Cost and Production 2¹

Fall 2017

 $^{^{1}}$ These lectures notes are adapted from earlier lecture notes originally used by Jan de Loecker. All errors are mine. $\langle \Box \rangle \land \langle \Xi \land \langle \Xi \rangle \land \langle \Xi \land \langle \Xi \rangle \land \langle \Xi \land \langle \Xi \land \langle \Xi \rangle \land \langle \Xi \land \land \langle \Xi \land$

Overview Lectures:

- 1. Introduction to production/cost analysis
- 2. Estimating Production Functions I (Olley Pakes)
- 3. Estimating Production Functions II (OP Extensions)

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4. Examples of applications

Introduction

- The focus is on the estimation of production and cost functions as tools to analyze firm performance. In this respect it is an important part of IO, but got divided into subfield and into other fields.
- All of the models will rely on symmetric information, and we will discuss the primitive on the supply side: productivity.

Estimation of OP model

They start out with a homogeneous good producer with a Cobb-Douglas (value added) production function (in logs)²

$$y_{it} = \beta_0 + \beta_l I_{it} + \beta_k k_{it} + \omega_{it} + \eta_{it}$$
(1)

- Here η is defined to be part of the error that is not known when input decision are made, whereas ω is crucially observed by the firm (not by the econometrician).
- Bias in coefficient using OLS: simultaneity especially in labor (no material here!).
- Selection bias due to E(ω_t|ω_t > <u>ω</u>(k_t), ω_{t-1}, χ_t = 1). Conditional on ω_{t-1} this function is decreasing in k_{t-1}. I.e. the value function is increasing in both, so if k is higher we will continue with a lower ω. This would lead to a negative capital coefficient bias.

Estimating labor: stage 1

OP work on condition that we can invert investment policy function that is solution to the dynamic problem outlined before. There is actually a bit of work to proof this (see Pakes, 1994).

$$i_t = i_t(k_t, \omega_t, a_t) \tag{2}$$

$$\omega_t = h_t(i_t, k_t, a_t) \tag{3}$$

where we now have a model that proxies (controls for) productivity and note the time subscript, i.e. market structure. Now we can substitute this into the production function and collect terms on capital, investment into non parametric function φ_t(.).

$$y_{it} = \beta_I I_{it} + \phi_t(i_{it}, k_{it}, a_{it}) + \eta_{it}$$

$$\tag{4}$$

where $\phi_t(i_t, k_t, a_t) \equiv \beta_0 + \beta_k k_{it} + h_t(i_{it}, k_{it}, a_{it})$

We simply estimate this as a partial linear model (Robinson, 1988) and get estimate for β_l and φ_{it}. Practically, just use polynomial expansion in state variables or use LWLS.

Selection control: stage 2

We now want explicitly control for non random exit of firms, i.e. firms with lower productivity (conditional on capital stock) have higher probability to exit the market. Or

$$Pr(\chi_{t+1} = 1 | \underline{\omega}_{t+1}(.), J_t) = Pr(\omega_{t+1} > \underline{\omega}_{t+1}(.) | .)$$
(5)

$$= 1 - F(\underline{\omega}_{t+1}(.)|\omega_t) \equiv P_t(i_t, a_t, k_t) = P_{it}$$
(6)

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This is again a non parametric function of the exit probability in the state variables (generated by the model) and we can estimate this using kernel estimation techniques or probit in polynomial of state variables. This will generate estimate for P_{it}.

Estimating fixed input coefficients β_k : stage 3

▶ We now have the following estimates (b_l, φ̂_{it}, P̂_{it}) and let us consider one period ahead, where we realize that this is only observed for surviving firms (Exit rule!). This implies that we know that

$$E(y_{t+1} - \beta_I I_{t+1} | a_{t+1}, k_{t+1}, \chi_{t+1} = 1) = \beta_0 + \beta_k k_{t+1} + E(\omega_{t+1} | \omega_t, \chi_{t+1} = 1)$$
(7)

• where now we will use the non parametric evolution of the productivity process, i.e. an AR(1) for ω would be a special case. We have that

$$E(\omega_{it+1}|\omega_t,\chi_{t+1}=1) =$$
(8)

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$$\int_{\underline{\omega}_{t+1}} \omega_{t+1} \frac{F(d\omega_{t+1}|\omega_t)}{\int_{\omega_{t+1}} F(d\omega_{t+1}|\omega_t)}$$
(9)

$$\equiv g(\underline{\omega}_{t+1}, \omega_t) = g(P_{it}, \phi_{it} - \beta_k k_{it})$$
(10)

3rd stage

Now using the Markovian assumption and its implication for expected productivity into the production function one period ahead, subtracting the know variation in labor (or whatever the input is you are estimating in the first stage).

$$y_{t+1} - b_{t}I_{t+1} = c + \beta_{k}k_{t+1} + g(\widehat{P}_{it}, \widehat{\phi}_{it} - \beta_{k}k_{it} - \beta_{0}) + \xi_{it+1} + \eta_{it+1}$$
(11)

- where crucially ξ is the innovation in the Markov process of productivity, this is exactly what forces us to use a first stage to estimate labor since ω_{it} = g(ω_{it-1}) + ξ_{it} and induces correlation between labor and productivity, when labor is optimally chosen each period.
- Labor can respond to ξ and therefore / is a function of it, that is why we need to subtract it out.
- Questions:
 - 1. Think about what if labor has adjustment cost and is a dynamic input.
 - 2. What if doing RD impacts future productivity (probabilistic vs deterministic)?

Implementing 3rd stage

- We will estimate this equation using NLLS while using a series approximation, say order 4 (typically you expand until no change).
- Recall that from the first stage we have an estimate for $\phi_{it} = y_{it} b_l l_t$, which no longer includes η due to estimation. Under the model's structure measurement error is purged.
- Estimating capital coefficient (or any coefficient of an input that has dynamics!) on the following

$$y_{t+1} - b_l l_{t+1} = \beta_0 + \beta_k k_{t+1} + \sum_{r=0}^{4-s} \sum_{s=0}^{4} \widehat{P}_{it}^s (\widehat{\phi}_{it} - \beta_k k_{it})^r + \xi_{it+1} + \eta_{it+1}$$
(12)

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Apply NNLS with OLS starting values to search for β_k, in stata nl for instance.

Alternative estimation: GMM

- Alternatively we can estimate the capital coefficient with GMM and rely on only one moment, while adding and testing for overidentifying restrictions.
- Introduce method here as will be useful for later (LP, ACF, DL)
- From first stage we have an estimate of productivity given parameter β_k

$$\omega_{it}(\beta_k) = \phi_{it} - \beta_k k_{it} \tag{13}$$

- Relying on productivity evolution process, ω_{it+1} = g(ω_{it}) + ξ_{it+1}, we can recover ξ_{it+1}(β_k) by non parametrically regressing ω_{it+1} on ω_{it}.
- ▶ Now we have $\xi_{it+1}(\beta_k)$ and can form moments to identify β_k

$$E\left\{\xi_{it+1}(\beta_k)\left(\begin{array}{c}k_{it+1}\\k_{it}\end{array}\right)\right\}=0$$
(14)

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Alternative proxy estimators

- We will now turn to recent modifications and extensions of the OP-estimator. A very good handbook chapter on this is Ackerberg, Benkard, Berry and Pakes (2007).
- We cover the Levinsohn and Petrin (2003, LP henceforth), the Ackerberg, Caves and Frazier (2006, ACF henceforth) extensions.

Issues

- Monotonicity of investment: modifications of OP require revisiting proof.
- Investment is lumpy and reduces sample size (efficiency), i.e. economic reality.
- Adjustment costs in labor and therefore dynamics in labor.
- Identification in LP and OP: what variation is left conditional on proxy.

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LP proxy estimator

- The OP estimator crucially relies positive investment data for firms in a given industry. In most developing countries this data is hard to come by. Furthermore, even in developed regions we might see lumpy investment and therefore a reduction in sample size and question of which firms we can analyze.
- ► Technical comment. We can use only-investment sample to estimate true underlying parameters and force them on entire sample when computing productivity. If we want to rely on cleaner measures of productvity we can only use restricted sample $\omega_{it} = \phi_{it} \beta_k k_{it}$.
- LP estimator replaces investment policy function by a static intermediate input demand equation, monotonicly increasing in productivity and proceed from there.

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We will revisit issues with this estimator later (assumptions on competition, identification).

Estimator

The estimation procedure is identical to OP, except for a different control function. We can rely on simple static profit maximization to establish the following condition.

$$m_t = m_t(\omega_t, k_t) \tag{15}$$

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This paper relies on a monotonic increasing function in ω to invert equation and have the unobserved state as a function of two observables. Assumes perfect competition. We can now rely on a similar approach

$$\omega_t = h_t(k_t, m_t) \tag{16}$$

Simply substitute this in for productivity term and in case of a sales generating production function recover estimate on materials in a third stage.

we estimate first stage as

$$y_{it} = \beta_l I_{it} + \phi_t(m_{it}, k_{it}) + \eta_{it}$$
(17)

- ► Second (or third if we do selection control which is different as well) stage is similar to GMM implementation of OP 3rd stage. Now, a moment condition on capital is formed, E(k_{it+1}ξ_{it+1}).
- Write down the estimation algorithm for a sales generating production function, and establish the moment conditions for estimating (β_m, β_k)

Problems of identification

- We will rely on the ACF paper to talk through some of the identification problems proxy estimators may suffer from.
- The assumption on perfect competition is key and can be relaxed but important for approach in LP (invertibility of *m* requires that as ω increases so does *m*).
- ▶ Immediate problem of collinearity between *l* and *m*.
- What if labor has adjustment costs as well, i.e. a dynamic input. Then labor is part of the state space.

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Towards a general way of dealing with all this issues is ACF.

ACF

- The main point of ACF is about realizing that every input into the production function can be partioned into
 - 1. a fixed or variable input, that is whether it is correlated with the current shock in ω , ξ . If the input is variable, but static, we can estimate them in the first stage.
 - 2. a dynamic or static input, whether they are a state variable in the dynamic model or not. If they are state variables, then investment decisions depend on them, and their coefficients cannot be identified without the third stage (like capital or age!). We could have all inputs be dynamic and therefore have $i_t = i_t(k_t, l_t, m_t, \omega_t)$.

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Relaxing assumptions on inputs

- Let us start with an inversion of productivity to investment conditional on the state variables of the problem.
- ACF note the two dimensions to every input (variable/fixed and static/dynamic) and it will have implications for the properties of the estimator. Let us consider a general case where we have one of each of the four possible inputs.

$$y_{it} = \beta_0 + \beta^{vs} x_{it}^{vs} + \beta^{vd} x_{it}^{vd} + \beta^{fs} x_{it}^{fs} + \beta^{fd} x_{it}^{fd} + \omega_{it} + \eta_{it}$$
(18)

Interpretation and identification

- ▶ In the original OP model, labor was x^{vs} and can be estimated in the first stage whereas capital was x^{fd} and had to be estimated in a second stage relying on $E(k_{it+1}\xi_{it+1}) = 0$. Note that we could also consider instruments in GMM approach and there we could include l_t and k_{t+1}, k_t , respectively as instruments for β_l and β_k
- Coefficients on x^{vd} cannot be identified in a first stage but can be estimated in second stage relying on instrument x^{vd}_{t-1}.
- Lastly, coefficients on fixed but static inputs can be estimated in a first stage. Which input would satisfy this condition?

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testing

Technical comment. Once there is a static or fixed input, we have overidentifying restrictions. This can be potentially interesting in testing some of the input timing assumptions. For instance, start with capital as a fixed dynamic input and use both k_{t+1} and k_t. Now test overidentifying restriction and k_t is only valid instrument if capital is fixed (other instrument always valid), rejection of specification might be evidence on capital's fixedness.

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Identification in 1st stage

- ACF further discuss the plausibility of the underlying data generating process for identification in OP and LP. The essential argument lies in the timing of inputs (I,m) with respect to shocks in productivity.
- Under the assumption that labor is indeed variable and static, in order for it to have independent variance, there must be a variable z_{it} that impacts the firms' choices of labor l_{it}, but does not impact the investment choice i_{it}.
- Therefore z_{it} must have some variance that is independent of (k, ω). If this were not the case, we would have that l_{it} = f(ω_{it}, k_{it}), and therefore perfect collinearity between l and φ(.) and no identification of β_l.

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So in order to follow the OP procedure, we need

$$I_{it} = f(\omega_{it}, k_{it}, z_{it}) \tag{19}$$

- where z_{it} are additional factors that impact labor demand with nonzero conditional variance.
- Note that z_{it} cannot be serially correlated or it would end up in the state space. As long as it is unobservable, it will break inversion. If observable, still problematic as it is part of φ(.).

Possible z variables

▶ We can come up with two different candidates for *z_{it}* to have identification:

- 1. input price differences across plants that are iid. This is at odds with other assumption in model that prices are given to all firms
- 2. iid random draws to the environment that cause differences in the variance of η_{it} over time. Like upcoming strikes, machine breakdown, maintenance periods.
- ACF propose solution to this and show identificaton of OP under a slightly different timing assumption. The argument revisits Nadiri and Rosen (1974) on timing of inputs.
- Suppose that *l_{it}* is not perfectly variable, but is chosen at some point in time between *t* − 1 and *t*. Denote this point at which it is decided as *t* − *b*, where 0 < *b* < 1.</p>

▶ Now let ω follow a Markov process between subperiods (t-1, t-b, t). In this case, labor is not a function of ω but of ω_{it-b} and therefore:

$$I_{it} = f(\omega_{it-b}, k_{it}) \tag{20}$$

- ▶ What breaks collinearity is the movement in productivity after t − b, therefore allowing independent variation to identify labor coefficient.
- Note, ACF also propose run first stage in all inputs, then identify labor in second stage with lagged variable as instrument.

LP revisited

- The same critique applies to the LP procedure, however, it more severe due to the use of variable input as proxy (intermediate inputs).
- It is hard to think of such a variable z_{it} that would affect a firms's labor choice but not its material choice, either directly or indirectly through the labor choice.

- Input demand equations for *I*, *m* with simple Cobb-Douglas will show forcefully. Labor will drop out!
- (LP can be fixed by either estimating all in second stage or using Wooldridge GMM version).

- ACF go on and draw similarity with other literature in dynamic panel estimation. Insight in impossibility to identify variable static inputs.
- ACF procedure is

$$y_{it} = \phi_t(m_{it}, k_{it}, l_{it}) + \eta_{it}$$
(21)

- Construct ξ_{it+1} from first stage, and set moments for *I*, *k* where we have more instruments and can test overidentifying restrictions and therefore whether our model makes sense.
- Know your industry and the institutional details to guide this analysis!

Wooldridge GMM

- Wooldrigde (2009) proposes an alternative implementation that deals with the identification of the production function coefficients and is robust to the criticism of Ackerberg, Caves and Frazier (2006).
- ► The approach relies on a joint estimation of a system of two equations using *GMM*, by specifying different instruments for both equations.
- Let's consider the Wooldrigde approach while relying on materials m_{it} to proxy for productivity (i.e. Wooldrigde/LP).
- This implies the following moments for identification

$$E(\eta_{it} + \xi_{it} | \mathcal{I}_{it}) = 0 \tag{22}$$

Wooldridge Ctd.

- where \mathcal{I}_t is the conditioning set at t.
- The condition on η_{it} is related to ACF first stage where η_{it} = y_{it} − β_lI_{it} − β_mm_{it} − β_kk_{it} − h(I_{it}, m_{it}, k_{it}; β_{h,t}), and β_{h,t} are the polynomial coefficients.
- The second moment is related to the second stage in the procedure described above and

 $\begin{aligned} \xi_{it+1} &= h(I_{it+1}, m_{it+1}, k_{it+1}; \beta_{h,t+1}) - g\left(h(I_{it}, m_{it}, k_{it}; \beta_{h,t})\right); \beta_g\right), \text{ and } \\ \beta_g \text{ are the polynomial coefficients on } g. \end{aligned}$

- The sample analogue of these moments will generate estimates for (β_l, β_m, β_k) in addition to the polynomial coefficients on the function h_t(l_{it}, m_{it}, k_{it}) and g(ω_{it}).
- Advantage: bootstrapping is not required for se, and more efficient estimators by using cross-equation correlations.
- Disadvantage: cost of searching over a larger parameter space, i.e. over production function coefficients, all polynomial coefficients used to approximate the functions h_t(.) and g(.).

Scalar unobservable

We can extend the model to allow for 2 unobservables that impact investment, e.g. unobserved demand conditions µ_{it}, now we have

$$i_t = i_t(k_t, \omega_t, \mu_t) \tag{23}$$

 we cannot invert and need a second control variable to invert and proceed (also see De Loecker, 2007). Call this variable s_t, and we have a bijection (it is onto) Υ(.)

$$\frac{I_{it}}{s_{it}} = \Upsilon_t(k_{it}, \omega_{it}, \nu_{it})$$
(24)

$$\omega_{it} = \Upsilon_{1,t}^{-1}(k_{it}, i_{it}, s_{it})$$
(25)

- ▶ and go from here. Again, this might help as it creates addiditonal variation to identify φ(.).
- Note, we need independent Markov processes for two unobservables otherwise we can no longer identify coefficient. The problem comes from the fact that the expectation of productivity becomes dependent on both lagged productivity and lagged demand shock. No more identifying variation left due to law of motion on capital.

Conclusions Methodology

- Different data and industry setting will require you to worry about the various aspects and dimensions of the problem
- No general way to go about this, except for thinking through how the various unobservables enter the model and your econometric procedure.

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> You will always get results, question is on identification believability!

Applications and Extensions

- Pavcnik (2002) relies on the OP method to investigate the productivity gains from trade liberalization in Chilean manufacturing sector. She then uses estimates for productivity to investigate reallocation due to exit and finds that reducing tariffs lead to a productivity increase for the industry.
- Finds same bias on labor and capital, and have impact on returns to scale and productivity estimates.
- Run simple probit regressions on productivity and tariffs, and run the OP decomposition which shows reallocation effecs while trade liberalization occured.

Extending the framework

- If you want to study impact of a firm-level decision (like exporting, RD, ownership structure, etc.) on productivity, you have to verify how it enters the information structure and whether it is correlated with the innovation in productivity.
- ▶ De Loecker (2007 JIE) modifies OP model to incorporate different market structures for exporters, in order to separate out the learning by exporting from the self-selection hypothesis offered by theory. This paper now has $i_t = i_{t,e}(k_t, \omega_t)$ or alternatively have $i_t = i_t(k_t, \omega_t, e_t)$. He finds very different coefficients of production function, and importantly different estimates for LBE different productivity trajectories relying on these techniques.
- ▶ De Loecker (2013) in addition, relaxes exogenous Markov process for productivity, by allowing $\omega_{it} = g(\omega_{it-1}, e_{it-1})$ while estimating production function.

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 De Loecker (2011 ECMA) applies two unobservables (productivity-demand)

Control functions: OP, LP and ACF

The main insight of OP (and subsequent modifications) is to rely on a FOC that relates an observable decision d to unobserved productivity ω and other observables (in OP capital k), and keep it as a vector k_{it}.

$$d_{it} = d(\mathbf{k}_{it}, \omega_{it}) \tag{26}$$

The properties of d(.) depend on the exact FOC and in the case of OP this is an investment policy function. The properties will then determine under which conditions we: 1) obtain such a function, and 2) when this is invertible such that:

$$\omega_{it} = d^{-1}(d_{it}, \boldsymbol{k}_{it}) \tag{27}$$

Identification pieces

- The entire literature has focussed on the exact conditions under which d(.) correctly proxies for productivity and its implications for identification.
- Input classification into variable/fixed and static/dynamic (ACF).

	variable	fixed
static	no adj/pbp	no adj/ across periods
dynamic	state var	adj cost/state var

- Crucial regardless above:
 - 1. Capital I.o.m.: $K_t = (1 \delta)K_{t-1} + I_{t-1}$,
 - 2. Productivity process: $\omega_t = g(\omega_{t-1}) + \xi_t$.
- ► Determines moment conditions: $\mathbb{E}(\xi_t x) = 0$, with x inputs and either t or t 1.
- Reality: survey questions related to end of year/ retrospective, etc. Puts the emphasis on *when* do firms make decisions, and *what* do they know when do so – i.e. information set content *I_{it}*.

Control functions: OP, LP and ACF: ctd.

- This control function replaces productivity and gives rise to so-called two step (OP, LP and ACF) or one-step procedure (due to Wooldrigde (2009)).
 - 1. Two-stage appproach:

$$y_{it} = \underbrace{\alpha_{l}l_{it} + \alpha_{k}k_{it} + d^{-1}(d_{it}, \mathbf{k}_{it})}_{\phi(l_{it}, k_{it}, d_{it})} + \epsilon_{it} \quad (28)$$

$$y_{it} = \alpha_{l}l_{it} + \alpha_{k}k_{it} + g(\phi_{it-1} - \alpha_{l}l_{it-1} - \alpha_{k}k_{it-1}) + \xi_{it} + \epsilon_{it} \quad (29)$$

$$\mathbb{E}\left[\left(\xi_{it} + \epsilon_{it}\right) \begin{pmatrix} l_{it-1} \\ k_{it} \\ \hat{\phi}_{it-1} \end{pmatrix}\right] = 0 \quad (30)$$

2. One-step approach:

$$y_{it} = \alpha_l l_{it} + \alpha_k k_{it} + g(d^{-1}(d_{it-1}, k_{it-1})) + \xi_{it} + \epsilon_{it}$$
(31)

Control functions: OP, LP and ACF: synthesis

- One could now spend rest of the day discussing details under which DGP moment conditions are valid – i.e. identification holds.
- I refer to ACF for an excellent treatment of the problem and summarize by:
 - OP relies on dynamic control, investment, and requires all relevant state variables in k. Implications: invertibility proof is non-trivial and in practice becomes bindnig when environment is slightly different (R&D, export, FDI, etc.)
 - LP relies on static control, intermediate input, and requires fully and correctly specified input demand equation but invertibility almost for free. Holds in large class of models of imperfect competition (important later).
 - ► ACF refines the exact moment conditions guaranteeing identification.

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Selection matters but unbalanced panel goes a long way.

Control functions: special case

Intuition behind approach comes from a special case (2× OLS):
 ω_{it} = ω_{it-1} + ξ_{it} and LP (m(.) log-linear)

$$y_{it} = \alpha_l I_{it} + \alpha_k k_{it} + \underbrace{\gamma_1 d_{it} + \gamma_2 k_{it}}_{\omega_{it}} + \epsilon_{it}$$

1st stage: $y_{it} = \phi(I_{it}, k_{it}, d_{it}) + \epsilon_{it}$

$$y_{it} = \alpha_l l_{it} + \alpha_k k_{it} + \omega_{it-1} + \xi_{it} + \epsilon_{it}$$
$$y_{it} = \alpha_l l_{it} + \alpha_k k_{it} + \underbrace{\phi_{it-1} - \alpha_l l_{it-1} - \alpha_k k_{it-1}}_{\omega_{it-1}} + \xi_{it} + \epsilon_{it}$$
$$2nd \text{ stage:} \quad \Delta y_{it} = \alpha_l \Delta l_{it} + \alpha_k \Delta k_{it} + \xi_{it} + \Delta \epsilon_{it}$$

If both labor and capital are set a period ahead for example, running OLS provides consistent estimates of α.

Active improvements to performance

- The literature, and practice, by and large relies on an exogenous process for ω which is often internally inconsistent but moreover counterfactual.
- Firms spend lots of resources to actively improve performance (investments, R&D, HR practices, advertizing, training, etc.) or their environment changes (trade liberalization, deregulation, taxes, etc.)
- At some level we miss this action \mathcal{A} in the process: $g(\omega_{t-1}, \mathcal{A}_{t-1})$:
 - 1. misspecified process leads to omitted variable bias
 - 2. Incorrect conclusions about drivers of productivity growth
- Take simple process before: bias comes from error

$$\Delta y_{it} = \alpha_I \Delta I_{it} + \alpha_k \Delta k_{it} + \xi_{it} + \mathcal{A}_{it-1} + \Delta \epsilon_{it}$$
(32)

► Evaluate 𝔼(𝔅_{t-1}Δ𝑥_t) = 0? and therefore heterogeneity becomes important – otherwise subsumed in g_t(.).

What have we learned?

- In the absence of obvious instruments (input prices; sources of random input choices) and/or imposed time series properties (fixed effects in efficiency/demand) we have an alternative approach to use optimal behavior (input demand or investment).
- Great deal of flexibility to the applied researcher provided that underlying model is spelled out.
- For now note that we can allow for $q_{it} = f(\mathbf{x}_{it}; \beta) + \omega_{it}$ in theory at least.
- Departures from exogenous productivity are important and matter a lot!

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TFP, TFPR, TFPQ and aggregation

- Across a variety of fields (macro-micro) economists talk about TFP, both its measurement and drivers.
- Traditional approach in macro and industry-wide data relied on correct price indices to correct for inflation.
- Move to micro data invalidates this approach if we care about the heterogeneity and *anything but* the aggregate.
- Recently use of TFPR and TFPQ (cfr Intro); and there equal confusion on welfare relevance – i.e., we do not want more resources towards high TFPQ firms unless homogeneous goods setting! In other words we need demand and leads to TFPR.
- Tension comes from origins of production functions (ag econ with product homogeneity) to its applications to highly differentiated manufacturing and service sectors – the real challenge ahead.