For example, for quadratic investment costs $I(s) = \frac{s^2}{2}$, $s_L(P) = \frac{(P-w)q(P)}{2+(P-w-\alpha c)q(P)}$ and $s_H(P) = \frac{(P-w-c)q(P)}{2+(P-w-\alpha c)q(P)}$.

3.3 MAP

MAP can have the benefit of the minimum RPM scheme described above in dulling competition between retailers and allowing them higher profit margins, thereby encouraging investment in service. However, MAP has the benefit of allowing some retailers to drop the actual price below the advertised minimum. Retailers may want to do this because their monopoly price may be below the MAP price. Moreover, this action can be in the interest of the manufacturer because it increases the sales per customer for customers who are aware of the minimum RPM policy and can increase the low-cost retailer's level of investment and the overall number of customers aware of the product.

Specifically, MAP at the MAP price P plays a different role from RPM when the high-cost retailer charges this price and the low-cost retailer charges a different (lower) price.⁴⁵

Under MAP, half of the consumers who are aware of the good purchase from either retailer (irrespective of its costs) since the retailers advertise the same price. These consumers then respond to the price that they observe at the retailer. It is clear that the low-cost retailer would charge $p^m(w)$, which maximizes $\frac{q(p^m(w))}{2}(p^m(w) - w)$. Consequently, service for low cost and high cost realization can be easily implicitly characterized through the first-order conditions as follows:⁴⁶

$$I'(s_L(P)) = (1 - \alpha s_H - (1 - \alpha) s_L)(p^m(w) - w) \frac{q(p^m(w))}{2}, \text{ and} \quad (13)$$

$$I'(s_H(P)) = (1 - \alpha s_H - (1 - \alpha) s_L)(P - w - c_H) \frac{q(P)}{2}. \quad (14)$$

These differ from the equations that characterize investment under RPM at the same price P (Equations (11) and (12)), since the low-cost retailer sells at a price $p^m(w)$ rather than P and sells $q(p^m(w))$ to each consumer who arrives at the retailer rather than $q(P)$. However, as these equations show, although greater investment from a low-cost retailer might be anticipated as a result of the imposition of MAP, a consequence would be a force that dampens a high-cost retailer's investment. As in the case with no restrictions, the manufacturer optimally sets T equal to a retailer's

⁴⁵At a sufficiently high MAP price, both a high-cost retailer and a low-cost retailer would charge a lower price than the MAP price. However, since a high-cost retailer's price can be predicted—it is simply its monopoly price (given w)—the manufacturer could set the MAP price at this level. Consequently, it is without loss of generality to suppose that the MAP price is always binding for a high-cost retailer in equilibrium. For expositional simplicity, we assume $P \leq p_H^M(w)$.

⁴⁶For example, with quadratic investment costs $I(s) = \frac{s^2}{2}$, a unique closed-form solution is assured. Specifically, $s_L(P) = \frac{(P-w)q(P)}{2+(P-w-\alpha c)q(P)}$ and $s_H(P) = \frac{(P-w-c)q(P)}{2+(P-w-\alpha c)q(P)}$.

expected profits and chooses w to maximize expected profits. A formal treatment can be found in Online Appendix B.3.

We argue that the different economics of MAP as compared to RPM, by allowing the low-cost retailer to charge a lower price than the MAP price, lead to higher industry profits and more effective investments. We establish this via the following proposition, and a subsequent example.

Proposition 3 *MAP can earn the manufacturer strictly higher profits than minimum RPM. When investment costs are sufficiently convex or if the firm is unlikely to be low cost (α is high enough), then MAP can never earn less than minimum RPM.*

Proof. We prove the first statement through Example 2 below.

For the second statement, denote the optimal RPM two-part tariff by (T^*, w^*) and the associated level of the price minimum P . Following the discussion in Section 3.3, for RPM to have any effect it must bind on both cost realizations to that $P > c + w^*$. It is clear that it is suboptimal from the manufacturer's perspective to set $P > p_H^m(w^*)$ as this leads to lower profits per aware consumer and can only reduce service.

Rather than considering the optimal MAP scheme, suppose that MAP is imposed at P and with a wholesale price of w^* ; clearly, a retailer with a high cost realization would choose a retail price equal to P .

If it is also the case that $P \geq p_L^m(w^*)$ then MAP would achieve the same outcome as RPM. However, it is possible that $p_L^m(w^*) < P$. This potentially affects M 's profits in several ways:

First, there may be a direct effect through sales by increasing the sales per aware consumer (since a low-cost retailer charges a lower price). Second, retailers' investments may change with an impact on sales: Holding fixed the service level of a high-cost retailer, it is clear that a lower-cost retailer would invest more. However, the higher investment of a low-cost retailer reduces a high-cost retailer's gains from investment. Overall, the combined effect is ambiguous. With sufficiently convex costs (or high α), the effects on the low-cost retailer's investment is negligible, leading to a positive overall effect on profits.

Finally, there is an effect through a (potentially) higher fixed fee T^{MAP} , which might also reflect different service levels, and therefore, in principle, the sign of this effect might be ambiguous. Again, when the investment cost function is sufficiently convex or α is high enough, the impact on service levels is small, such that this effect is also positive. Thus, M 's overall profits rise.. ■

It is worth noting that MAP may under-perform RPM from the perspective of the manufacturer; this is (trivially) the case when all consumers can search costlessly ($\sigma_l = \sigma_h = 1$) and MAP serves no useful function.

While Proposition 3 provides sufficient conditions for the manufacturer to earn higher profits with MAP, it is noteworthy that these conditions are somewhat strong, and, in effect, ensure that the effects through the investment of a retailer with low-cost realization are negligible. Example 2, below, provides a numerical complement to Propositions 2 and 3. In particular, the example demonstrates that settings do indeed exist in which, by allowing for greater investment in efficient service and simultaneously allowing more sales to be realized, MAP can be pro-competitive (whether judged according to a consumer surplus or a total surplus criterion).

Example 2 *Let $q(p) = 1 - p$, $\alpha = 0.9$ and $c_H = 0.4$ and $I(s) = \frac{s^2}{2}$. Table 1 shows the characteristics of the optimal policies in the cases of no restrictions, of RPM and of MAP.⁴⁷*

First note that minimum RPM leads to higher profits for the manufacturer than no restrictions, as in Proposition 2. Moreover MAP generates highest profits for the manufacturer, consistent with Proposition 3. It is noteworthy, that MAP also generates highest total surplus and consumer surplus. This arises with higher investment in service. In this way, the example highlights the argument in this section that MAP may play a pro-competitive role in leading to higher levels of investment in service.

Note that the RPM price is fixed and is slightly lower than the monopoly price for high-cost retailers (which would be 0.7 at the wholesale price $w = 0$) since this price is too high for the low cost retailers. Instead, the MAP price is indeterminate; as long as it is higher than 0.7, both high cost and low cost retailers can charge the monopoly price. Thereby, retailers earn the high margins that induce relatively high investment levels. In turn, this accounts for the positive impact of MAP on surplus.

Example 2, together with Propositions 2 and 3, allows us to establish the following:

Proposition 4 *MAP can be optimal from the manufacturer's perspective and generate higher consumer and total surplus than either RPM or no vertical restraints.*

4 MAP and collusion

Vertical restrictions, and RPM in particular, have long been regarded as potentially collusive mechanisms. That is, the manufacturer's binding of retailers to a common

⁴⁷Matlab code for generating the results in Table 1 can be found at www.johnasker.com/MAPservice.zip

Table 1: Results for example 2: $\alpha = 0.9$, $c_H = 0.4$, $q(p) = 1 - p$. $I(s) = \frac{s^2}{2}$

	No restrictions	RPM	MAP
Manufacturer expected profits	0.0057	0.0077	0.0084
Retail expected profits	0.0020	0.0039	0.0041
Low-cost R profits	0.0203	0.0109	0.0125
High-cost R profits	0	0.0031	0.0032
P	-	0.6723	≥ 0.7
w	0.0805	0	0
s_L	0.1836	0.1048	0.1140
s_H	0	0.0424	0.0428
Consumer surplus	0.0061	0.0051	0.0065
Total surplus	0.0159	0.0205	0.0231

price can look, at first glance, like a price-fixing agreement.⁴⁸ In the modern era, the concern that RPM facilitates collusion continues to shape policy, as supported by research, most notably, the central contribution of Jullien and Rey (2007).⁴⁹ Jullien and Rey (2007) demonstrate that RPM facilitates collusion by making deviations more transparent at the cost of reducing responsiveness to local idiosyncratic demand fluctuations.⁵⁰ The analysis below shows that MAP can similarly soften competition, again by inducing demand patterns akin to a market division scheme, without reducing responsiveness to local market demand fluctuations. This situation results in MAP facilitating a more profitable and stable cartel.

The idea can be seen heuristically in the following simple example. Two manufacturers sell to a market via dedicated (non-shared) retailers. Consider a market in which there is always a high-value consumer who would pay h . In addition, with probability μ , there is an additional consumer who has a low valuation l . This

⁴⁸See Overstreet (1983) for a comprehensive historical survey.

⁴⁹Jullien and Rey (2007) provide a formal foundation for ideas that have existed, albeit in only heuristic form, since at least Telser (1960) and Yamey (1954).

⁵⁰Jullien and Rey (2007) note other elements of the tradeoff, notably that RPM can make punishment more severe.

additional consumer is local to one of the two manufacturers' retailers (with equal probability). That is, with probability $1 - \mu$, there is only a high-value consumer, and with probability $(\mu/2)$, there is also a low-value consumer available to manufacturer 1 and with probability $(\mu/2)$, a low-value consumer available to manufacturer 2. Suppose that $2l > h$, so that if a low-value consumer is available, the industry would earn higher profits by selling to both consumers than to the high-value consumer alone.⁵¹

In the simple environment outlined above, RPM does not help manufacturers collude (with this simple unit demand specification, the manufacturers can agree on a common (high) retail price by setting $w_1 = w_2 = h$), and observing retail prices allows manufacturers to monitor their compliance with this scheme. MAP, or equivalently an advertising ban, accommodates heterogeneity in this case by allowing for responses to local demand shocks but without intensifying competition (since for a retailer, there is always a 50% chance of selling to the high-value consumer, and if there is a low-cost consumer, then the relevant retailer will want to sell to him and does so). Hence, here, MAP can increase the scope for collusion relative to no vertical arrangements or, indeed, relative to RPM.

In the following, we embed this logic in a formal model that builds on the Jullien and Rey (2007) framework. The modeling objective is to allow price advertising to have an explicit role in the market, while retaining tractability and enriching the economic forces illustrated in Jullien and Rey (2007). The resulting framework is one in which RPM facilitates a more stable cartel (in the sense of admitting collusion for a wider range of discount rates than a cartel without restraints) at the cost of flexibility. MAP, by contrast, can facilitate an even more stable cartel and does not sacrifice flexibility, leading to higher cartel profits.

4.1 Model details

There are two manufacturers, each of whom distributes to the local market via a dedicated (non-shared) retailer; that is, there is an M_1 that sells through R_1 and an M_2 that sells through R_2 . In this section, as in Section 2.1, we suppose that consumers have unit demand. However, instead of assuming that the two goods are homogeneous, we suppose that consumers value retailer R_j 's good at $v + \xi_j$, where $\xi_j \sim U[0, 1]$ is a manufacturer-retailer specific demand shock that reflects idiosyncratic match values. It follows that consumers will choose to visit retailer R_j

⁵¹Note that this also implies that each manufacturer would rather sell with certainty at a price l than sell at a price of h with probability $(1/2)$.

if

$$v + \xi_j - E(p_j | p_j^a) \geq 0 \quad (15)$$

and

$$v + \xi_j - E(p_j | p_j^a) > v - \xi_k - E(p_k | p_k^a), \quad (16)$$

where p_j^a is R_j 's advertised price and $E(p_j | p_j^a)$ is the expectation of the retail price, conditional on the advertised price.⁵²

The first inequality requires that the consumer must expect non-negative surplus, and the second requires that the consumer expects greater surplus from visiting R_j than R_k . If the first inequality holds and the second condition holds with equality, then we suppose that the consumer purchases from whichever retailer offers a higher ξ . To keep exposition simple, $\sigma = 0$ so that consumers visit at most only one store (search costs are high).

Furthermore, we suppose that ξ_j is not observed by any manufacturer or by the rival retailer; it is only privately observed by R_j after contracting with the manufacturer and before setting the retail price. Each manufacturer sells to its dedicated retailer using a two-part tariff $\{w_j, T_j\}$ (which is not publicly observed).⁵³ Lastly, $v > 1$, which ensures that w_j is always set such that all ξ_j realizations are served. Advertised prices are observed by all—including rival manufacturers.

Timing within a stage is otherwise as in Section 2.1.1. There is an infinite repetition of such stages where manufacturers discount the future at a common rate δ and retailers and consumers are myopic in their behavior.

In the remainder of this section, we focus on symmetric equilibria and characterize the policies that attain the same profits that an individual monopolist selling a single good could attain and the range of discounts for which these can be sustained. First, we do so under no vertical restrictions, then RPM (where the corresponding benchmark is such a monopolist who charges a single retail price), and, finally, MAP. We then compare these and show that MAP allows for both more profitable and more sustainable collusion. In fact, in the cases of no vertical restraints and RPM, a cartel can earn higher profits since a cartel involves two goods and so two potential match values; in Online Appendix B.4, we show that this involves somewhat more involved expressions, but the qualitative results are unaffected and collusion under MAP is both more profitable and more sustainable than under the alternatives.

⁵²In the absence of a MAP restraint, $E(p_j | p_j^a) = p_j$. With a MAP restraint, this expectation will be determined by the equilibrium described in Section 4.4.

⁵³These assumptions reflect those in Jullien and Rey (2007). The assumptions on ξ_j give a reason to provide some pricing discretion to the retailer. The assumptions on $\{w_j, T_j\}$ make the cartel's monitoring of compliance non-trivial (and keeps IR constraints relatively simple).

4.2 Collusion with no vertical restrictions

Here, we sketch the elements of the equilibrium and provide a characterization that is further fleshed out in the appendix. Once characterized, collusion to attain monopoly profits in the absence of vertical restraints provides a benchmark against which to compare the much simpler RPM- and MAP-facilitated schemes.

We begin by presenting a preliminary result that will prove useful. Specifically, we begin by examining the pricing game between retailers, supposing (as will turn out to be the case in equilibrium) that manufacturers choose the same input price; that is, supposing that $w_1 = w_2 = w$. In this case, the pricing game between retailers is a minor variant on the standard IPV symmetric first-price sealed-bid auction in which each retailer solves

$$p_j = \arg \max_{w \leq p \leq v + \xi_j} (p - w) \Pr(\xi_j - p > \xi_k - p_k) \quad (17)$$

Equilibrium pricing is stated in Proposition 5 and derived following the steps in, for instance, Krishna (2002).

Proposition 5 *Assume that the manufacturers charge the same wholesale price w ($w_j = w_k = w$). In a monotone perfect Bayesian equilibrium, each retailer's equilibrium price is equal to its advertised price and is given by $p_j^a = p_j = w + \frac{\xi_j}{2} - \frac{w-v}{2} 1_{v < w}$.*

Proof. See the appendix. ■

Proposition 5 means that if $w = 0$, as would arise in the static version of the game, then retailers price using the rule $p_j = \frac{\xi_j}{2}$. That is, prices fluctuate by half the actual fluctuations in local demand. The retailers' expected profit, which is then transferred to the manufacturer through the fixed fee, T_j , is equal to the expectation of $\frac{\xi_j^2}{2}$, which is $\frac{1}{6}$.⁵⁴

Instead, the scheme that attains an individual monopolist's profits in this setting involves both manufacturers setting $w = v$ and $T = \frac{1}{6}$. That is, the manufacturers set w as high as possible without excluding any consumers,⁵⁵ and then, retailers compete by accounting for idiosyncratic market conditions. The manufacturers extract the remaining expected surplus from retailers using the lump sum component of the two-part tariff. This is the scheme that a monopolist selling a single good would find optimal, although as we describe in Online Appendix B.4, the cartel can earn higher profits than a monopolist could since a cartel of two manufacturers involves two match realizations rather than one.

⁵⁴That is, the retailer's expected profit is $\int_0^1 \frac{\xi_j}{2} \Pr(\xi_j > \xi_k) d\xi_j$.

⁵⁵This follows from the assumption that $v > 1$.

This collusive scheme is enforced using a variant of a grim trigger strategy in which manufacturers set $w_t = v$ if all past advertised pricing has been in the interval $[v, v + \frac{1}{2}]$, and $w_t = 0$ otherwise. This results in an expected per-period collusive profit for each manufacturer of $\frac{v}{2} + \frac{1}{6}$. This characterization is complicated by the fact that determining the optimal deviation is not immediate. In particular, if a manufacturer charges a slightly lower wholesale price in some period t , because of the local demand ξ_j demand variations, this may lead to only a marginal gain but will have only a marginal probability of detection. Alternatively, a manufacturer might prefer a more drastic deviation that leads to a substantive gain, albeit at the cost of a higher probability of detection. These two forms of deviation can, in principle, lead to two locally optimal deviations, which then need to be compared. Full details appear in the appendix. The following proposition provides the lower bound on the discount factor, δ , required for this collusive scheme to be supportable in equilibrium.

Proposition 6 *A collusive equilibrium in which manufacturers set $w_t = v$ if $p_j^a \in [v, v + \frac{1}{2}]$ for all j in all past periods, and $w_t = 0$ otherwise, is supportable if $\delta > \frac{6v-2}{9v-2}$. In such an equilibrium, the per-period profit earned by each manufacturer, π_{NR}^c , is $\frac{v}{2} + \frac{1}{6}$.*

Proof. See the appendix. ■

4.3 Collusion with RPM

When RPM is available, the manufacturers can set the price at which the retailers sell to customers. As discussed in Section 2.1, this will also be the advertised price. The value of RPM is that it removes any uncertainty on the part of the cartel members as to whether there has been a deviation or not, albeit at the expense of losing some flexibility in adjusting to local market conditions (that is, adjusting for ξ_j realizations). Thus, collusion can now take the following form. Set $w_t = p^{RPM} = v$ if the same advertised pricing has been observed in all past periods, and $w_t = 0$ otherwise.⁵⁶ Proposition 7 describes this much simpler, collusive environment.⁵⁷

Proposition 7 *A collusive equilibrium in which manufacturers set $w_t = p^{RPM} = v$ as long as $p_{j,t}^a = v$ for all j in all past periods, and $w_t = 0$ otherwise, is supportable*

⁵⁶An alternative punishment phase may involve using RPM and setting $w_t = p^{RPM} = 0$. This is a more extreme form of punishment. With this punishment stage equilibrium, $\pi^p = 0$ in the notation of the proof of Proposition 7. Thus, in Proposition 7, the threshold would adjust to $\delta > \frac{1}{2}$.

⁵⁷Just as for the case of no vertical restraints, the optimal collusive scheme exploits that the cartel that brings together two manufacturers generates two match values. The optimal scheme is characterized in Online Appendix B.4.

if $\delta \geq \underline{\delta}^{RPM} = \frac{3v}{6v-1}$. In such an equilibrium, the per-period profit earned by each manufacturer, π_{RPM}^c , is $\frac{v}{2}$.

Proof. The proof is standard. It involves solving for δ such that $\frac{\pi_{RPM}^c}{1-\delta} \geq \pi_{RPM}^D + \frac{\delta}{1-\delta}\pi^p$, where $\pi_{RPM}^c = \frac{v}{2}$, $\pi_{RPM}^D = v$ and $\pi^p = \frac{1}{6}$. ■

4.4 Collusion with MAP

Collusion, facilitated by MAP, involves coordinating on the advertised price but allowing the transaction price (as usual under MAP) to be constrained only to be less than or equal to that advertised price. The value of this action to the colluding manufacturers is that it obfuscates pricing in the market for consumers, effectively introducing a form of market division while retaining flexibility on the part of retailers to adjust prices to local idiosyncratic conditions; that is, it is a scheme that partially accommodates the realized demand heterogeneity. In this environment, a natural collusive strategy to consider is one where each manufacturer operates as independent monopolist on half of the market. This can be achieved when each manufacturer sets $p_{j,t}^a = p_{k,t}^a = v + 1$. Retailers, having retained the flexibility to adjust prices downward, are now free from competition and can extract all consumer surplus by setting $p_{j,t} = v + \xi_{j,t}$ (in contrast, with no restraints, there is competition so that $p_{j,t} = v + \frac{\xi_{j,t}}{2}$). Thus, the MAP provision in this environment allows the retailer to extract all available surplus from consumers they service (assuming a rationing rule in which consumers purchase the good for which they have the highest value, given their indifference between goods after expected pricing is taken into account).⁵⁸ Proposition 8 formalizes this intuition.

The proposition supposes that if the cartel agreement breaks down, the manufacturers revert to playing the static equilibrium policies.⁵⁹

Proposition 8 *A collusive equilibrium in which manufacturers enforce $p_{j,t}^{MAP} = v+1$ as long as $p_{j,t}^a = v+1$ for all j in all past periods, and $w_t = 0$ without MAP otherwise, is supportable if $\delta \geq \underline{\delta}^{MAP} = \frac{1}{2}$. In such an equilibrium, the per-period profit earned by each manufacturer, π_{MAP}^c , is $\frac{v}{2} + \frac{1}{3}$.*

⁵⁸In any other rationing rule, the surplus extraction is inefficient because when competition is constrained, consumers are not directed toward products that maximize gains from trade. In the case where consumers who are indifferent are equally likely to visit either firm, the corresponding expressions for those that appear in Proposition 8 are $\pi_{MAP}^c = \frac{v}{2} + \frac{1}{4}$ and $\underline{\delta}^{MAP} = \frac{6v+3}{12v+4}$.

⁵⁹An alternative punishment phase may still involve using MAP and setting $w = p^{MAP} = 0$. This is a more extreme form of punishment. With this punishment stage equilibrium, $\pi^p = 0$. In this case, Proposition 8 would instead require $\delta > \frac{1}{2}$.

Proof. The proof is standard, requiring solving for δ such that $\frac{\pi_{MAP}^C}{1-\delta} \geq \pi_{MAP}^D + \frac{\delta}{1-\delta}\pi^p$, where $\pi_{MAP}^C = \frac{v}{2} + \frac{1}{3}$, $\pi_{MAP}^D = v + \frac{1}{2}$ and $\pi^p = \frac{1}{6}$. ■

4.5 Vertical restraints can facilitate collusion

A comparison of Propositions 6, 7 and 8 makes it clear that the collusive profits with MAP dominate those from the scheme without restrictions, which in turn dominate those from the RPM scheme. That is, $\pi_{MAP}^c > \pi_{NR}^c > \pi_{RPM}^c$.

Further, employing a vertical restraint (whether MAP or RPM) for the cartel can increase stability in the sense that the force of future retribution for a deviation coupled with the prospect of losing collusive gains becomes a stronger incentive relative to the gains from deviating. This increase in the strength of dynamic incentives is captured by comparing the lower bound of the range of discount factors under which collusion can be supported. It can be easily verified that for a high enough v (a sufficient condition is that $v > \frac{2+2\sqrt{2}}{3}$ so that $\frac{3v}{6v-1} < \frac{6v-2}{9v-2}$), a cartel can be more easily sustained with RPM than with no restraints. MAP, by contrast, always increases stability relative to no restraints since we assume $v > 1$ and, so, $\frac{1}{2} = \underline{\delta}^{MAP} < \underline{\delta}^{NR} = \frac{6v-2}{9v-2}$ in this range. The intuition for these results follows arises from observing that it is easier to monitor a manufacturer's deviation from the cartel arrangement when this arrangement involves a vertical restraint. When v is low, then an RPM arrangement may be sacrificing too much in terms of profits, as in Jullien and Rey (2007), and the flexibility to respond to local demand conditions is relatively valuable. MAP, on the other hand, maintains flexibility and therefore does not demand the same trade-off.

Finally, as mentioned above, $\underline{\delta}^{MAP} < \underline{\delta}^{RPM}$, which suggests that MAP policies may help in stabilizing cartel agreements relative to RPM, in addition to providing higher cartel profits.⁶⁰ This follows because, even though in both arrangements it is equally easy to monitor deviations from the cartel agreements, the MAP arrangement allows for a higher cartel profit since it accommodates the heterogeneity associated with local demand variation.

Thus, MAP policies can allow a manufacturers' cartel to attain greater surplus extraction than a cartel that does not use MAP or RPM and strengthens the dynamic incentives that give the cartel stability. As such, in this setting with search frictions, MAP appears to be a more effective cartel facilitation device than RPM. Although, for slightly easier exposition, above, we compare MAP to collusive schemes under RPM and no restraints that replicate the profits of a single-good monopolist,

⁶⁰As already noted above, in Footnotes 56 and 59, if MAP or RPM is used in the punishment stage, then $\underline{\delta}^{MAP} = \underline{\delta}^{RPM}$.

Appendix B.4 shows that the qualitative results—that MAP leads to a more stable and more profitable cartel—are robust to considering optimal cartels.

5 Discussion

This paper has discussed both pro- and anti-competitive features of MAP policies. MAP policies are distinct from RPM in that they can serve as a more effective way to incentivize service and a more effective way to facilitate collusion. As such, this paper suggests that to the extent that controversy surrounds the appropriate policy treatment of vertical price restraints, the same, or greater, controversy should surround vertical information restraints, of which MAP is a prominent example. At the very least, carefully considering the nature and impact of information constraints in markets where search is a central feature seems warranted.

Of course, another way of restraining information is a ban on retail price advertising, which may be an interesting restriction in its own right. Formally, in the MAP price discrimination discussion in Section 2, this ban on advertising would lead to equivalent retail pricing decisions and economic outcomes. Similarly, in the case of upstream collusion in Section 4, a ban on price advertising would lead retailers to charge their monopoly prices (which manufacturers can extract through fixed fees), leading to similar profits and conditions to sustain a collusive agreement to ban price advertising. In richer environments with downward-sloping demand rather than unit demand, as in Section 3, the level of the advertising restriction can be used to affect the double marginalization concern raised by Spengler (1950) in addition to the other considerations outlined. Further, in the presence of additional upstream and retail (imperfect) competitors, advertising directed at inter-brand competition would still be valuable and would lead the “monopoly” prices on which we focus to be interpreted as best responses to such competitors’ strategies. By contrast, when faced with an advertising ban and sufficiently high search costs (still retaining inter-brand competition), retailers could not commit to charge anything other than their monopoly price.⁶¹ Thus, a MAP policy can allow a manufacturer to reassure consumers that they will not suffer this kind of hold-up and, as a consequence, help the manufacturer engage in inter-brand competition.

This paper leaves open at least three areas of enquiry. First, aside from the collusion model, the frameworks presented here do not consider the impact of competition at the manufacturer level. Given that information restraints will likely change the

⁶¹Note that advertising is a way to make this commitment due to the requirement that the price be at or below the advertised price.

cross-price elasticities between competitors, even absent collusion, this area seems rich for further investigation.

Second, the models presented here limit the role of advertising to pure price advertising. This limitation is analytically helpful in creating a clear mapping between the advertised price and the transaction price but suppresses aspects that may be important in a richer model. An obvious issue in the price discrimination and collusion models is that unit demand means that prices do not influence quantities to the extent they might in a real market. A more delicate, and interesting, issue is that advertised prices may signal quality—an issue that is claimed to be a concern in some MAP policy statements.

Finally, the extent to which MAP, like other vertical restraints, can exclude a rival is not explicitly investigated, although it is conjectured that, as in other situations, this effect can be generated.

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A Appendix A: Proofs

Lemma 1 *There exists a unique equilibrium. The equilibrium features symmetric pricing strategies, such that, for $j \in \{1, 2\}$,*

1. if $c_j = c$, $p_H = w + c$
2. if $c_j = 0$, price (p_L) is drawn from the distribution $F_L(p) = \frac{(p-w)q(p) - \alpha(\bar{p}-w)q(\bar{p})}{(1-\alpha)(p-w)q(p)}$ with the support $[\underline{p}, \bar{p})$, where $\bar{p} = \min\{p_H, p_L^M(w)\}$ and p_L^M is the monopoly price a low-cost retailer would charge; i.e., $p_L^M = \arg \max(p-w)q(p)$; and \underline{p} is implicitly defined by $q(\underline{p})(\underline{p}-w) = \alpha(\bar{p}-w)q(\bar{p})$.

Proof. By standard Bertrand reasoning, in case of high cost realisation, that is, if $c_j = c$, then profits are competed away; that is, $\pi(p_H) = 0$. In turn, this requires $p_H = c + w$.

For $c_j = 0$, the equilibrium price distribution resembles that in Stahl (1989) or Spulber (1995) where standard arguments necessitate an atomless, continuous pricing distribution. The maximal price is the minimum of the monopoly price, given w and $c_j = 0$, denoted $p_L^M(w)$, and $c + w - \varepsilon$ such that $\varepsilon \rightarrow 0$. That is, if $p_L^M(w) \geq c + w$, then the maximal price of the price distribution will be arbitrarily close to $c + w$. Hence, if $p_L^M(w) \geq c + w$, $\bar{p} = c + w$. Otherwise, $\bar{p} = p_L^M(w)$.

Since a mixed strategy requires that $\alpha(\bar{p}-w)q(\bar{p}) = [\alpha + (1-\alpha)(1-F_L(p))](p-w)q(p)$ for all p in the support, it follows that $F_L(p) = \frac{(p-w)q(p) - \alpha(\bar{p}-w)q(\bar{p})}{(1-\alpha)(p-w)q(p)}$.

The lower bound of the support is found by setting $F_L(p) = 0$.

Uniqueness follows since the distribution of types has bounded support, following Maskin and Riley (1984).⁶² ■

Lemma 3 *Given minimum RPM set at the level $P < w + c$, there exists a unique equilibrium in the pricing game. The equilibrium features symmetric pricing strategies, such that for $j \in \{1, 2\}$,*

1. if $c_j = c_H$, $p_H = w + c_H$;

⁶²See also Kuniavsky (2013) who considers asymmetric equilibria in the Stahl (1989) model which require sufficient “loyals,” who do not feature in our environment.

2. if $c_j = 0$ then if $\underline{p} \geq P$ then p_L , is drawn from a truncation of $F_L(p)$ on $[\underline{p}, \bar{p}]$ (where these are all as described in Lemma 1); and if $\underline{p} < P$ p_L , is drawn from a truncation of $F_L(p)$ on $(\underline{p}_{\min}, \bar{p})$ and the remaining probability $F_L(\underline{p}_{\min})$ assigned to an atom at P ; where \underline{p}_{\min} satisfies

$$(\underline{p}_{\min} - w)q(\underline{p}_{\min}) = \frac{\alpha(P - w)q(P)(\bar{p} - w)q(\bar{p})}{(P - w)q(P) - 2\alpha(\bar{p} - w)q(\bar{p})}.$$

Proof. The logic is similar to the proof of Lemma 1.

Bertrand reasoning uniquely determines the behavior of retailers with high cost realization.

Indifference ensures that when a retailer with a low cost realization employes mixed strategies then it must be the case for $p \in (\underline{p}_{\min}, \bar{p})$ that the price distribution is $F_L(p)$,⁶³ for some \underline{p}_{\min} . In case $\underline{p} \geq P$ it is trivial that $\underline{p}_{\min} = \underline{p}$.

Finally, in case there is a mass at P , pricing marginally here would bring a discrete benefit, thus \underline{p}_{\min} the lower bound to the mixing range must be such that a retailer with a low cost realization is indifferent between choosing P or any other price. That is,

$$(\bar{p} - w)q(\bar{p}) = (P - w)q(P) \left[1 - (1 - \alpha) \frac{F_L(\underline{p}_{\min})}{2} \right], \quad (18)$$

where the square bracket corresponds to the probability of making a sale when charging a price P ; the only circumstance in which this doesn't happen is in half the cases when the other retailer is also low cost and charges a price of P .

Substituting for $F_L(\underline{p}_{\min})$ and rearranging gives the form of \underline{p}_{\min} that appears in the statement of the result. Note that (18) can be rewritten as $\left[1 - (1 - \alpha) \frac{F_L(\underline{p}_{\min})}{2} \right] = \frac{(\bar{p} - w)q(\bar{p})}{(P - w)q(P)}$, since we have assumed that $\bar{p} = p_L^M(w)$ and that the price cap is not everywhere binding, the right hand side is in $(0, 1)$ and so also $\frac{F_L(\underline{p}_{\min})}{2}$ is in $(0, 1)$ so $\underline{p}_{\min} < \bar{p}$.

Finally $\underline{p}_{\min} > P$ (since both are below $p_L^M(w)$) requires that $(P - w)q(P) < \frac{\alpha(P - w)q(P)(\bar{p} - w)q(\bar{p})}{(P - w)q(P) - 2\alpha(\bar{p} - w)q(\bar{p})}$ iff $\frac{(P - w)q(P)}{(\bar{p} - w)q(\bar{p})} < (1 + 2\alpha)$ which is necessarily the case. ■

Proposition 5 Assume that $w_j = w_k = w$. In a monotone perfect Bayesian equilibrium, $p_j = w + \frac{\xi_j}{2} - \frac{w - v}{2} \mathbf{1}_{v < w}$.

⁶³As for Lemma 1, this is unique.

Proof. The proof follows the proof of equilibrium in an IPV symmetric first-price sealed-bid auction. See Krishna (2002) p.16ff for omitted details, which are easily adapted to this setting. The retailers solve

$$p_j(\xi_j) = \arg \max_{w \leq p \leq v + \xi_j} (p - w) \Pr(\xi_j - p > \xi_k - p_k) \quad (19)$$

Let $u_k(\xi_k) = \xi_k - p_k(\xi_k)$ and denote the equilibrium strategy to be $\beta_k(\xi) = u(\xi_j)$, and $\beta^{-1}(u_k) = \xi_k$. Assume that this strategy is monotone and increasing. To begin, assume that $w \leq v$. Recall that ξ is uniformly distributed on $[0, 1]$. The retailers' problem is then

$$p_j = \arg \max_p (p - w) \beta^{-1}(\xi_j - p) \quad (20)$$

First-order conditions yield

$$\beta^{-1}(\xi_j - p) - (p - w) \frac{\partial \beta^{-1}(\xi_j - p)}{\partial (\xi_j - p)} = 0 \quad (21)$$

Imposing the equilibrium condition that $\beta(\xi_j) = u(\xi)$ yields

$$\xi_j - (p(\xi_j) - w) \frac{1}{1 - p'(\xi)} = 0 \quad (22)$$

$$p(\xi) + p'(\xi)\xi = \xi + w \quad (23)$$

$$\frac{\partial}{\partial \xi}(p(\xi)\xi) = \xi + w \quad (24)$$

$$p(\xi)\xi = \int (\xi + w) d\xi \quad (25)$$

Noting that $u(0) = 0$ allows the integral to be evaluated, yielding

$$p(\xi) = \frac{\xi}{2} + w \quad (26)$$

It remains to address the case where $w \geq v$. In this case, some measure of ξ are excluded. The measure of included realizations is now $U[w - v, 1]$, which implies that the boundary condition is now $u(w - v) = 0$, yielding the pricing part of the proposition. $p_j^a = p_j$ follows from Lemma 1. ■

Proposition 6 *A collusive equilibrium in which manufacturers set $w_t = v$ if $p_j^a \in [v, v + \frac{1}{2}]$ for all j in all past periods, and $w_t = 0$ otherwise, it is supportable if*

$\delta > \frac{6v-2}{9v-2}$. In such an equilibrium, the per-period profit earned by each manufacturer, π_{NR}^c , is $\frac{v}{2} + \frac{1}{6}$.

Proof. There are three possible deviations. The first is $w \in [0, v - 1]$, denoted D1. The second is $w \in (v - 1, v - \frac{1}{2})$, denoted D2. The third is $w \in [v - \frac{1}{2}, v)$, denoted D3.

Given these three types of deviation, there are three conditions that are necessary for collusion to be able to be sustained. These are

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D1} + \Pr(p_j < v|w^{D1}) \frac{\delta}{1-\delta} \pi^p + (1 - \Pr(p_j < v|w^{D1})) \frac{\delta}{1-\delta} \pi_{NR}^c, \quad (27)$$

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D2} + \Pr(p_j < v|w^{D2}) \frac{\delta}{1-\delta} \pi^p + (1 - \Pr(p_j < v|w^{D2})) \frac{\delta}{1-\delta} \pi_{NR}^c; \text{ and,} \quad (28)$$

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D3} + \Pr(p_j < v|w^{D3}) \frac{\delta}{1-\delta} \pi^p + (1 - \Pr(p_j < v|w^{D3})) \frac{\delta}{1-\delta} \pi_{NR}^c; \quad (29)$$

where π_{NR}^c denotes the per-period collusive profit. Following the discussion in the text $\pi_{NR}^c = \frac{v}{2} + \frac{1}{6}$. If the collusion breaks down, manufacturers earn the one-shot profit, which is denoted by $\pi^p = \frac{1}{6}$.

Note that it is immediate that deviating to $w > v$ cannot be optimal.

The proof proceeds by establishing properties of the optimal deviation of each type, first by characterizing the retailer's pricing when $w_j < w_k = v$ and then examining the manufacturer's problem in setting w_j for each type of deviation. Finally, a δ that is sufficient for none of the deviations to be attractive is derived.

First, consider the pricing of a retailer with a wholesale unit price of w_j facing a rival that prices in line with the cartel rule, such that $p_k = v + \frac{\xi_k}{2}$.⁶⁴ The retailer's pricing problem is

$$p_j^*(\xi_j) = \arg \max_p (p - w_j) \Pr(\xi_j - p > \xi_k - v - \frac{\xi_k}{2}) \quad (30)$$

Given that ξ_k is uniformly distributed on $[0, 1]$, this amounts to maximizing

$$(p - w) (2\xi_j + 2v - 2p) \quad (31)$$

in the region where $(2\xi_j + 2v - 2p) \in [0, 1]$ (outside of this region, either there is no chance of winning or the retailer wins for certain and is merely forgoing revenue by dropping the price). Ignoring this constraint results in the pricing rule $p^*(\xi_j) :=$

⁶⁴Recall that retailers are not part of any cartel agreement and compete in a static game.

$\frac{v+w_j}{2} + \frac{\xi_j}{2}$. If $2\xi_j + 2v - 2p^* > 1$, then the pricing rule is $p = v - \frac{1}{2} + \xi_j$.⁶⁵ Further, if $2\xi_j + 2v - 2p^*(\xi_j) > 1$, then the same inequality holds for all $\xi > \xi_j$. This expression results in the following retailer pricing rule when $w_j < w_k = v$: $p_j = \max \left\{ \frac{v+w_j}{2} + \frac{\xi_j}{2}, v - \frac{1}{2} + \xi_j \right\}$.

Next, we turn to considering the manufacturer's strategy. The first style of deviation (*D1*) involves choosing the optimal deviation in the interval $w \in [0, v - 1]$. In this interval, by inspecting the retailer's pricing rule derived above, the probability of detection is equal to 0.5, regardless of w . Hence, the optimal deviation can be derived by maximizing the combined retailer-manufacturer-pair deviation payoff (since the manufacturer can extract the retailer's profit through the fixed fee). It is clear that this action is exactly what the retailer will do when $w_j = 0$. Hence, the profit maximizing *D1* deviation arises when the retailer sets prices such that $p_j = v - \frac{1}{2} + \xi_j$. From the manufacturer's point of view, this retailer pricing policy will arise for any $w \in [0, v - 1]$ and can therefore be implemented in a variety of ways, which all result in the same deviation profit of v (the lump-sum component of the manufacturer's two-part tariff will be used to extract the remaining expected profits from the retailer). Thus, the *D1* deviation yields $\pi^{D1} = v$ and $\Pr(p_j < v | w^{D1}) = \frac{1}{2}$.

Therefore, we can solve for the minimum δ such that a *D1* deviation is not attractive. The *D1* condition requires that

$$\left(\frac{v}{2} + \frac{1}{6} \right) \frac{1}{1 - \delta} \geq v + \frac{1}{2} \frac{\delta}{1 - \delta} \frac{1}{6} + \frac{1}{2} \frac{\delta}{1 - \delta} \left(\frac{v}{2} + \frac{1}{6} \right) \quad (32)$$

which implies that when $\delta \geq \frac{6v-2}{9v-2}$, a *D1* deviation is not attractive.

The second style of deviation (*D2*) leaves the probability of detection unchanged at 0.5. To see this, note that for all $w \in (v - 1, v - \frac{1}{2})$, $p_j = \max \left\{ \frac{v+w_j}{2} + \frac{\xi_j}{2}, v - \frac{1}{2} + \xi_j \right\} = v$ when $\xi = \frac{1}{2}$ and that p is monotonic in ξ ; therefore, the deviation is detected for $\xi \leq \frac{1}{2}$ and undetected otherwise. Additionally, recall that when $w = 0$, the retailer chooses $p_j = v - \frac{1}{2} + \xi_j$. Hence, by setting $w \in (v - 1, v - \frac{1}{2})$, the manufacturer diminishes stage profits, with no compensating return in terms of adjusting the probability around detection (that is, leaving the continuation value unchanged). Hence, a *D2* deviation must always be dominated by a *D1* deviation.

The third style of deviation (*D3*), involves choosing the optimal deviation in the interval $w \in [v - \frac{1}{2}, v)$. In this interval, a change in w will affect the probability of detection; specifically, in this range, $\Pr(p_j < v | w) = \Pr(\frac{v+w}{2} + \frac{\xi_j}{2} < v) = \Pr(\xi_j <$

⁶⁵Recall, $v > 1$, making this event relevant. In this event, the pricing rule is derived by raising p until the chance of winning is equal to 1.

$v - w) = v - w$. Trivially, $\frac{d\Pr(p_j < v|w_j)}{dw_j} = -1$. Given that $w + 1 - v$ is the value of ξ_j such that $\frac{v+w}{2} + \frac{\xi_j}{2} = v - \frac{1}{2} + \xi_j$, the deviation profit can be written as

$$\pi^{D3} = \int_0^{w_j+1-v} \left(\frac{v+w_j}{2} + \frac{x}{2} \right) (v+x-w_j) dx + \int_{w_j+1-v}^1 \left(v - \frac{1}{2} + x \right) dx. \quad (33)$$

It will be useful to note that $\frac{\partial \pi^{D3}}{\partial w_j} = -w_j(w_j + 1 - v)$ and that Equation (33) also describes π^{D2} .

The optimal $D3$ deviation is the solution to

$$\max_{w_j} D3(w_j) \equiv \max_{w_j} \pi^{D3} + \Pr(p_j < v|w_j) \frac{\delta}{1-\delta} \pi^p + (\Pr(1-p_j < v|w_j)) \frac{\delta}{1-\delta} \pi_{NR}^c \quad (34)$$

which, taking the derivative with respect to w_j , yields

$$\frac{\partial D3(w_j)}{\partial w_j} = \frac{\partial \pi^{D3}}{\partial w_j} - \frac{\partial \Pr(p_j < v|w_j)}{\partial w_j} \frac{\delta}{1-\delta} (\pi_{NR}^c - \pi^p) \quad (35)$$

$$= -w_j(w_j + 1 - v) + \frac{\delta}{1-\delta} \frac{v}{2} \quad (36)$$

where the last equality follows by substituting for the two derivatives, as calculated above. Note that the second derivative is $\frac{\partial^2 D3(w_j)}{\partial w_j^2} = v - 1 - 2w_j < 0$ in the range $w \in [v - \frac{1}{2}, v)$.

Necessary conditions for the existence of an optimal $D3$ deviation (i.e., an interior solution in $[v - \frac{1}{2}, v)$) are that $\frac{\partial D3(w_j)}{\partial w_j} \Big|_{w_j=v-\frac{1}{2}} \geq 0$ and $\frac{\partial D3(w_j)}{\partial w_j} \Big|_{w_j=v} < 0$.

From Equation (36), $\frac{\partial D3(w_j)}{\partial w_j} \Big|_{w_j=v-\frac{1}{2}} \geq 0$ implies that $\delta \geq \frac{2v-1}{4v-1}$. Similarly, $\frac{\partial D3(w_j)}{\partial w_j} \Big|_{w_j=v} < 0$ implies $\delta < \frac{2}{3}$. Hence, for a $D3$ deviation to exist, it must be that $\delta \in [\frac{2v-1}{4v-1}, \frac{2}{3})$.

Since $v > 1$, $\delta > \frac{6v-2}{9v-2}$ implies that $\delta > \frac{2}{3}$ and so in the region $\delta > \frac{6v-2}{9v-2}$, there can be no such optimal deviation. ■

B Appendix B: Online appendix: Not for publication

B.1 Strategic consumers and advertising

In the text, we do not allow consumers to be strategic but simply suppose that high-search-cost consumers choose to purchase from the retailer with the lowest advertised price unless prices are the same, in which case, they randomize. In this appendix, we argue that such behavior is an equilibrium of a game that allows for strategic consumers who engage in reasoning consistent with a perfect Bayesian equilibrium.

Specifically, a retailer’s advertised retail price is denoted p_j^a . We assume that this price is flexible, and therefore, p_j^a is set contemporaneously with the retailer’s transaction price. We make the following assumption, which ensures that the advertised price is not simply cheap talk.

A1. Advertising cannot be fraudulent in the following sense: the advertised price can be no lower than the retail transaction price p_j , i.e., $p_j^a \geq p_j$.

In all other respects, we keep the game form the same as in Section 2.1. Given that some consumers can visit only a single retailer, beliefs as to p_j given p_j^a need to be considered, which results in the relevant equilibrium concept being (a refinement of) perfect Bayesian equilibrium.

In principle, there is considerable flexibility in assigning off-equilibrium beliefs and thereby inducing perverse equilibrium behavior. However, Assumption A1 specifies that a retailer cannot advertise a price below the price that it charges imposes real constraints. For example, suppose that a retailer charging a price of \$4 or charging a price of \$6 were expected to advertise the same price. The advertised price would have to be a price at or above \$6 to conform with this restriction. Suppose that this advertised price is \$20 and that these are the only two kinds of retailers expected to advertise at \$20 in equilibrium. Then, consumers expect some price above \$4 on average (if it were equally likely that retailers charge \$4 and \$6, consumers would anticipate \$5 on average).⁶⁶ Suppose that the retailer charging \$4 instead of advertising a price of \$20 advertises a price of \$4. Given the restriction that retailers cannot advertise at prices below what they charge, it must be that consumers anticipate paying a price at or below \$4, *even if this is an off-equilibrium advertised*

⁶⁶Recall that retailers are indistinguishable to consumers, so that if $p_j^a = p_k^a$ then $E(p_j|p_j^a) = E(p_k|p_k^a)$.

price; subsequently, this advertised price attracts more consumers, as their expected surplus must be strictly greater and would, therefore, be more profitable.⁶⁷

When there are no advertising restrictions, the simple logic in the example above has considerable bite. The underlying point is that for every advertised price arising in equilibrium, there is a unique retail transaction price. This situation creates advertised prices that are unambiguous and are *equivalent* to requiring that retailers will advertise the price that they actually charge.⁶⁸ The following result makes this argument precise for pure strategy equilibria.⁶⁹

Proposition 9 *Suppose that there are no advertising restrictions. Then, in any (pure strategy) perfect Bayesian equilibrium, each p^a maps to a unique p on the equilibrium path.*

Proof. Suppose, toward a contradiction, that there is an equilibrium in which this is not the case. Then, the equilibrium must involve (at least) two retailers, who charge different transaction prices, choosing the same advertised price a . The other possibility is that one retailer sets the same advertised price for two different transaction prices, but this is ruled out by virtue of each retailer being able to set only one price.

First, it must be that all retailers advertising at a attract some customers. If not, then they can always set their advertised price equal to a level that does attract customers and still make a profit since the manufacturer cannot discriminate at the wholesale level.

Now, consider all retailers who advertise at a . Of this set, let $p_{\min}(a)$ denote the price of the retailer with the lowest actual price. Note that since there is more than one retailer that advertises at a , it must be the case that consumers anticipate a non-zero probability of an actual price strictly greater than $p_{\min}(a)$ and therefore expect a surplus smaller than that generated by receiving $p_{\min}(a)$ with certainty.⁷⁰

⁶⁷If, in equilibrium, competing retailers advertise and charge a price of \$6 but the retailer of interest has an actual price of \$4, any advertised price in the interval $[4, 6)$ is considered equivalent to advertising a price of \$4.

⁶⁸Two mappings from advertised prices to transaction prices are equivalent if changing the set of prices from which advertised prices can be drawn makes no difference to the realized transaction prices or consummated transactions. That is, the language does not matter as long as the message is the same.

⁶⁹The result can easily be extended to mixed strategy equilibria, albeit with a much stronger notation.

⁷⁰Recall that from consumers' point of view, retailers are identical (aside from their advertised prices). Hence, consumers cannot condition their expectations of p_j on j .

Further, for all retailers that advertise at a , high-search-cost consumers randomize over these retailers.

The restriction that a retailer cannot charge a price higher than its advertised price implies that if the retailer charging $p_{\min}(a)$ advertised its actual price $p_{\min}(a)$, consumers could not put any probability on the retailer charging a higher price than $p_{\min}(a)$ (and might even put some probability on a lower price). That is, consumers anticipate an actual price that is equal to or lower than $p_{\min}(a)$. Expected surplus similarly strictly increases.

Taking together the observations in each of the three paragraphs above, we observe that setting the advertised price at $p_{\min}(a)$ would attract strictly greater demand when compared to choosing the advertised price a . This situation generates the required contradiction. ■

In particular, trivially, with no advertising restrictions, there is no loss in supposing that equilibria will have the intuitive property that consumer beliefs are monotone in the following sense.

Definition 1 *Consumers' beliefs are **monotone** if, $\forall p_j^a > \widehat{p}_j^a, E(p_j|p_j^a) > E(p_j|\widehat{p}_j^a)$.*

Definition 2 *A **monotone perfect Bayesian equilibrium** is a perfect Bayesian equilibrium in which consumers' beliefs are monotone.*

In the case of advertising restrictions, a retailer may not be able to set the advertised price equal to its actual price. Moreover, when the MAP price is high, the logic in the example above that underlies Proposition 9 can have little bite. Returning to the example above, suppose that there was a MAP price of \$10 and consumers expected that retailers advertising a price of \$20 were equally likely to actually charge \$4 or \$12, supported by the (off-equilibrium) beliefs that any retailer advertising anything between \$10 and \$20 actually charges \$9. Here, since the MAP restriction prevents the retailer charging \$4 from advertising at a sufficiently low price, this retailer does not gain from advertising at a price below \$20. A focus on monotone perfect Bayesian equilibrium rules out such perverse outcomes. Restricting equilibrium to requiring monotone beliefs leads retailers to advertise at as low a price as possible. The following result is immediate.

Proposition 10 *In a MAP regime, in all monotone perfect Bayesian equilibria, each retailer advertises its actual price unless the MAP restriction binds, and in this case, it advertises the MAP price; i.e., $p_j^a = \max\{p^{MAP}, p_j\}$.*

In the paper, we assume that firms advertise a price $p_j^a = p_j$, or, if a MAP restriction exists, $p_j^a = \max\{p^{MAP}, p_j\}$. In this appendix, in the absence of fraudulent advertising (as under Assumption A1), we have shown that imposing MAP restrictions can capture all the economic richness that a richer strategy space would deliver with only minimal (reasonable) restrictions on the set from which consumer beliefs are drawn. Further, these restrictions are required only when MAP policies bind.

B.2 Additional material for Section 2

The model in Section 2, while providing a clear intuition, is of course rather stylized in several respects. Here, we allow for greater generality in a few respects in order to explore the robustness of this intuition. We characterize the optimal MAP-based discriminatory price scheme by allowing for marginal cost differences between retailers and a richer correlation structure between consumer type (h or l) and search cost. We then show, via numerical simulation, that MAP-based discrimination is the optimal pricing policy over a range of parameter values. Further, the incentive constraints analogous to Equations (1) and (2) need not both bind in this richer setting.

In particular, while Section 2 supposes that only low-value consumers might be “shoppers” who visit both retailers at no cost, here, we also allow high-value consumers to be shoppers. Further, while Section 2 assumes that retailers are homogeneous, it may be more realistic to allow for some heterogeneity. There are, of course, many potential sources of heterogeneity; for example, some retailers with both a physical and an online presence may have different marginal costs than other retailers who are only online. Further, “non-shopper” consumers may find some retailers more salient than others,⁷¹ in contrast to our assumption that non-shoppers are equally divided between the two retailers. Of course, this salience of a well-known retailer is likely to be reinforced by a MAP restriction that prevents rivals from advertising a lower price.

Adding heterogeneity along all these dimensions requires further notation. We therefore focus on the case in which retailers differ only in their marginal costs, in line with our analysis in Section 3. Specifically, we suppose that one of the retailers is a low-cost retailer (whom incurs no cost beyond payments to the manufacturer; and the other retailer incurs an additional cost of c_H per unit ($c_H > 0$)). It is convenient to refer to the retailers as R_L and R_H , respectively. Recall that we suppose that a

⁷¹For example, De Los Santos et al. (2012) observe that in their data, “Amazon was visited in 74 percent of book transactions and that in only 17 percent did Amazon buyers browse any other bookstore.” (p.2961)

fraction λ of consumers have low valuation of the good. We suppose that a fraction of σ_L of low-value consumers are searchers and σ_H of high-value consumers are searchers. Otherwise, we assume the same timing and strategies as in Section 2.1.

The optimal MAP-based discriminatory price scheme is characterized as follows:

Proposition 11 *In the optimal MAP-based discriminatory price scheme (when feasible), the manufacturer sets*

$$T^*(w) = \min \left\{ \begin{array}{l} (h - w - c_H)(1 - \sigma_H)\frac{1-\lambda}{2}, \\ (l - w) \left(\sigma_L\lambda + \sigma_H(1 - \lambda) + (1 - \sigma_L)\frac{\lambda}{2} + (1 - \sigma_H)\frac{1-\lambda}{2} \right) \end{array} \right\}.$$

and

$$w^* = \arg \max_w w \left(1 - (1 - \sigma_L)\frac{\lambda}{2} \right) + 2T^*(w)$$

s.t.

$$\begin{aligned} \left(\sigma_L\lambda + \sigma_H(1 - \lambda) + (1 - \sigma_L)\frac{\lambda}{2} + (1 - \sigma_H)\frac{1 - \lambda}{2} \right) (l - w) &\geq (1 - \lambda)\frac{1 + \sigma_H}{2}(h - w) \\ \left(\sigma_L\lambda + \sigma_H(1 - \lambda) + (1 - \sigma_L)\frac{\lambda}{2} + (1 - \sigma_H)\frac{1 - \lambda}{2} \right) (l - w - c_H) &\leq (h - w - c_H)(1 - \sigma_H)\frac{1 - \lambda}{2} \\ w &\leq \min\{l, h - c_H\}. \end{aligned}$$

R_L sets $p = l$ and R_H sets $p = h$.

Proof. The proof extends the intuition developed in Proposition 1. It is immediate that the discriminatory MAP scheme should involve the retail prices set at l and h . Since R_H has higher marginal costs of production than R_L , more industry profit is generated when the R_L sets a price of l (and sells a greater quantity) and R_H sets a price of h . As argued below, the manufacturer can ensure that this is the equilibrium outcome.

The fixed fee is set to extract as much surplus as possible, subject to both the low-cost retailer and high-cost retailer being willing to take up the contract; that is, it is equal to the minimum of their (gross of fixed fee) profits.

The manufacturer chooses the input price w^* to maximize its profits.

The penultimate two inequalities at the end of the statement of the proposition correspond to the incentive constraints that guarantee that the two retailers do not wish to deviate from their equilibrium pricing strategies. The first ensures that the low-cost retailer prefers charging a price of l to charging a price of h , and the second ensures that the high-cost retailer prefers charging a price of h to undercutting the

low-cost retailer and attracting all searchers. The final inequality ensures that both retailers prefer to make positive sales. ■

Note that Example 1, in the main text suffices to demonstrate that there are parameter values in which the optimal MAP-based discriminatory scheme is feasible. We show that this finding is not a knife-edge result by considering a range of parameters through extending the example.

Figure B.1 below sets $h = 2$, $l = 1$ and $\lambda = 0.5$, as in Example 1. It sets $c_H = 0.2$ and varies σ_L and σ_H . In this example, the optimal non-MAP scheme is realized by setting $w = h$, resulting in a profit to the manufacturer of 1. The black region indicates where the optimal MAP-based discriminatory price scheme dominates the optimal non-MAP scheme for the manufacturer. In this region, it is never the case that both incentive constraints bind in the optimal MAP-based discriminatory price scheme.

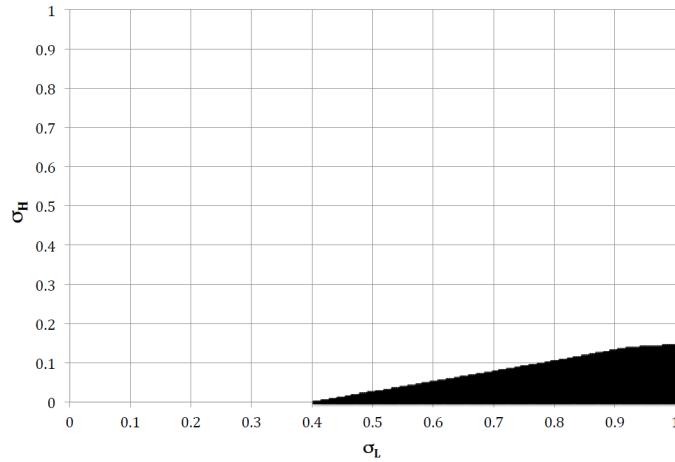


Figure B.1: Region with MAP-based discrimination being optimal, by (σ_H, σ_L)

B.3 Additional material for Section 3

Section 3.2: The manufacturer’s problem with RPM

As described in the text, RPM can only play a role in inducing service provision only if it binds for both low and high cost realizations; that is, if $P > w + c_H$.

In this case, where both high- and low-cost firms set a price equal to $P > w + c_H$, demand for a retailer irrespective of its costs is given by $\frac{q(P)}{2}$. Service for low cost and high cost realization can be easily implicitly characterized through the first-order conditions described in the main text in (12) and (11). Finally, we can turn to the problem of the manufacturer who sets T equal to a retailer's expected profits so

$$T = \alpha \left[(P - w - c) \frac{q(P)}{2} (1 - (1 - s_H)(1 - \alpha s_H - (1 - \alpha)s_L)) - I(s_H) \right] \quad (37)$$

$$+ (1 - \alpha) \left[(P - w) \frac{q(P)}{2} (1 - (1 - s_L)(1 - \alpha s_H - (1 - \alpha)s_L)) - I(s_L) \right],$$

and chooses w and $P > w + c_H$ to maximize

$$2T + wq(P) \left[\alpha^2(1 - (1 - s_H)^2) + 2\alpha(1 - \alpha)(1 - (1 - s_L)(1 - s_H)) + (1 - \alpha)^2(1 - (1 - s_L)^2) \right]. \quad (38)$$

Section 3.3: The manufacturer's problem with MAP

The manufacturer sets T as follows

$$T = \alpha \left[(P - w - c) \frac{q(P)}{2} (1 - (1 - s_H)(1 - \alpha s_H - (1 - \alpha)s_L)) - I(s_H) \right] \quad (39)$$

$$+ (1 - \alpha) \left[(p^m(w) - w) \frac{q(p^m(w))}{2} (1 - (1 - s_L)(1 - \alpha s_H - (1 - \alpha)s_L)) - I(s_L) \right],$$

and chooses w and $P > w + c_H$ to maximize expected profits: the sum of the fixed fee T and expected revenue from the per-unit fee:

$$2T + w \left[\begin{array}{l} \alpha^2(1 - (1 - s_H)^2)q(P) + (1 - \alpha)^2(1 - (1 - s_L)^2)q(p^m(w)) \\ + \alpha(1 - \alpha)(1 - (1 - s_L)(1 - s_H))(q(P) + q(p^m(w))) \end{array} \right]. \quad (40)$$

B.4 Additional material for Section 4

While the main text considers a natural collusive scheme: the cartel mimics the same behavior that a monopolist supply chain selling a single good would set. However, colluding manufacturers could potentially achieve higher profits. This is because, with two retailers through which the manufacturers could sell, the industry is less likely to lose a sale by raising the wholesale price above v than it would in case there is only one retailer. With two retailers a consumer's maximal valuation is

$v + \max\{\xi_1, \xi_2\}$ rather than $v + \xi_1$ with only a single retailer. The optimal collusive scheme would take account of this fact.

We begin characterizing the solution to this form of cartel behavior by considering what would maximize cartel profits.

Proposition 12 *Suppose that the two manufacturers merge but continue to sell through the two retailers. The merged manufacturer optimally sets*

$$w_{NR}^* = \frac{3v + \sqrt{v^2 + 4}}{4}$$

and

$$T_{NR}^* = \frac{4v(v^2 + 3) - 4(1 + v^2)\sqrt{v^2 + 4}}{384} + \frac{1}{6}$$

the merged manufacturer earns $\frac{16+36v+2v^2\sqrt{v^2+4}+8\sqrt{v^2+4}-2v^3}{48}$. In this case observed retail prices will be in the interval $[w_{NR}^*, w_{NR}^* + \frac{1}{2}]$.

Proof. First suppose that the manufacturer sets $w < v$ then following Proposition 5, the retailers set $p_j = w + \frac{\xi_j}{2}$.

Since $v > w$ it is immediate that the manufacturer sells with probability 1 and would benefit from raising w . Thus the manufacturer optimizes by setting $w \geq v$.

When it does so then its profits from the per-unit charge can be calculated by considering the probability of sale as a function of w . It is easier to consider the probability of no sale at either retailer, which requires that $\xi_1 < p_1 - v$ and $\xi_2 < p_2 - v$ or equivalently $\xi_i < \frac{w+v}{2} + \frac{\xi_i}{2} - v$ iff $\xi_i < w - v$ for $i = 1, 2$. It follows that the Probability that there is no sale is that $(w - v)^2$ and the probability that there is a sale is $1 - (w - v)^2$.

Next consider each retailer's profit (excluding any fixed fee) when the wholesale price is w . This is $E\left[\frac{\xi_j}{2} \mid \xi_j > \max\{\xi_{-j}, w - v\}\right]$ since retailer j makes a sale only when the consumer prefers this retailer to the other (which requires $\xi_j > \xi_{-j}$) and to the outside option (which requires that $w - v$); further, when retailer j makes a

sale its profit is $\frac{\xi_j}{2}$.

$$\begin{aligned}
 E \left[\frac{\xi_j}{2} \mid \xi_j > \max \{ \xi_{-j}, w - v \} \right] &= \frac{1}{2} E \left[\xi_j \mid \xi_j > \max \{ \xi_{-j}, w - v \} \right] \\
 &= \frac{1}{2} \int_0^1 \int_{\max\{w-v, z\}}^1 y dy dz \\
 &= \frac{1}{2} \int_{w-v}^1 \int_z^1 y dy dz + \frac{1}{2} \int_0^{w-v} \int_{w-v}^1 y dy dz \\
 &= \frac{v^3 - w^3 + 1}{6} - \frac{v^2 w + v w^2}{2}.
 \end{aligned}$$

This can be extracted as a fixed fee and the manufacturer earns a per unit profit of w . Thus in the range $w \geq v$ (which we assume) and $w \leq v + 1$ (which we will later verify must be the case) the manufacturer's overall profits are given by:

$$\Pi = w(1 - (w - v)^2) + 2 \left(\frac{v^3 - w^3 + 1}{6} - \frac{v^2 w + v w^2}{2} \right).$$

The SOC is given by $\frac{d^2}{dw^2} \Pi = 2v - 8w < 0$ where the inequality follows on noting that $w > v$.

The FOC is given by $\frac{d}{dw} \Pi = 0$ which has one solution with $w > v$ (and it is immediate that this solution involves $v + 1 > w$) namely $w = \frac{3}{4}v + \frac{1}{4}\sqrt{v^2 + 4}$.

Manufacturer profits and fixed fee can be calculated through simple substitution to complete the result. ■

Proposition 12 establishes the maximal cartel profits and the retailer contracts that implement them. We now turn to consider the conditions under which the cartel can sustain them.

Proposition 13 *A collusive equilibrium in which manufacturers set $w_t = w_{NR}^* = \frac{3v + \sqrt{v^2 + 4}}{4}$ if $p_j^a \in [w_{NR}^*, w_{NR}^* + \frac{1}{2}]$. for all j in all past periods, and $w_t = 0$ otherwise, is supportable if*

$$\delta > \underline{\delta}_{NR} = \max \left\{ \frac{12(w_{NR}^* - \pi_{NR}^c)}{6(w_{NR}^* - \pi_{NR}^c) + 6w_{NR}^* - 1}, \frac{w_{NR}^*}{\pi_{NR}^c - \pi^p + w_{NR}^*} \right\}$$

where π_{NR}^c is the per-period profit earned by each manufacturer, in such an equilibrium and is given by $\pi_{NR}^c = \frac{16 + 36v + 2v^2\sqrt{v^2 + 4} + 8\sqrt{v^2 + 4} - 2v^3}{96}$.

Proof. There are three possible deviations. The first is $w \in [0, w_{NR}^* - 1]$, denoted D1, note that this also implies that $w < v$. The second is $w \in (w_{NR}^* - 1, w_{NR}^* - \frac{1}{2})$, denoted D2 and again here $w < v$. The third is $w \in [w_{NR}^* - \frac{1}{2}, w_{NR}^*]$, denoted D3.

Given these three types of deviation, there are three conditions that are necessary for collusion to be able to be sustained. These are

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D1} + \Pr(p_j < w_{NR}^* | w^{D1}) \frac{\delta}{1-\delta} \pi^p + (1 - \Pr(p_j < w_{NR}^* | w^{D1})) \frac{\delta}{1-\delta} \pi_{NR}^c \quad (41)$$

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D2} + \Pr(p_j < w_{NR}^* | w^{D2}) \frac{\delta}{1-\delta} \pi^p + (1 - \Pr(p_j < w_{NR}^* | w^{D2})) \frac{\delta}{1-\delta} \pi_{NR}^c \quad (42)$$

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D3} + \Pr(p_j < w_{NR}^* | w^{D3}) \frac{\delta}{1-\delta} \pi^p + (1 - \Pr(p_j < w_{NR}^* | w^{D3})) \frac{\delta}{1-\delta} \pi_{NR}^c \quad (43)$$

where π_{NR}^c denotes the per-period collusive profit, following Proposition 12 $\pi_{NR}^c = \frac{16+36v+2v^2\sqrt{v^2+4}+8\sqrt{v^2+4}-2v^3}{96}$; if the collusion breaks down (which requires that deviation is observed—that is, the observed retail price is below w_{NR}^*), manufacturers earn the one-shot profit, which is denoted by $\pi^p = \frac{1}{6}$.

Note that it is immediate that deviating to $w > w_{NR}^*$ cannot be optimal.

The proof proceeds by establishing properties of the optimal deviation of each type, first by characterizing the retailer's pricing when $w_j < w_k = w_{NR}^*$ and then examining the manufacturer's problem in setting w_j for each type of deviation. Finally, a δ that is sufficient for none of the deviations to be attractive is derived.

Retailer pricing

First, consider the pricing of a retailer with a wholesale unit price of w_j facing a rival that prices in line with the cartel rule, such that $p_k = w_{NR}^* + \frac{\xi_k}{2}$.⁷² The retailer's pricing problem is

$$p_j^*(\xi_j) = \arg \max_p (p - w_j) \Pr \left(v + \xi_j - p > \max \left\{ v + \xi_k - w_{NR}^* - \frac{\xi_k}{2}, 0 \right\} \right) \quad (44)$$

Note that $v + \xi_k - w_{NR}^* - \frac{\xi_k}{2} > 0$ iff $\xi_k > 2(w_{NR}^* - v) = 2(\frac{3v+\sqrt{v^2+4}}{4} - v) =$

⁷²Recall that retailers are not part of any cartel agreement and compete in a static game.

$\frac{1}{2}\sqrt{v^2+4} - \frac{1}{2}v$ so we can write this as

$$\begin{aligned}
 p_j^*(\xi_j) &= \arg \max_p (p - w_j) \Pr \left(v + \xi_j - p > v + \xi_k - w_{NR}^* - \frac{\xi_k}{2} \mid \xi_k > \frac{1}{2}\sqrt{v^2+4} - \frac{1}{2}v \right) \\
 &\quad + (p - w_j) \Pr (v + \xi_j - p > 0) \left(\frac{\sqrt{v^2+4}}{2} - \frac{v}{2} \right) \\
 &= \arg \max_p (p - w_j) \Pr \left(2(w_{NR}^* + \xi_j - p) > \xi_k > \frac{1}{2}\sqrt{v^2+4} - \frac{1}{2}v \right) \\
 &\quad + (p - w_j) \Pr (v + \xi_j - p > 0) \left(\frac{\sqrt{v^2+4}}{2} - \frac{v}{2} \right) \\
 &= \arg \max_p (p - w_j) (\max\{1, 2(w_{NR}^* + \xi_j - p)\} - \frac{1}{2}\sqrt{v^2+4} - \frac{1}{2}v) \\
 &\quad + (p - w_j) \left(\frac{\sqrt{v^2+4}}{2} - \frac{v}{2} \right) 1_{v+\xi_j-p>0}
 \end{aligned}$$

Note that at $\max\{1, 2(w_{NR}^* + \xi_j - p)\} = 1$ then the retailer would choose to set $p = w_{NR}^* + \xi_j - \frac{1}{2}$ and so $v + \xi_j - p = v + \frac{1}{2} - w_{NR}^* = v - \frac{3v+\sqrt{v^2+4}}{4} + \frac{1}{2} > 0$. Trivially then at lower p where $\max\{1, 2(w_{NR}^* + \xi_j - p)\} \neq 1$ then $1_{v+\xi_j-p>0} = 1$. It follows that it is without loss of generality to write

$$p_j^*(\xi_j) = \arg \max_p (p - w_j) \Pr \left(v + \xi_j - p > v + \xi_k - w_{NR}^* - \frac{\xi_k}{2} \right) \quad (45)$$

Given that ξ_k is uniformly distributed on $[0, 1]$, this amounts to maximizing

$$(p - w) (2\xi_j + 2w_{NR}^* - 2p) \quad (46)$$

in the region where $(2\xi_j + 2w_{NR}^* - 2p) \in [0, 1]$ (outside of this region, either there is no chance of winning or the retailer wins for certain and is merely forgoing revenue by dropping the price). Ignoring this constraint results in the pricing rule $p^*(\xi_j) := \frac{w_{NR}^* + w_j}{2} + \frac{\xi_j}{2}$. If $2\xi_j + 2w_{NR}^* - 2p^* > 1$, then the pricing rule is $p = w_{NR}^* - \frac{1}{2} + \xi_j$. Further, if $2\xi_j + 2w_{NR}^* - 2p^*(\xi_j) > 1$, then the same inequality holds for all $\xi > \xi_j$. This expression results in the following retailer pricing rule when $w_j < w_{NR}^*$: $p_j = \max \left\{ \frac{w_{NR}^* + w_j}{2} + \frac{\xi_j}{2}, w_{NR}^* - \frac{1}{2} + \xi_j \right\}$.

Case D1

Next, we turn to considering the manufacturer's strategy. The first style of deviation (*D1*) involves choosing the optimal deviation in the interval $w \in [0, w_{NR}^* - 1]$.

In this interval, by inspecting the retailer's pricing rule derived above, the probability of detection is equal to 0.5, regardless of w . Hence, the optimal deviation can be derived by maximizing the combined retailer-manufacturer-pair deviation payoff (since the manufacturer can extract the retailer's profit through the fixed fee). It is clear that this action is exactly what the retailer will do when $w_j = 0$. Hence, the profit maximizing $D1$ deviation arises when the retailer sets prices such that $p_j = w_{NR}^* - \frac{1}{2} + \xi_j$. From the manufacturer's point of view, this retailer pricing policy will arise for any $w \in [0, w_{NR}^* - 1]$ and can therefore be implemented in a variety of ways, which all result in the same deviation profit of w_{NR}^* (the lump-sum component of the manufacturer's two-part tariff will be used to extract the remaining expected profits from the retailer). Thus, the $D1$ deviation yields $\pi^{D1} = w_{NR}^*$ and $\Pr(p_j < v | w^{D1}) = \frac{1}{2}$.

Therefore, we can solve for the minimum δ such that a $D1$ deviation is not attractive. The $D1$ condition requires that

$$\pi_{NR}^c \frac{1}{1-\delta} \geq w_{NR}^* + \frac{1}{2} \frac{\delta}{1-\delta} \frac{1}{6} + \frac{1}{2} \frac{\delta}{1-\delta} \pi_{NR}^c \quad (47)$$

which implies that when $\delta \geq \frac{12(w_{NR}^* - \pi_{NR}^c)}{6(w_{NR}^* - \pi_{NR}^c) + 6w_{NR}^* - 1} = \frac{1}{6} \frac{18v - v^2\sqrt{v^2+4} + 8\sqrt{v^2+4} + v^3 - 8}{54v - v^2\sqrt{v^2+4} + 20\sqrt{v^2+4} + v^3 - 16}$, a $D1$ deviation is not attractive.

Case D2

The second style of deviation ($D2$) leaves the probability of detection unchanged at 0.5. To see this, note that for all $w \in (w_{NR}^* - 1, w_{NR}^* - \frac{1}{2})$, $p_j = \max \left\{ \frac{w_{NR}^* + w_j}{2} + \frac{\xi_j}{2}, w_{NR}^* - \frac{1}{2} + \xi_j \right\} = w_{NR}^*$ when $\xi = \frac{1}{2}$ and that p is monotonic in ξ ; therefore, the deviation is detected for $\xi \leq \frac{1}{2}$ and undetected otherwise. Additionally, recall that when $w = 0$, the retailer chooses $p_j = w_{NR}^* - \frac{1}{2} + \xi_j$. Hence, by setting $w \in (w_{NR}^* - 1, w_{NR}^* - \frac{1}{2})$, the manufacturer diminishes stage profits, with no compensating return in terms of adjusting the probability around detection (that is, leaving the continuation value unchanged). Hence, a $D2$ deviation must always be dominated by a $D1$ deviation.

Case D3

The third style of deviation ($D3$), involves choosing the optimal deviation in the interval $w \in [w_{NR}^* - \frac{1}{2}, w_{NR}^*]$. In this interval, a change in w will affect the probability of detection; note that $\frac{w_{NR}^* + w_j}{2} + \frac{\xi_j}{2} > w_{NR}^* - \frac{1}{2} + \xi_j$ iff $w_j - w_{NR}^* + 1 > \xi_j$

and so

$$\begin{aligned}
 \Pr(p_j < w_{NC}^*|w) &= \Pr(w_{NR}^* > \frac{w_{NR}^* + w}{2} + \frac{\xi_j}{2} \text{ and } w - w_{NR}^* + 1 > \xi_j) \\
 &\quad + \Pr(w_{NR}^* > w_{NR}^* - \frac{1}{2} + \xi_j \text{ and } w - w_{NR}^* + 1 < \xi_j) \\
 &= \Pr(w_{NR}^* - w > \xi_j, w - w_{NR}^* + 1 > \xi_j) + \Pr(\frac{1}{2} > \xi_j > 1 + w - w_{NR}^*) \\
 &= \Pr(w_{NR}^* - w > \xi_j, w - w_{NR}^* + 1 > \xi_j) \\
 &= w_{NR}^* - w
 \end{aligned}$$

Consequently, $\frac{d\Pr(p_j < w_{NC}^*|w)}{dw} = -1$.

Given that $w + 1 - w_{NR}^*$ is the value of ξ_j such that $\frac{w_{NR}^* + w}{2} + \frac{\xi_j}{2} = w_{NR}^* - \frac{1}{2} + \xi_j$, the deviation profit can be written as

$$\pi^{D3} = \int_0^{w_j+1-w_{NR}^*} \left(\frac{w_{NR}^* + w_j}{2} + \frac{x}{2} \right) (w_{NR}^* + x - w_j) dx + \int_{w_j+1-w_{NR}^*}^1 \left(w_{NR}^* - \frac{1}{2} + x \right) dx. \quad (48)$$

It will be useful to note that $\frac{\partial \pi^{D3}}{\partial w_j} = -w_j(w_j + 1 - w_{NR}^*)$ and that Equation (48) also describes π^{D2} .

The optimal $D3$ deviation is the solution to

$$\max_{w_j} D3(w_j) \equiv \max_{w_j} \pi^{D3} + \Pr(p_j < w_{NR}^*|w_j) \frac{\delta}{1-\delta} \pi^p + (1 - \Pr(p_j < w_{NR}^*|w_j)) \frac{\delta}{1-\delta} \pi_{NR}^c \quad (49)$$

which, taking the derivative with respect to w_j , yields

$$\frac{\partial D3(w_j)}{\partial w_j} = \frac{\partial \pi^{D3}}{\partial w_j} - \frac{\partial \Pr(p_j < w_{NR}^*|w_j)}{\partial w_j} \frac{\delta}{1-\delta} (\pi_{NR}^c - \pi^p) \quad (50)$$

$$= -w_j(w_j + 1 - w_{NR}^*) + \frac{\delta}{1-\delta} (\pi_{NR}^c - \pi^p), \quad (51)$$

where the last equality follows by substituting for the two derivatives, as calculated above. Note that the second derivative is $\frac{\partial^2 D3(w_j)}{\partial w_j^2} = w_{NR}^* - 1 - 2w_j < 0$ in the range $w \in [w_{NR}^* - \frac{1}{2}, w_{NR}^*]$.

Necessary conditions for the existence of an optimal $D3$ deviation (i.e., an interior solution in $[w_{NR}^* - \frac{1}{2}, w_{NR}^*]$) are that $\frac{\partial D3(w_j)}{\partial w_j} \Big|_{w_j=w_{NR}^* - \frac{1}{2}} \geq 0$ and $\frac{\partial D3(w_j)}{\partial w_j} \Big|_{w_j=w_{NR}^*} < 0$.

From Equation (51), $\frac{\partial D3(w_j)}{\partial w_j} \Big|_{w_j=w_{NR}^*-\frac{1}{2}} \geq 0$ implies that $\delta \geq \frac{2w_{NR}^*-1}{2w_{NR}^*-1+4(\pi_{NR}^c-\pi^p)}$. Similarly, $\frac{\partial D3(w_j)}{\partial w_j} \Big|_{w_j=w_{NR}^*} < 0$ implies $\delta < \frac{w_{NR}^*}{\pi_{NR}^c-\pi^p+w_{NR}^*}$. Hence, for a $D3$ deviation to exist, it must be that $\delta \in \left[\frac{2w_{NR}^*-1}{2w_{NR}^*-1+4(\pi_{NR}^c-\pi^p)}, \frac{w_{NR}^*}{\pi_{NR}^c-\pi^p+w_{NR}^*} \right)$ and so no such deviation can exist if $\delta > \frac{w_{NR}^*}{\pi_{NR}^c-\pi^p+w_{NR}^*}$. This observation, together with the observation that $\delta \geq \frac{12(w_{NR}^*-\pi_{NR}^c)}{6(w_{NR}^*-\pi_{NR}^c)+6w_{NR}^*-1}$ is required to rule out a deviation of type $D1$, establishes the result. ■

B.4.1 RPM

Next we turn to consider the optimal collusive scheme under RPM (rather than the one that replicates a monopoly supply chain as in the main text).

If the manufacturers set $w = p$ then there is a sale unless both $p > v + \xi_1$ and $p > v + \xi_2$; this suggests that total cartel profits (when $p > v$ and under an interior solution) are given by $p(1 - (p - v)^2)$.

It is easy to verify that this is maximized at $p = \frac{2v+\sqrt{v^2+3}}{3}$ and that $\frac{2v+\sqrt{v^2+3}}{3} \in (v, v + 1)$. Moreover this implies that

$$\pi_{RPM}^c = \frac{1}{2}p(1 - (p - v)^2) = \frac{1}{2} \frac{2v + \sqrt{v^2 + 3}}{3} \frac{3 + v - \sqrt{v^2 + 3}}{3} \frac{3 - v + \sqrt{v^2 + 3}}{3}.$$

This allows us to consider when the optimal RPM collusive scheme is sustainable. Specifically, we can characterize $\underline{\delta}^{RPM}$ through the following:

$$\frac{\pi_{RPM}^c}{1 - \delta} \geq \pi_{RPM}^D + \frac{\delta}{1 - \delta} \pi^p,$$

where $\pi_{RPM}^D = 2\pi_{RPM}^c$. This yields $\delta \geq \underline{\delta}^{RPM} = \frac{6\pi_{RPM}^c}{12\pi_{RPM}^c-1}$ and, thereby, establishes the following.

Proposition 14 *A collusive equilibrium in which manufacturers set $w_t = p_t^{RPM} = \frac{2v+\sqrt{v^2+3}}{3}$ as long as $p_{j,t}^a = \frac{2v+\sqrt{v^2+3}}{3}$ for all j in all past periods, and $w_t = 0$ otherwise, is supportable if $\delta \geq \underline{\delta}^{RPM} = \frac{6\pi_{RPM}^c}{12\pi_{RPM}^c-1}$ where π_{RPM}^c is the per-period profit earned by each manufacturer in such an equilibrium and is given by, $\pi_{RPM}^c = \frac{1}{2} \frac{2v+\sqrt{v^2+3}}{3} \frac{3+v-\sqrt{v^2+3}}{3} \frac{3-v+\sqrt{v^2+3}}{3}$.*

B.4.2 MAP

Since MAP operates, in effect, as a market division scheme, the optimal collusive MAP contract involves each of the manufacturers operating as a monopolist. Thus, Proposition 8 implements the collusive MAP scheme and determines when it is sustainable.

B.4.3 Restraints facilitating collusion

First note that in comparing the sustainable of collusion through MAP rather than the RPM, trivially $\underline{\delta}^{RPM} = \frac{6\pi_{RPM}^c}{12\pi_{RPM}^c - 1} > \frac{1}{2}$ so the MAP scheme is more sustainable.

As for profitability of the two schemes, it can be shown that the cartel profits under MAP are necessarily higher than the cartel profits under RPM; equivalently $\frac{v}{2} + \frac{1}{3} - \left(\frac{1}{2} \frac{2v + \sqrt{v^2+3}}{3} \frac{3+v - \sqrt{v^2+3}}{3} \frac{3-v + \sqrt{v^2+3}}{3}\right) > 0$ in the range $v > 1$. To see this first note that the left hand side of this inequality is positive at $v = 0$ and that its derivative with respect to v is equal to $\frac{(2v^2+3)(\sqrt{v^2+3}-v)-3v}{18\sqrt{v^2+3}} > 0$ where this inequality follows on noting that the numerators is necessarily positive.

It remains to consider the no constraint case. We can compare $\underline{\delta}^{RPM}$ and $\underline{\delta}_{NR}$. Recall that

$$\begin{aligned} \underline{\delta}^{RPM} &= \frac{6\pi_{RPM}^c}{12\pi_{RPM}^c - 1} = \frac{6\left(\frac{1}{2} \frac{2v + \sqrt{v^2+3}}{3} \frac{3+v - \sqrt{v^2+3}}{3} \frac{3-v + \sqrt{v^2+3}}{3}\right)}{12\left(\frac{1}{2} \frac{2v + \sqrt{v^2+3}}{3} \frac{3+v - \sqrt{v^2+3}}{3} \frac{3-v + \sqrt{v^2+3}}{3}\right) - 1} \\ \underline{\delta}_{NR} &= \max \left\{ \frac{12(w_{NR}^* - \pi_{NR}^c)}{6(w_{NR}^* - \pi_{NR}^c) + 6w_{NR}^* - 1}, \frac{w_{NR}^*}{\pi_{NR}^c - \pi^p + w_{NR}^*} \right\} \\ &= \max \left\{ \frac{12\left(\frac{3v + \sqrt{v^2+4}}{4} - \frac{16 + 36v + 2v^2\sqrt{v^2+4} + 8\sqrt{v^2+4} - 2v^3}{96}\right)}{6\left(\frac{3v + \sqrt{v^2+4}}{4} - \frac{16 + 36v + 2v^2\sqrt{v^2+4} + 8\sqrt{v^2+4} - 2v^3}{96}\right) + 6\frac{3v + \sqrt{v^2+4}}{4} - 1}, \right. \\ &\quad \left. \frac{\frac{3v + \sqrt{v^2+4}}{4}}{\frac{16 + 36v + 2v^2\sqrt{v^2+4} + 8\sqrt{v^2+4} - 2v^3}{96} - \frac{1}{6} + \frac{3v + \sqrt{v^2+4}}{4}} \right\} \end{aligned}$$

These are functions of v and can be simply plotted, we plot the first (RPM) in black and the latter (NR) in red in the graph below.

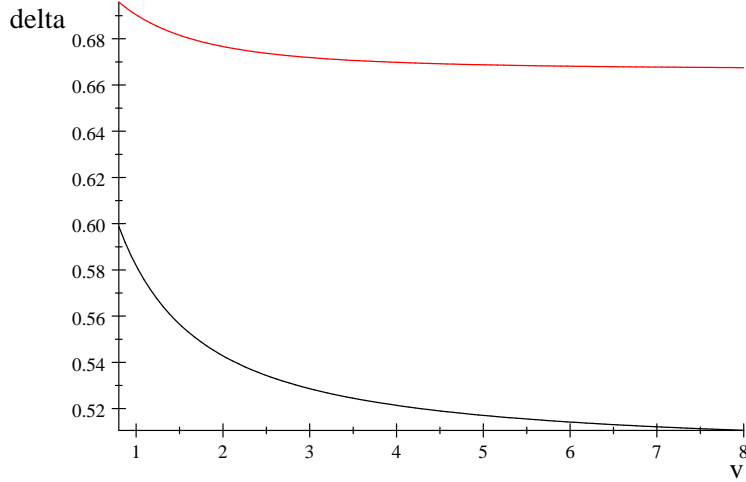


Figure B.2: $\underline{\delta}^{RPM}$ and $\underline{\delta}_{NR}$ against v

In the relevant range ($v > 1$) it follows that $\underline{\delta}_{NR} > \underline{\delta}^{RPM}$ and so it is easier to sustain collusion under RPM than under no constraint. Since as argued above $\underline{\delta}^{RPM} > \frac{1}{2} = \underline{\delta}^{MAP}$, it is also easier to sustain collusion under MAP than under no constraint.

Turning to compare profits, in contrast to the main text it is not immediate that $\pi_{MAP}^c > \pi_{NR}^c$ since under MAP there may be sales to the retailer with the lower-valued shock; however, we can write $\pi_{MAP}^c - \pi_{NR}^c = \frac{v}{2} + \frac{1}{3} - \left(\frac{4v(v^2+3) - 4(1+v^2)\sqrt{v^2+4}}{384} + \frac{1}{6} \right) = \frac{15}{32}v + \frac{1}{96}v^2\sqrt{v^2+4} + \frac{1}{96}\sqrt{v^2+4} - \frac{1}{96}v^3 + \frac{1}{6}$.

Since $v^2\sqrt{v^2+4} > v^3$ for $v > 0$, it follows that this is positive.

Thus MAP is both more profitable and more sustainable than either RPM or no constraints.