Subsidies, Entry and the Distribution of R&D Investment

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Abstract

We analyze the link between entry and R&D spending distribution. We consider a monopolistic competitive market with free entry in which firms can invest in cost-cutting R&D by paying a fixed cost first. For an intermediate level of fixed cost, there is a unique equilibrium in which the market segments into investing and non-investing firms. We show that the measure of R&D investing firms decreases as entry occurs. Using this result, we show how alternative government policies affect the R&D spending distribution. In particular, we characterize the cases in which incentives to promote R&D spending can result in exit. We show that while subsidy to entry may be welfare neutral from the consumers' point of view, R&D subsidies, despite promoting exit sometimes, are always welfare improving. Data motivating these results are drawn from the Taiwanese and Korean semiconductor industries.

Keywords: Entry, Subsidy, Research and Development, Product Differentiation.

JEL Codes: L11, L63, O2, O31, D24, D43

1 Introduction

This paper studies the link between changes in the number of firms active in an industry, and the distribution of R&D spending among them. In particular, we build a framework in which we analyze the effect of government policies (or other industry shocks) on the proportion of firms investing in R&D, as well as on their R&D intensity. In turn, changes in the R&D investment distribution cause entry or exit on the market and a new industry equilibrium to arise. Finally, we discuss the impact of these perturbations on consumers' welfare.

We analyze an industry in which a continuum of firms simultaneously invest in cost-cutting R&D, and then compete in the product market. First, in Proposition 1, we analyze the industry equilibrium in the short run, for a fixed market size. When investing in R&D involves a positive fixed cost K (e.g., establishment of an R&D department, investment in R&D capital, etc.), three types of industry equilibria may arise. In particular, for high and low K, the only equilibrium involves either no firm or all firms investing in R&D, respectively. For an intermediate interval of values of K, there is a unique equilibrium in which the market segments into two sets of firms. The firms in the first set pay the fixed cost K and invest in R&D. As a result, since they have lower costs, they charge lower prices in the product market and have higher economic profits. The firms in the second set do not invest in R&D, and have a higher price level and lower economic profits. The relative measures of the two sets are determined in equilibrium by the fact that all firms must be ex-ante indifferent between being in the first or in the second set. Next, in Proposition 2, we characterize the long-run equilibria in the industry, allowing for free-entry conditional on paying an entry cost. We show that if the entry cost is constant in the measure of firms active on the market, only symmetric equilibria arise in the long run, while if the entry cost is increasing in the measure of active firms (e.g., due to an upward sloping supply of entrepreneurial talent, or increasing advertising costs), asymmetric equilibria arise in the long run as well.

We use this analysis to address the effects of three forms of government subsidy (an entry subsidy, and subsidies to the fixed and the variable components of the R&D costs) on entry and on the distribution of R&D spending in the market.[1] The distribution of R&D spending in this model is described by two variables: (i) the measure of investing firms, and (ii) their level of R&D spending ("R&D intensity"). Both these variables affect market price levels and consumer welfare.

The flavor of our results is best described by focusing on the effects of an entry subsidy on an asymmetric industry equilibrium studied in Proposition 3. First, we find that, as the measure of

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[1] Note the government R&D subsidies could be reinterpreted as exogenous technology shocks on the fixed and variable components of R&D. In this case, our framework allows to identify the impact of such shocks on the industry equilibrium and the consumer welfare.

active firms in the industry increases due to the subsidy, the measure of firms investing in R&D *always decreases* in equilibrium. The intuition behind this result is the following: For a given measure of active firms in the industry, suppose that we are in the asymmetric equilibrium as described earlier. If more firms join the market and the set of investing firms remains the same, the profits of both types of firms decrease. However, the negative impact is greater for the investing firms than for the non-investing ones. Thus, to restore the equilibrium indifference condition between the two types of firms, we need to decrease the set of firms investing in R&D.

Second, we examine how an entry subsidy affects R&D intensity. In particular, we show that two distinct effects influence R&D intensity. First, the increase in the measure of active firms, via entry, tends to decrease the returns on R&D investment and, thus, to depress R&D intensity. We name this the *"entry effect."* Second, the decrease in the measure of investing firms could lead to lower market competition, higher R&D investment returns and, thus, a higher R&D intensity. This effect is novel in the literature and is named the *"concentration effect."* We show that if a subsidy to entry is introduced, the entry effect and the concentration effect offset each other in a non-zero measured set of parameters, and the R&D intensity may remain constant. This, together with the decrease in the measure of investing firms, suggests that entry leads to *more concentrated R&D spending*. In the product market, this change in the R&D distribution has the potential to lead to higher prices and a decrease in consumers' welfare after an increase in entry. As we show in Section 5, our model allows us to quantify this cost in consumers' welfare versus the direct benefit induced by entry. We show that if an industry equilibrium is perturbed by a subsidy to entry, the net effect is (weakly) beneficial to consumers. However, in the welfare analysis carried out in Section 5, we show that if the economy is in an asymmetric equilibrium, the shift of the R&D distribution caused by such subsidy eventually results in *entry being neutral from a consumer welfare's perspective.*

We conduct similar comparative static exercises to study the effect of the subsidization of the fixed and variable components of R&D expenditure on R&D distribution in the industry. Perhaps the most surprising finding is that the introduction of R&D subsidies can induce exit under some conditions. In particular, in Proposition 4, we show that while a (small) subsidy to the R&D fixed costs can only cause entry if the original industry equilibrium is symmetric, it causes exit if the original equilibrium is asymmetric. Furthermore, in Proposition 5, we show *a subsidy to the R&D variable costs can only cause exit*. Still, Proposition 6 shows that such subsidies, *despite the occurrence of exit,* are still *strictly improving consumers' welfare.*

The rest of the paper is organized as follows. After the empirical motivation and the literature review, we present the model in Section 2. In Section 3, we solve the model identifying the conditions for symmetric and asymmetric equilibria both in the short and the long run. Section 4 is devoted to comparative statics, and Section 5 to the welfare analysis of the previous results. In Section 6,

we conclude.

1.1 Empirical Motivation

The model explored in this paper is motivated by the striking distributional patterns in R&D expenditure observed in data from industries experiencing significant expansions. Table 1 shows plant level data on the Taiwanese Semiconductor Industry (SIC 3211) drawn from manufacturing surveys conducted by the Ministry of Economic Affairs.[2] During the 1980s, the Taiwanese semiconductor industry grew 646% (in terms of revenue) and 144% (in terms of establishments). As such, it is an example of an industry undergoing significant size changes.

{INSERT HERE: TABLE 1 FROM DOC FILE WITH LABEL: Table 1: R&D in the Taiwanese Semiconductor Industry 1981-93}

Panel 1 of Table 1 shows, for each survey year, the sum of (nominal) revenue attributed to each plant, the total number of plants, the number and proportion of plants that recorded some expenditure on R&D, and the average (across all plants) and total (nominal) expenditures on R&D. The period between 1981 and 1986 is particularly interesting. The number of plants increased from 1279 to 2808 during this period. However, despite this rapid industrial expansion, the proportion of plants engaged in R&D steadily decreased from 26.5% to 18.4%.

Panels 2 and 3 of Table 1 show the percentiles of expenditures on R&D. Panel 2 shows the level of expenditure corresponding to each percentile of expenditure, so that in 1981, 95% of plants had R&D expenditure equal to or less than 14,729. Panel 3 examines the proportion of total R&D expenditures conducted by plants at or below each percentile level, so that in 1981 firms with R&D expenditure equal to or less than 14,729 (i.e., corresponding to those firms at or below the 95th percentile) accounted for 18.4% of total R&D expenditure. Reading together Panels 2 and 3 shows a striking shift in the distribution of R&D expenditure, with R&D expenditure becoming more concentrated during the period of dramatic industry expansion occurring between 1981 and 1986. That is to say, those establishments in the far right tail of the distribution of R&D expenditures increased their R&D intensity markedly, while other establishments decreased their expenditures. It is notable that these trends became reversed in the years after 1986.[3]

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[2]The data in these surveys cover between 88% and 94% of employment in the manufacturing sector (depending on the survey year). We are greatful to Daniel Xu for his help in accessing these data. The data are avaliable at http://pages.stern.nyu.edu/~jasker/

[3]During the early- to mid-80s, the Taiwanese semiconductor industry was expanding in areas such as chip design, testing, packaging and some specialist fabrication. Significantly, the large DRAM (memory chip) market during this period was dominated by Japan and the U.S., with Taiwan not being a participant. A significant structural

{INSERT HERE: TABLE 2 FROM DOC FILE WITH LABEL: Table 2: R&D in the Korean Semiconductor Industry 1993-96}

Similar patterns in R&D expenditure emerge in other data. Table 2 shows an abbreviated version of Table 1, reporting plant-level data on the Korean semiconductor industry drawn from manufacturing surveys conducted by the Korean Government (see Xu (2008) for a description of these surveys). The same patterns seen in the Taiwanese data emerge in the Korean data during the period 1993-1996, which also corresponds to a period of dramatic industry expansion. The emergence of similar patterns in the Korean and Taiwanese data is all the more noteworthy due to the differences in their governments' policy toward the semiconductor industry and the much higher concentration of the Korean semiconductor industry relative to the Taiwanese industry (see Matthews and Cho (2000) for a comparative history).

The model developed in this paper offers a unified framework to study the R&D spending distribution in an industry. In particular, it seeks to provide a market-based explanation for these contemporaneous industry expansions and shifts in R&D expenditures. First, it captures the fact that, while some establishments do engage in R&D, many do not. Second, it develops an intuition for why the distribution of R&D expenditures may change with the number of active firms. Third, it suggests conditions under which the maximum level of R&D expenditure in the industry increases (a topic of interest in earlier literature (e.g., Sutton (2001)). Lastly, it suggests some intuition behind the drop in the proportion of establishments engaged in R&D.[4]

It is worth stating explicitly that the model is intentionally stylized and does not purport to explain the Taiwanese and Korean experiences specifically or completely. The purpose of the model is to carefully develop a set of intuitions for how changes in the number of active firms, whether via government intervention or exogenous technological shocks, affect the distribution of R&D. The contribution of the model is to provide a set of comparative statics linking these shocks and R&D patterns, which will provide a framework for applied researchers to understand the incentives provided by (say) R&D subsides, industry-specific assistance or trade policies together with the incentives provided by the market. It is designed to be broadly applicable to a large class of industrial settings. That said, the model is constructed with the patterns evident in the data firmly in mind.

The Taiwanese and Korean case studies, from which empirical motivation is drawn, contribute

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break occurred in the industry in 1986 with the development of large-scale foundry facilities by TSMC, heralding the beginning of Taiwan's entry into DRAM fabrication (see Matthews and Cho (2000)).

[4]To capture the drop in the proportion but the (weak) increase in the number of firms doing R&D requires a minor extension of the basic model. As an example, a demand shift along with industry expansion can do this.

to several modeling decisions. First, the model focuses on cost-cutting innovations rather than demand-shifting innovations. Fitting any innovation into this taxonomy is always problematic, but necessary for parsimony. Since the Taiwanese and Korean semiconductor industries were never defining the frontier of technological accomplishment during the two periods of industry expansion, we have chosen to examine cost-cutting innovation. Second, the data examined here are plant-level while the theory is firm-level. This is of particular relevance to the Korean data, since the Korean industry is highly concentrated. Sadly, data on ownership structures are unavailable. Our concern about this aspect of the data leads us to examine both the Korean and Taiwanese data, hoping that seeing similar patterns across industries with very different structures would mitigate measurement problems arising from this data constraint. Similarly, the data are more aggregated than we would like, covering firms at several levels in the industry's vertical chain.[5] Lastly, we consider a model in which firms produce differentiated products. Servati and Simon (2005) describe the various dimensions on which products in the semiconductor industry are differentiated.

1.2 Related Literature

Market structure was first identified by Shumpeter as one of the key determinants of R&D spending. This observation has generated a vast literature. In particular, Dasgupta and Stiglitz (1980) studied the impact of market structure on the value of innovation, while Loury (1979), Lee and Wide (1980) and Reinganum (1982, 1985) analyzed the impact of entry on R&D in the contest of patent races.

More recently, seminal work by Sutton (2001) studied the lower bound to both the equilibrium market concentration and R&D intensity of the highest spender in the market.[6] This approach, by design, puts no structure on the actual distribution of R&D activity across industry participants. By analyzing the strategic decisions of all firms in an industry, we are able to investigate the forces that shape the equilibrium distribution of R&D activity across all firms, as opposed to just the investment intensity of the highest R&D spender. As a consequence, our model provides a complement to the Sutton approach.

There are elements in common between this paper and Fisman and Rob (1999). In particular, Fisman and Rob's work is connected with ours in the sense that they present a model in which ex-ante identical firms make, in equilibrium, different R&D spending decisions. However, the focus

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[5] These considerations limit the extent to which the model can be meaningfully tested using these data. Testing-while possibly feasible using these, or similar, data-would require a data-collection exercise beyond the scope of the current paper.

[6] In Sutton's analysis, firms have the possibility to invest in several R&D trajectories. The scope economies across these trajectories and the effectiveness of R&D technology, together, determine a measure of how much a firm that outspends its rival in R&D can steal consumers away from other firms. This measure sets a lower bound to both the equilibrium market concentration and R&D intensity of the highest spender in the market

of their paper is different from ours since they study the link between the firm customer base (i.e., the firm size) and R&D spending in a search model in which customers have to pay a fixed cost to switch product. Instead, we focus on the properties of the equilibrium R&D distribution (e.g., measure of investing firms and R&D intensity) and how this distribution changes as entry occurs, as a result of government interventions or industry environmental changes.

An alternative modeling approach to that taken in this paper may have been to adopt the fully dynamic framework of Ericson and Pakes (1995) and Pakes and McGuire(1994). Setting aside the expositional advantages of an analytic model, the asymmetric equilibrium that is explored in this paper does not easily translate to the Markov-Perfect equilibrium concept exploited in these dynamic frameworks.

2 The Model

2.1 Firms and Consumers

Consider a monopolistic competitive market with free entry populated by an interval of active firms $\mathcal{N} = [0, n]$. The preferences of the representative consumer are described by a Dixit-Stiglitz utility function[7]

$$u\left(y\right) = m + W \ln \left[\int_0^n y\left(i\right)^\alpha di\right]^{1/\alpha},$$

where $y\left(i\right)$ is the consumption of the good produced by firm $i \in \mathcal{N}$, m is the numeraire, $W > 0$, and $\alpha \in (0, 1)$ measures the substitutability of the consumption goods. As is well known, utility maximization subject to the budget constraint $m + \int_0^n p(i)y\left(i\right) di \leq E$, yields the following demand function for good i, given its price $p(i)$:

$$y\left(i\right) = Mp\left(i\right)^{-\frac{1}{1-\alpha}},$$

with $M \equiv \frac{W}{\int_0^n p(j)^{-\frac{\alpha}{1-\alpha}} dj}$.

Suppose that, in order to enter the market and stay active, a firm has to pay an *entry cost*. We model the entry cost $C(n)$ as:

$$C(n) = \begin{cases} c + f(n) & \text{if } c + f(n) > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant and $f'\left(n\right) > 0$ for all n.[8] While the term c captures the part of entry costs

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[7]See Dixit and Stiglitz (1977).

[8]Note that we assume that the entry cost is a fixed cost for staying active that firms have to pay *periodically*.

which is unaffected by the measure of active firms in the economy, the second term, $f(n)$, reflects the economic cost of necessary inputs for new firms such that $f'(n) > 0$.[9]

Examples of environments that yield an entry cost increasing in n include the following ones: (a) If each firm requires an entrepreneur and the opportunity cost of entrepreneurs' labor is heterogenous, this will imply an upward sloping supply of entrepreneurial input (and, hence, upward sloping entry cost curve); (b) Similarly, heterogeneity in the economic cost of other necessary, fixed, resources (e.g. location or specific capital) yields the same upward sloping entry cost curve; (c) If new firms need to advertise to generate brand awareness, advertising cost will typically increase in the number of brands competing for consumer attention. All these examples motivate what is essentially an upward sloping supply of entrants in the market.

Second, suppose that each active firm $i \in \mathcal{N}$ can decide whether to invest in cost-cutting R&D. R&D investment requires a fixed cost $K > 0$, and a variable cost $k(i) \in R_+$, which we refer to as *R&D intensity* of firm i. R&D investment has the effect of reducing the marginal cost of production of the firm. In particular, the (constant) marginal cost of a firm is given by a function $v : R_+ \rightarrow (0,1]$ defined as $v(k) = (1+k)^{-\rho}$ with $\rho \in (0,1)$.[10]

To guarantee that the optimal investment problem has an interior solution, and to focus the analysis on the most interesting cases, we assume that the parameters of the model satisfy the following requirement:

Assumption 1 α *and* ρ *satisfy* $1 - \alpha - \alpha\rho > 0$.

2.2 Timing

The timing of the game is as follows:

(1) Infinitely many potential entrant firms, simultaneously decide whether or not to enter this market. If they enter, they become an element in the set of active firms, $\mathcal{N}$, and they pay the entry cost $C(n)$, where n is the measure of the set $\mathcal{N}$.

(2) All active firms in the set $\mathcal{N}$ simultaneously decide whether and how much to invest in R&D by paying $K + k(i) \in R_+$. If a firm $i \in \mathcal{N}$ decides not to invest in R&D, that firm does not pay K and $k(i) = 0$.

(3) Each firm $i \in \mathcal{N}$ adopts the technology $v(i) = (1+k(i))^{-\rho}$ and decides how much to produce by choosing $q(i) \in R_+$.

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[9] As a limit case, we comment on an entry cost constant in n at the end of Section 3.

[10] Note that we do not require all the firms to develop the same technology, but rather, we assume that two technologies developed with the same investment k cut the costs to the same level $v(k)$. Also, we focus on a particular functional form for $v(\cdot)$ for the sake of simplicity. The quality of the results of the paper does not depend on the specific characteristics of this functional form.

(4) The production is sold on the market and profits are realized.

In this paper, we adopt *Subgame Perfect Nash Equilibrium* as solution concept and focus on pure strategy equilibria only.

2.3 Payoffs and Industry Equilibrium

Conditional on entry, *having paid the cost* $C(n)$, the total profit of a generic firm $i \in \mathcal{N}$ in the monopolistic competitive market is:

$$\pi(i) = [p(i) - v(i)] y(i) - k(i) - K1_{\{k(i)>0\}},$$

where $[p(i) - v(i)] y(i)$ is the economic profit, $K + k(i)$ is the investment in R&D.[11] After solving for equilibrium in the final product market, because the firms' optimal mark-up rule in this model is $p(i) = \frac{v(i)}{\alpha}$, it is easy to check that

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\begin{aligned}
\pi(i) &= \frac{W(1-\alpha)}{\int_{\mathcal{N}} v(j)^{\frac{-\alpha}{1-\alpha}} dj} v(i)^{-\frac{\alpha}{1-\alpha}} - k(i) - K1_{\{k(i)>0\}} \\
&= \frac{W(1-\alpha)}{\int_{\mathcal{N}} (1+k(j))^{\frac{\alpha\rho}{1-\alpha}} dj} (1+k(i))^{\frac{\alpha\rho}{1-\alpha}} - k(i) - K1_{\{k(i)>0\}}.
\end{aligned}
\tag{1}
$$

It is easy to see that the payoff of firm i is decreasing in the technology level of its competitors. This is because the technology level of the other firms affects the other firms' prices and, via monopolistic competition, the demand that firm i faces on the final product market.

Note that for this market to be in equilibrium, it must be the case that $\pi(i) = \pi(j)$ for all $i, j \in \mathcal{N}$ (otherwise a firm would have a profitable deviation in mimicking the strategy of a rival). Thus, we can denote by $\widehat{\pi}(n)$ the total profits of (all) the firms on the market as a function of the measure of active firms n. Thus, an equilibrium is described by a measure of active firms n^* and a function $k^*(i)$, which describes the R&D investment for all $i \in [0, n^*]$.

Industry Equilibrium An industry is in a long-run equilibrium if:

(a) The marginal firm entering the market gains zero total profits-that is $\widehat{\pi}(n^*) = C(n^*)$;

(b) For all $i \in [0, n^*]$ such that $k^*(i) > 0$, $k^*(i)$ maximizes (1).

3 Equilibrium Analysis

In this section, we start the equilibrium analysis with a short-run perspective, i.e., for a fixed measure of firms active in the industry. Then, we consider the long-run perspective and we endogenize

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[11] We denote by 1_E the indicator variable equal to 1 if the event E occurs, and zero otherwise.

n in Section 3.2.

3.1   Short-Run Equilibrium

We begin by considering equilibrium in the short-run when the measure of active firms n is fixed. Observe that, for a fixed n, given that a firm $i \in \mathcal{N}$ decides to pay the fixed cost K and invest in R&D, the optimal R&D intensity of firm i is given by the solution of the problem

$$\max_{k(i) \in R_+} A(1 + k(i))^{\frac{\alpha\rho}{1-\alpha}} - k(i), \tag{2}$$

where $A \equiv \frac{W(1-\alpha)}{\int_{\mathcal{N}} (1+k(j))^{\frac{\alpha\rho}{1-\alpha}} dj}$ is a function of the other firms' R&D intensities. By Assumption 1, problem (2) has a unique solution, $k^*(i) = \left(\frac{A\alpha\rho}{1-\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} - 1$. Also, note that, for a given measure n of active firms and a measure μ of investing firms, since any investing firm solves the same optimal investment problem, *the equilibrium R&D intensity of all investing firms is the same*. Thus, it will be useful at points in the paper to adopt the notation $k(\mu, n)$ to indicate the equilibrium R&D intensity for given μ and n. Similarly, A can be written as

$$A(\mu, n) = \frac{W(1 - \alpha)}{\mu(1 + k(\mu, n))^{\frac{\alpha\rho}{1-\alpha}} + (n - \mu)}.$$

Thus, an industry equilibrium can always be described by a triplet (n^*, μ^*, k^*), where n^* is the measure of firms active on the market, μ^* is the measure of investing firms, and k^* is their R&D intensity.

We are now ready to characterize the industry equilibria in the short-run in the next proposition.

Proposition 1 *For any fixed n, there are $\underline{K}$ and $\overline{K}$, $\underline{K} < \overline{K}$, such that:*[12]

(i) *If $K \leq \underline{K}$ and $\frac{W\alpha\rho}{n} > 1$, all firms invest. If $K < \underline{K}$, this equilibrium is unique;*

(ii) *If either $K \geq \underline{K}$ or $\frac{W\alpha\rho}{n} \leq 1$, no firms invest. If either $K > \underline{K}$ or $\frac{W\alpha\rho}{n} \leq 1$, this equilibrium is unique;*

(iii) *If $K \in (\underline{K}, \overline{K})$ and $\frac{W\alpha\rho}{n} > 1$, there exists a unique equilibrium. In this equilibrium, only a positive measure $\mu \in (0, n)$ of firms invest in R&D.*

Parts (i) and (ii) of Proposition 1 describe symmetric equilibria in which all firms in the industry take the same action. In particular, $\underline{K}$ is defined as the maximal K such that, if all other firms

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[12] The equilibria described are not unique only when either $K = \underline{K}$ or $K = \underline{K}$. However, in these cases, other equilibria differ from the one described only in the strategies followed by zero-measured sets of firms.

in the market invest in R&D, a firm is still better off investing itself. Similarly, $\overline{K}$ is defined as the minimal K such that, in a situation in which no firm on the market is investing, a firm is still better off not investing itself.[13]

Part (iii) of Proposition 1 describes an asymmetric equilibrium in which firms split into two positive-measured sets: one set in which firms invest in R&D and another in which firms do not. This equilibrium is built as follows. Let μ be the measure of firms making a positive investment in R&D. As noted above, since all investing firms face the same problem (2), their R&D intensity has to be the same. For any μ, let this R&D intensity be $k(\mu)$.[14] Let us now denote by $\pi_I(\mu)$ the equilibrium profits of a firm investing $k(\mu)$ in R&D when a measure μ of firms are investing the same in R&D. Similarly, denote by $\pi_{NI}(\mu)$ the equilibrium profits of a firm not investing in R&D when a measure μ of firms are investing $k(\mu)$ in R&D. In equilibrium, μ should guarantee that the payoff of an investing firm is equal to the payoff of the non-investing firms-that is, the equilibrium μ^* must satisfy

$$\pi_I(\mu^*) = \pi_{NI}(\mu^*). \tag{3}$$

This is portrayed graphically in Figure 1.

In order to guarantee that such μ^* exists in the range $(0, n)$, let us analyze the extremes of this interval. Suppose that $\mu = 0$, and observe that part (ii) of Proposition 1 implies that, if $K < \overline{K}$, then $\pi_I(0) > \pi_{NI}(0)$. On the other hand, suppose that $\mu = n$. Because of part (i) of Proposition 1, we have that if $K > \underline{K}$, then $\pi_I(n) < \pi_{NI}(n)$. Thus, if $K \in (\underline{K}, \overline{K})$, the functions $\pi_I(\mu)$ and $\pi_{NI}(\mu)$ have to cross at at least once in the interval $(0, n)$.[15] The uniqueness part of the proof follows from the fact that, under our assumptions, the function $\pi_I(\mu)$ decreases faster than $\pi_{NI}(\mu)$ at any μ. This implies that the two functions can cross, at most, once.

To summarize, part (iii) of Proposition 1 describes the sufficient and necessary conditions for the existence of an asymmetric equilibrium in which a measure $\mu^* > 0$ of firms invest $k(\mu^*)$ in R&D, achieve a better technology and charge lower prices on the product-market. On the other hand, a measure $n - \mu^* > 0$ of firms do not invest in R&D, save in R&D fixed costs and charge higher prices on the product market.

{INSERT HERE: FIG 1 FROM DOC FILE WITH LABEL: Figure 1: An asymmetric equilibrium for a given n. Parameter values set at $n = 1, \alpha = 0.5, \rho = 0.5, W = 10, K = 1$.}

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[13]Analytical expressions for $\underline{K}$ and $\overline{K}$ can be found in the Appendix.

[14]Wherever the measure of active firms n is fixed, with a slight abuse of notation, we drop the n in $k(\mu, n)$ and the subsequently defined function $\pi_I(\mu, n)$, $\pi_{NI}(\mu, n)$, and we write $k(\mu)$, $\pi_I(\mu)$ and $\pi_{NI}(\mu)$ instead.

[15]We refer to the Appendix for the necessity of the conditions in part (iii) for the existence of an asymmetric equilibrium. Also, note that if $K = \underline{K}$ or $K = \overline{K}$, asymmetric equilibria may exist in which $\mu = n$ or, respectively, $\mu = 0$.

3.2 Entry

We conclude the equilibrium analysis by addressing the entry process in this industry and identifying the equilibrium measure of the set of active firms n^*. Recall that, from the definition of an industry equilibrium, n^* is determined by requiring that the marginal firm entering the market gains zero total profits-that is $\pi(n^*) = C(n^*)$.

The following result completes the characterization of the long-run equilibria.

Proposition 2 *(i) For any c such that $C(n) > 0$ for some n, there is an equilibrium in which a positive measure of firms n^* enter the market. (ii) There are $\overline{c}, \underline{c} > 0$, $\underline{c} < \overline{c}$ such that if $c > \overline{c}$, all firms in the market invest in equilibrium, and if $c < \underline{c}$, no firm in the market invests in equilibrium. (iii) If $c \in (\underline{c}, \overline{c})$, a unique asymmetric equilibrium exists.*

Proposition 2 guarantees that, for any $C(n) > 0$, a market with a positive measure of firms arises in equilibrium. To see this, let us consider the function $\widehat{\pi}(n)$, which describes the equilibrium profits of each firm as a function of the measure of active firms n. By Proposition 1, if $n \geq W\alpha\rho$ or $K > \overline{K}$, no firms invest in equilibrium, and it is easy to check that $\widehat{\pi}(n) = \frac{W(1-\alpha)}{n}$. Note that, since $\overline{K}$ is decreasing in n, the condition $K \geq \overline{K}$, for a given K, corresponds to $n \geq \widetilde{n}$ for some $\widetilde{n} > 0$ that satisfies $K \geq \overline{K}$ with equality. This implies that we have $\widehat{\pi}(n) = \frac{W(1-\alpha)}{n}$ for all $n \geq \overline{n} \equiv \min[W\alpha\rho, \widetilde{n}]$, which converges to zero as n goes to infinity.

Consider, now, the case in which $n < W\alpha\rho$ and $K \leq \underline{K}$. Since $\underline{K}$ is decreasing in n, for a given K the condition $K \leq \underline{K}$ corresponds to $n \leq \underline{n}$, where $\underline{n}$ satisfies $K \leq \underline{K}$ with equality. Proposition 1 guarantees that, in this case, for any $n \leq \underline{n}$, all firms invest $k^* = \frac{W\alpha\rho}{n} - 1$ in equilibrium, and the equilibrium profit is

$$\widehat{\pi}(n) = \frac{W(1 - \alpha - \alpha\rho)}{n} + 1 - K.$$

By Assumption 1, $\widehat{\pi}(n)$ is decreasing in n, and diverges to infinity as $n \to 0$. It is easy to check that $\underline{K} \leq \overline{K}$ implies that $\underline{n} \leq \overline{n}$.

We can now focus on the case in which an asymmetric equilibrium arises-that is, $K \in (\underline{K}, \overline{K})$, or equivalently $n \in (\underline{n}, \overline{n})$. In this region, Proposition 1 guarantees that the equilibrium profit is uniquely defined for any n. Moreover, in the proof of Proposition 2 (in the Appendix), we show the overall continuity of the function $\widehat{\pi}(n)$ and that $\widehat{\pi}(n)$ *is constant in the $(\underline{n}, \overline{n})$ region.*

Thus, if $C(n)$ cuts $\widehat{\pi}(n)$ at a point where $\widehat{\pi}(n) < \widehat{\pi}(\overline{n})$, n^* is uniquely defined, such that $n^* > \overline{n}$, and in equilibrium no firm invests in R&D. Second, if $C(n)$ cuts $\widehat{\pi}(n)$ at a point where $\widehat{\pi}(n) > \widehat{\pi}(\underline{n})$, n^* is uniquely defined, we have that $n^* < \underline{n}$, and in equilibrium all firms invest in R&D. Finally, if $C(n)$ cuts $\widehat{\pi}(n)$ in the region $n \in [\underline{n}, \overline{n}]$, the continuity of the function $\widehat{\pi}(n)$ guarantees that an

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industry equilibrium always exists and an asymmetric equilibrium arise. Figure 2 displays how the resulting equilibria vary as the supply of entrants changes.

Finally, note that *if $C(n)$ is constant in n*,there is only one level of c, denoted by $\widetilde{c}$, such that asymmetric equilibria can survive in the long-run. Thus, if $c = \widetilde{c}$, any $n \in [\underline{n}, \overline{n}]$ can constitute a long-run equilibrium measure of active firms. Otherwise, the long-run industry equilibrium will always be symmetric.

{INSERT HERE: FIG 2 FROM DOC FILE WITH LABEL: Figure 2: Equilibrium profit $\widehat{\pi}(n)$ as a function of the measure of active firms n. Parameter values set at $\alpha = 0.5, \rho = 0.5, W = 10, K = 1$.}

4   Comparative Statics

In this section, we examine the impacts of a series of supply shocks that, in various ways, affect entry and R&D investment decisions in the model we analyzed before. More specifically, we investigate changes in the cost of entry $C(\cdot)$, the fixed R&D cost, K, and the variable R&D cost, k. Each of these supply shocks is motivated by appealing to examples of government subsidy programs that exist in various jurisdictions.[16],[17]

While, in each section, we give specific examples of government programs that correspond to the subsidies investigated, it is useful to cast this exercise in terms of our motivating empirical example: the semiconductor industries in Taiwan and South Korea. Some evidence exists that South Korea, in particular, has engaged in subsidization of the semiconductor industry. In 2005, the Office of the U.S. Trade Representative won a case at the WTO in which it was alleged that "subsidies [had been] provided by the Government of Korea to Hynix, a Korean manufacturer of memory semiconductors."[18] The nature of the subsidy found to exist was somewhat novel in that it arose due to the government "pressuring private companies to extend credit or make investments on non-commercial terms" in circumstances in which the recipient firm, Hynix, was suffering financial distress. Characterizing this form of subsidization along conventional lines is difficult: If the subsidy is anticipated, it may amount to an entry subsidy; If the political context of the subsidy is a result

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[16]These perturbations could equally be interpreted as exogenous technology shocks.

[17]In the context of the semiconductor model it seems reasonable to attribute a significant proportion of industry change to growth in demand (W). In this model, it is straightforward to show that such an increase in demand will (weakly) induce entry and, at least for a sufficiently large demand increase, always increase firms' R&D investment. We focus on supply side effect largely due to the more economically interesting effect that are generated.

[18]Details are contained in a Press Release from USTR dated 06/27/2005 titled 'United States Wins WTO Semiconductor Case' avaliable at http://www.ustr.gov/ Document_Library/Press_Releases/2005 /Section_Index.html.

of R&D performance, it may amount to a form of R&D subsidy. As a result, we investigate a range of subsidy forms.

4.1   Lowering Entry Costs

Most jurisdictions have government policies aimed at lowering the cost of entry of new firms. For example, in the U.S. the Small Business Administration helps Americans start, build and grow businesses by providing financial assistance, in the form of loan guarantees, access to specific venture capital funds and other, more targeted, programs. The SBA also provide advisory services. State programs, such as the Small Business Development Centers in California, provide access to business counseling, planning, marketing and training services that *de facto* reduce the cost of starting businesses.

We model the effect of subsidizing entry by allowing $C(n)$ to shift downward in response to the subsidy. That is, pre-subsidy, $C(n) = c + f(n)$, while post-subsidy, $C(n) = c' + f(n)$ where $c' < c$. Appealing to Figure 2, note first that, by Proposition 2, when $c \leq \underline{c}$, as c decreases, the industry grows arbitrarily large, and the only equilibrium is a symmetric one in which no firm invests. Thus, as c decreases, eventually all firms stop investing in R&D. Suppose, now, that c increases in the range $c \geq \overline{c}$. In that region, as c increases, the industry becomes an arbitrarily small one in which all firms invest. The following result describes the implications of a decrease of c.

Proposition 3 *(i) If $c \leq \underline{c}$, as c decreases, n increases and no firm invests in R&D; (ii) As c decreases in the region $c > \overline{c}$, n increases, all firms invest in R&D, and the R&D intensity decreases; (iii) Finally, if $c \in (\underline{c}, \overline{c})$, as c decreases, n increases, the equilibrium measure of investing firms μ decreases, and the R&D intensity stays constant.*

First, note that Proposition 3 implies that, as c decreases due to an entry subsidy, *more entry always occurs*. Second, let us turn to the effect of an entry subsidy on the R&D distribution on the market. The first variable that describes the R&D distribution on a market is the measure of investing firms in an asymmetric equilibrium, μ. Point (iii) of Proposition 3 implies that, as entry occurs due to lower entry costs, the measure of investing firms μ strictly decreases. To see why, for a given c, consider an equilibrium measure of active firms n^*, and suppose that we are in an asymmetric equilibrium as described in Proposition 1 (part (iii)), and that μ^* is the equilibrium measure of investing firms. Let us now decrease c. As noted above, a decrease in c will always induce more entry. As entry occurs, let us look at the profits of investing and non-investing firms if the measure of investing firms is still μ^*. While the profits of both types of firms decrease, the negative impact is greater on the investing firms than on the non-investing ones. Thus, *the equilibrium indifference condition between the two types of firms must be satisfied at a $\mu < \mu^*$.*

Also, Proposition 3 addresses the second variable that determines the R&D spending distribution in the industry: the R&D intensity, k. Recall that $k(\mu, n)$ is the equilibrium R&D intensity for given measures of investing firms μ, and active firms n. Since a decrease in c always increases the measure of active firms n, the impact of a change in n on the equilibrium R&D intensity $k(\mu(n), n)$ is captured by $\frac{\partial k(\mu(n), n)}{\partial n}$. Let us decompose $\frac{\partial k(\mu(n), n)}{\partial n}$ into two components:

$$\frac{\partial k(\mu(n), n)}{\partial n} = \frac{\partial k(\mu, n)}{\partial n} + \frac{\partial k(\mu, n)}{\partial \mu} \frac{\partial \mu}{\partial n} = EE + CE.$$

We name

$$EE = \frac{(1-\alpha)(1 + k(\mu(n), n))}{(\alpha\rho + \alpha - 1)(n - \mu(n)) - \mu(n)(1 + k(\mu(n), n))^{\frac{\alpha\rho}{1-\alpha}}}$$

the *"entry effect."* Since, by Assumption 1, $\alpha\rho + \alpha - 1 < 0$, then $EE < 0$. The entry effect measures how the increase in measure of active firms affects the R&D intensity, *keeping the measure of investing firms μ constant.* Since an increase in n, via an increase in the level of market competition, decreases the returns on the R&D investment, it is always the case that the entry effect affects the R&D intensity negatively. On the other hand, let

$$CE = \frac{(1-\alpha)(1 + k(\mu(n), n))\left[(1 + k(\mu(n), n))^{\frac{\alpha\rho}{1-\alpha}} - 1\right]\frac{\partial \mu(n)}{\partial n}}{(\alpha\rho + \alpha - 1)(n - \mu(n)) - \mu(n)(1 + k(\mu(n), n))^{\frac{\alpha\rho}{1-\alpha}}} > 0$$

be the *"concentration effect."* Note that, by Proposition 3, $\frac{\partial \mu(n)}{\partial n} < 0$, implying $CE > 0$. The concentration effect is novel and measures how the increase in n affects the R&D intensity via a decrease in the measure of investing firms $\mu(n)$. Since a decrease in investing firms increases R&D investment returns, the concentration effect affects the R&D intensity positively. Point (iii) of Proposition 3 implies that, in a region where the industry is in an asymmetric equilibrium, the entry and the concentration effects exactly offset each other, making the R&D intensity constant in c.

4.2 R&D Technology

We now focus on the implications of an improvement in the R&D production function, such as a subsidy to R&D investment. First, we focus on a lump-sum subsidy, which is captured in our model by a reduction in K. Second, we consider a subsidy proportional to the variable component of R&D expenditure, k.[19]

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[19]Note that if the government subsidy is a percentage of the *total* R&D investment-that is, the ex-post R&D cost is $\gamma(K + k)$ for some $\gamma \in (0, 1)$- its impact is simply the combination of the impacts of subsidies to the fixed and variable components of R&D costs, as described in Sections 4.2.1 and 4.2.2.

Many jurisdictions subsidize industrial R&D. In the U.S., the Research and Experimentation Tax Credit provides companies that qualify for the credit the ability to deduct from corporate income taxes an amount equal to 20% of qualified research expenses above a base amount (similar, complementary, state initiatives exist). In the U.K., the R&D Tax Credit allows companies to deduct up to 150% of qualifying expenditure on R&D activities when calculating their profit for tax purposes. Small or medium enterprise can, in certain circumstances, surrender this tax relief to claim payable tax credits in cash from HM Revenue & Customs. Alongside these broad brush schemes, industry-specific programs can exist, such as the Australian Space Concession Program, which provides duty-free entry of goods imported for use in a space project.[20] Each of these examples has elements that could affect both the fixed and variable costs of R&D. Hence, we consider the effects of a subsidy on each.

4.2.1 Lowering Fixed R&D Costs

Suppose that all firms that invest in R&D receive a subsidy from the government that decreases the R&D fixed cost from K to $K' < K$. In order to study how the subsidy affects the industry structure, it is necessary to study how the R&D distribution and, in turn, the equilibrium profits $\widehat{\pi}(n)$ will change at the original measure of active firms, n^*. The change in the equilibrium profit determines the effect of the policy on entry or exit and, eventually, the new measure of firms n^{**} determines the new R&D distribution.[21]

In particular, recall that $\underline{n}$ and $\overline{n}$ are the measures of active firms such that, prior to any subsidy, a firm is indifferent between investing and not investing when all firms invest or, respectively, do not invest. Similarly, we can define $\underline{n}'$ and $\overline{n}'$ to be the measures of active firms such that, when all firms invest, or do not invest, respectively, a firm is indifferent between investing and not investing when a fixed subsidy is introduced. Note that $\underline{n}' > \underline{n}$ and $\overline{n}' > \overline{n}$. Lastly, let $\widehat{\pi}(n)$ and $\widehat{\pi}'(n)$ be the pre- and post-subsidy equilibrium total profit of the active firms in the market for a measure of active firms n.

{INSERT HERE: FIG 3 FROM DOC FILE WITH LABEL: Figure 3: Pre- and post-subsidy equilibrium profits as a function of n. Parameter values set at $\alpha = 0.5, \rho = 0.5, W = 10, K = 1, K' = 0.5$ }

Figure 3 illustrates the effect of the subsidy on the market. The effects can be summarized

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[20]Australia has an R&D Tax concession that operates like the U.K. credit. Australia has no government space program.

[21]Note that we will follow the same methodology in the subsequent comparative statics exercise as well (lowering in R&D variable cost).

by reference to five regions, depending on the pre-subsidy measure of active firms, n^*. Region 1 corresponds to the region such that $n^* < \underline{n}$. In this region, *fixing the measure of active firms at* n^*, all firms invest in equilibrium both in the pre-subsidy and the post-subsidy environment. As shown in Figure 3, since all firms choose to invest, the only effect of the subsidy is to raise profits since the burden of the fixed cost of R&D is lessened. This means $\widehat{\pi}'(n^*) > \widehat{\pi}(n^*)$, inducing entry if the constant component of $C(n)$ is at c_1 in Figure 3. At the other extreme, in Region 5, if $n^* > \overline{n}'$, then, since there is no R&D investment by any firm pre- or post-subsidy at n^*, $\widehat{\pi}'(n^*) = \widehat{\pi}(n^*)$ and the subsidy has no effect on entry or any other variable of interest (this will be the case if the the constant component of $C(n)$ is at c_3 in Figure 3).

Regions in which asymmetric equilibria exist have a richer economic intuition. For example, assuming that the subsidy is such that $[\underline{n}, \overline{n}] \cap [\underline{n}', \overline{n}']$ is non-empty, Region 3 is defined by $n^* \in [\underline{n}', \overline{n}]$.[22] This corresponds to the constant component of $C(n)$ being at c_2 in Figure 3. This implies that if the measure of active firms is n^*, we have an asymmetric equilibrium (as described in Proposition 3) arising for both K and K'. For a fixed n^*, let us investigate how the pre-subsidy equilibrium profit $\widehat{\pi}(n^*)$ and the post-subsidy equilibrium profit $\widehat{\pi}'(n^*)$ relate to each other. Recall that, in the original equilibrium, a measure of firms μ^* invest in R&D and a measure $n^* - \mu^*$ do not. Note that, since K and K' do not affect the optimal R&D investment problem, for any fixed n^* and μ^*, the profit of a non-investing firm will not change, while the profit of an investing firm will increase by $K - K'$. This causes the equilibrium in the post-subsidy environment to shift to a higher measure of investing firms $\mu^{**} > \mu^*$ (see Figure 4). *In turn, this implies a lower R&D intensity and profits—that is $\widehat{\pi}'(n^*) < \widehat{\pi}(n^*)$* . In other words, the subsidy has the effect of increasing the equilibrium measure of investing firms. In turn, this increases the level of competition on the market (more firms charge a lower price), which has the effect of decreasing the equilibrium profit and, thus, of *inducing exit in the long-run*.[23]

{INSERT HERE: FIG 4 FROM DOC FILE WITH LABEL: Figure 4: Pre- and Post-subsidy equilibrium μ and profit, holding n^* fixed. Parameter values set at $\alpha = 0.5, \rho = 0.5, W = 10, K = 1,$ $K' = 0.5$ }

Proposition 4 summarizes all the comparative statics results for the five regions of interest. Observe that Proposition 4 implies that the regions where a fixed subsidy may induce exit are the ones in which either the original equilibrium is asymmetric (as discussed above), or in which the subsidy causes the equilibrium at the original n^* to switch from a symmetric one, in which no firm

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[22] Note that a small enough subsidy always guarantees the non-emptiness of $[\underline{n}, \overline{n}] \cap [\underline{n}', \overline{n}']$.

[23] Recall that we assumed that the cost $C(\cdot)$ has to be paid periodically.

invests, to one in which some will. In Section 5, we discuss the welfare implications of this result. The proof of Proposition 4, which has economic content, is also provided, to the extent not covered in the text above.

Proposition 4 *Let (n^*, μ^*, k^*) be an equilibrium in the pre-subsidy economy. There is Δ such that, if $K - K' < \Delta$, a fixed subsidy induces exit (i) if the original equilibrium is asymmetric ($n^* > \mu^* > 0$), or (ii) if the original equilibrium is one on which no firms invest ($\mu^* = 0$) and the post-subsidy industry equilibrium at n^* is asymmetric.*

Proof of Proposition 4: We consider each region in Figure 3 in sequence.

(1) $n^* < \underline{n}$ (case c_1 in Figure 3). Pre-subsidy, each firm's R&D intensity is $k^* = \frac{W\alpha\rho}{n^*} - 1$ and profit is $\widehat{\pi}(n^*) = \frac{W(1-\alpha-\alpha\rho)}{n^*} + 1 - K$. Since $n^* < \underline{n} < \underline{n}'$, the post-subsidy equilibrium for the measure of active firms n^* is one in which R&D intensity is $k^* = \frac{W\alpha\rho}{n^*} - 1$ for all firms as well. Thus, a decrease in K leads to an increase in each firm's profit for the original size n^*. Thus, the new equilibrium will involve an increase in the measure of active firms, $n^{**} > n^*$ (and, hence, since $k = \frac{W\alpha\rho}{n} - 1$, a decrease in R&D intensity per firm).

(2) $n^* \in [\underline{n}, \underline{n}']$. In this case, the original equilibrium is asymmetric, but K' is such that the new equilibrium, for a fixed measure of active firms n^*, is one in which all firms invest in R&D. Note that if $(K - K') < \pi_I(\mu^*) - \frac{W(1-\alpha-\alpha\rho)}{n^*} - 1 + K \equiv \Delta$ (that is, the subsidy is small enough), then $\pi'_I(n^*) = \pi_I(n^*) + (K - K') < \pi_I(\mu)$. In this case, the post-subsidy equilibrium profit would be lower than the pre-subsidy equilibrium profit at n^*, *inducing exit*. However, if $(K - K') \geq \pi_I(\mu) - \frac{W(1-\alpha-\alpha\rho)}{n^*} - 1 + K$, then the post-subsidy equilibrium profit is higher than the pre-subsidy equilibrium profit, inducing entry.

(3) $n^* \in [\underline{n}', \overline{n}]$ (Case c_2 in Figure 3).[24] Fixing n^*, for any $\mu \in [0, n^*]$, observe that $\pi_{NI}(\mu)$ is not affected by a decrease in K, while, since the optimal R&D intensity does not depend on K, $\pi'_I(\mu) = \pi_I(\mu) + (K - K')$. This implies that at the original equilibrium μ^*, we have $\pi'_I(\mu^*) > \pi_I(\mu^*)$, which implies that the new equilibrium μ^{**}—that is, μ^{**} such that $\pi'_I(\mu^{**}) = \pi_{NI}(\mu^{**})$—is such that $\mu^{**} > \mu^*$. This is portrayed in Figure 4. Since $\pi_{NI}(\mu)$ is decreasing in μ, we also have that $\pi_{NI}(\mu^{**}) < \pi_{NI}(\mu^*)$. This implies that, for any $n \in [\underline{n}', \overline{n}]$, the post-subsidy equilibrium profit $\widehat{\pi}'(n)$ is lower than the pre-subsidy equilibrium profit $\widehat{\pi}(n)$. Thus, *exit is induced*.

(4) $n^* \in [\overline{n}, \overline{n}']$. Note that, in this case, fixing n^*, in the original equilibrium, no firm invests in R&D and the equilibrium profit was $\pi_{NI}(n^*)$. However, in the new equilibrium, a positive measure of firms invest in R&D, and the equilibrium profit is $\pi_{NI}(\mu)$. Since $\pi_{NI}(n^*) > \pi_{NI}(\mu)$ for any

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[24]Note that, if $\overline{n} < \underline{n}'$, and $n^* \in [\overline{n}, \underline{n}']$, then, in the pre-subsidy equilibrium, no firm invests while, in the post-subsidy equilibrium, all firms do. It is easy to check that this case is consistent with an increase in profit if $K' < 1 - \frac{W\alpha\rho}{n^*}$, and a decrease of profits otherwise.

$\mu < n^*$, this implies an decrease in equilibrium profits, which, in turn, *induces exit*.

(5) $n^* > \overline{n}'$ (case c_3 in Figure 4). See text above..　　　　　■

4.2.2　Lowering Variable R&D costs

Let us now consider a policy that reduces the variable cost of R&D— that is, suppose that for every dollar spent on R&D (over and above K), each firm gets a $(1 - \gamma)$ rebate from the government. That is, firm i's profit is now given by

$$\pi(i) = A(1 + k(i))^{\frac{\alpha\rho}{1-\alpha}} - \gamma k(i) - K.$$

Thus, for any fixed measure of active firms n and a measure of investing firms μ, since the marginal cost of a unit of investment is reduced from 1 to γ, the equilibrium R&D intensity of each investing firm increases. As in the fixed subsidy case, in order to study how a proportional subsidy affects the industry structure, it is necessary to study how the equilibrium profits will change for any fixed n. In particular, we can define $\underline{n}''$ and $\overline{n}''$ to be the measures of active firms such that, when all firms invest, or do not invest, respectively, a firm is indifferent between investing and not investing when the marginal cost of investing is $\gamma < 1$. Note that $\underline{n}'' > \underline{n}$ and $\overline{n}'' > \overline{n}$.

{INSERT HERE: FIG 5 FROM DOC FILE WITH LABEL: Figure 5: Pre- and post-(variable) subsidy equilibrium profits as a function of n. Parameter values set at $\alpha = 0.5$, $\rho = 0.5$, $W = 10$, $K = 1$, $\gamma = 0.6$.}

Figure 5 shows how subsidization of the variable cost of R&D affects profits as a function of the measure of active firms. In the first region, where the pre-subsidy measure of active firms is $n^* < \underline{n}$ (all firms always engage in R&D), the effect of the subsidy is, for a fixed n, to increase the R&D intensity. This intensifies competition in the product market as prices drop, leading to a drop in firm profits. If the constant component of $C(n)$ is at c_1, the subsidy induces exit, which further increases the intensity of R&D intensity of active firms. At the other extreme, where $n^* > \overline{n}''$ (no firms ever engage in R&D), the subsidy does not come into effect, and has no effect on the industry.

Proposition 5 collects the results for the subsidy of variable R&D expenditures. The proof of Proposition 5 is presented in the Appendix.

Proposition 5 *A variable R&D subsidy always decreases profits, and induces exit.*

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5 Welfare Analysis

In this section, we address the link between our previous results and welfare. In particular, in Section 4, we analyzed how some environmental changes such as government policies affect the measure of firms present on the market and, in turn, their R&D distribution. The goal of this section is to illustrate the effect of these policies on consumers' surplus.

Recall that the utility function of the representative consumer is

$$u\left(y\right) = m + W \ln \left[\int_0^n y\left(i\right)^\alpha di\right]^{1/\alpha}.$$

In equilibrium, there are two categories of products: those that are produced by firms that invest in R&D and those that are produced by the other firms. Together with the symmetric structure of the utility function, this means that utility can be summarized by

$$u\left(x, y\right) = m + W \ln \left[\mu x^\alpha + \left(n - \mu\right) y^\alpha\right]^{1/\alpha},$$

where x is the consumption level of each good that is produced by an R&D-performing firm and y is the consumption of each good whose production has not been subject to R&D.

Since the utility functions are quasi-linear, the Marshallian demand curves and the compensated demand curves of each good (other than the numeraire) coincide. This makes it easy to formulate the expenditure function, whose derivation is presented in the Appendix.

$$e\left(p_x, p_y, u\right) = u - W \ln W\alpha - \frac{W\left(1 - \alpha\right)}{\alpha} \ln \left[\mu \left(1 + k\right)^{\frac{\alpha\rho}{1-\alpha}} + \left(n - \mu\right)\right] + W.$$

Note that the expenditure function for regions of symmetric equilibria are attained by setting $\mu = n$ or 0 as required. For any policy change, the change in consumer surplus is given by

$$\Delta CS = e\left(p_x^0, p_y^0, u\right) - e\left(p_x^1, p_y^1, u\right),$$

where p_x^0 and p_x^1 are old and new prices, respectively. The next result evaluates the comparative statics studied in Section 4 from a consumer welfare perspective.

Proposition 6 *(i) After a subsidy to entry costs, if an industry is in a symmetric equilibrium, the consumer surplus always increases, while if an industry is in an asymmetric equilibrium, the consumers' surplus stays constant. (ii) After a fixed or variable R&D subsidy, the consumer surplus always strictly increases.*

Proposition 6 allows the impact of a change in the entry cost (Proposition 3) and subsidy of the fixed and variable cost of R&D (Proposition 4 and 5, respectively) to be evaluated from the point

of view of consumer welfare. Since c does not affect the expenditure function directly (that is, a change in c does not change the function $\widehat{\pi}(n)$), its only effect on consumer surplus is via changing n. Point (i) of Proposition 6 tells us that interventions affecting C in a way that promotes entry are weakly beneficial for consumer surplus. However, note that if the economy is in an asymmetric equilibrium, entry subsidies that promote entry via changes in c *do not affect consumers' welfare*. This is because entry will cause a readjustment of the R&D distribution in the market, described in Proposition 3, that eventually leaves the consumers' surplus unaffected.

Point (ii) of Proposition 6 analyzes the effect of both fixed and variable R&D subsidy on consumers' surplus. Recall that, by Propositions 4 and 5, we know that there are conditions under which such subsidies may cause exit from the market (in particular, by Proposition 5, a variable subsidy always causes exit when it has an impact). *Despite the occurrence of exit*, Point (ii) of Proposition 6 implies that *such subsidies are always welfare improving from the consumers' point of view*.

6   Conclusion

The model presented in this paper sheds light on some of the determinants of the R&D distribution in an industry and, in particular, it identifies the proportion of firms in a market that decide to engage in R&D at all, as well as the R&D intensity of these firms. As such, it examines an important sub-set of the determinants of the broader distribution of R&D activity within an industry.

The focus of the paper on the interplay between the measure of active firms affects R&D investment is motivated by the striking changes in R&D patterns observed in data from the Taiwanese and Korean semiconductor industries during periods of dramatic industry expansion. While the model is much more generally applicable, this motivating example provides an empirical setting that illustrates the relevance and usefulness of the framework provided. The model can capture several elements of the data including: the division of establishments into those that do and do not engage in R&D and the increase in the intensity of R&D conducted by the top percentile of firms engaged in R&D.[25] Ultimately, the model gives some sense of the equilibrium forces that may have contributed to decrease the proportion of firms engaged in R&D. Lastly, we show how a range of government interventions can affect the incentives for entry and R&D investment within this framework, and we provided the evaluation of these policies from a consumers' perspective.

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[25] This last point can be easily generated by an outward shifting demand schedule, as seems likely in this case.

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Appendix

The following two Lemmas are used in several subsequent proofs.

Lemma 1 *Given a measure of active firms n, and a measure of firms with a positive R&D intensity, μ, we have $\frac{\partial A(\mu,n)}{\partial \mu} < 0$.*

Proof of Lemma 1. To show that $\frac{\partial A(\mu,n)}{\partial \mu} < 0$ for all μ, let us first compute the derivative $\frac{\partial k^*(\mu,n)}{\partial \mu}$. Recall that, from the optimal investment problem and by symmetry, we have

$$\frac{W\alpha\rho}{(n-\mu)+\mu\left(1+k^*\right)^{\frac{\alpha\rho}{1-\alpha}}}\left(1+k^*\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} - 1 = 0,$$

which implies

$$\frac{\partial k^*(\mu,n)}{\partial \mu} = \frac{\left[\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]}{\frac{\alpha\rho-1+\alpha}{(1-\alpha)(1+k^*(\mu,n))}\left\{n+\mu\left[\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]\right\} - \frac{\alpha\rho\mu}{1-\alpha}\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}}. \tag{4}$$

Note that $\frac{\partial k^*(\mu,n)}{\partial \mu}$ is negative. Now, we have that

$$\frac{\partial A(\mu,n)}{\partial \mu} = -W(1-\alpha)\frac{\left[\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} - 1\right] + \frac{\mu\alpha\rho}{1-\alpha}\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\partial k^*(\mu,n)}{\partial \mu}}{\left[n+\mu\left[\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]\right]^2},$$

which implies that $\frac{\partial A(\mu,n)}{\partial \mu} < 0$ if and only if

$$\left[\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} - 1\right] + \frac{\mu\alpha\rho}{1-\alpha}\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\partial k^*(\mu,n)}{\partial \mu} > 0. \tag{5}$$

By plugging (4) into (5) and after some manipulations, one can see that condition (5) reduces to

$$\frac{\mu\alpha\rho}{1-\alpha}\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} < \frac{\mu\alpha\rho}{1-\alpha}\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}$$
$$+\frac{1-\alpha-\alpha\rho}{(1-\alpha)(1+k)}\left[n+\mu\left[\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]\right],$$

which is satisfied because, by Assumption 1, we have $\frac{1-\alpha-\alpha\rho}{1-\alpha} > 0$. ∎

Lemma 2 Recall that $\pi_I(\mu,n) = A(\mu,n)\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} - k^*(\mu,n) - K$ and $\pi_{NI}(\mu,n) = A(\mu,n)$, where I and NI index investing and non-investing firms, respectively, and $k^*(\mu,n)$ is the optimal R&D intensity conditional on investing. Then, $\frac{\partial \pi_I(\mu,n)}{\partial \mu} < \frac{\partial \pi_{NI}(\mu,n)}{\partial \mu} = \frac{\partial A(\mu,n)}{\partial \mu} < 0$.

Proof of Lemma 2:

$$\frac{\partial \pi_I(\mu,n)}{\partial \mu} = \frac{\partial A(\mu,n)}{\partial \mu}\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} < \frac{\partial A(\mu,n)}{\partial \mu}.$$

By the envelope theorem. Given that $\left(1+k^*(\mu,n)\right)^{\frac{\alpha\rho}{1-\alpha}} > 1$, it must be that the inequality holds. ∎

Proof of Proposition 1. The proof proceeds in a series of steps. We begin by deriving expressions for $\underline{K}$ and $\overline{K}$ (steps 1 and 2). Then we show that $\overline{K} > \underline{K}$ (step 3). Steps 4 and 5 then complete parts (i) and (ii) of the proposition with proofs of uniqueness. Step 6 addresses the existence and uniqueness of the asymmetric equilibrium (part (iii) of the proposition).

Step 1: Let us assume that all firms invest in R&D. In this situation, the equilibrium investment of all firms is $k^* = \frac{W\alpha\rho}{n} - 1$. Thus, if a firm invests, the profit is $\frac{W(1-\alpha)}{n} - \frac{W\alpha\rho}{n} + 1 - K$, while if it doesn't, the profit is $\frac{W(1-\alpha)}{n\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha}}} = \left(\frac{W}{n}\right)^{\frac{1-\alpha-\alpha\rho}{1-\alpha}}(1-\alpha)(\alpha\rho)^{\frac{-\alpha\rho}{1-\alpha}}$. This implies that all firms investing in R&D is an equilibrium if

$$\frac{W(1-\alpha-\alpha\rho)}{n} + 1 - K \geq \left(\frac{W}{n}\right)^{\frac{1-\alpha-\alpha\rho}{1-\alpha}}(1-\alpha)(\alpha\rho)^{\frac{-\alpha\rho}{1-\alpha}}, \tag{6}$$

or

$$K \leq \frac{W}{n}[1-\alpha-\alpha\rho - \left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho}{1-\alpha}}(1-\alpha)] + 1 \equiv \underline{K}. \tag{7}$$

Step 2: Suppose that no firm invests in R&D. In this case, if a firm invests, the optimal investment is $k^* = \left(\frac{W\alpha\rho}{n}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} - 1$. By Assumption 1, $\frac{W\alpha\rho}{n} > 1$ guarantees k^* to be positive. Thus, a firm is better off not investing if

$$\frac{W(1-\alpha)}{n} \geq \frac{W(1-\alpha)}{n}(1+k^*)^{\frac{\alpha\rho}{1-\alpha}} - k^* - K. \tag{8}$$

Hence:

$$K \geq \overline{K} \equiv \left(\frac{W\alpha\rho}{n}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}}\left[\frac{1-\alpha}{\alpha\rho} - 1\right] - \frac{W(1-\alpha)}{n} + 1. \tag{9}$$

Step 3: To establish that $\overline{K} > \underline{K}$, we note that a sufficient condition for $\overline{K} > \underline{K}$ is

$$\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha-\alpha\rho}}[1-\alpha-\alpha\rho] - (1-\alpha) \; > \; 1-\alpha-\alpha\rho - \left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho}{1-\alpha}}(1-\alpha)$$

$$= \; \left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho}{1-\alpha}}\left[\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha}}(1-\alpha-\alpha\rho) - (1-\alpha)\right].$$

First, note that $\left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho}{1-\alpha}} < 1$ since $\frac{W\alpha\rho}{n} > 1$. Second, $\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha}} < \left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha-\alpha\rho}}$. Hence,

$$\left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho}{1-\alpha}}\left[\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha}}(1-\alpha-\alpha\rho) - (1-\alpha)\right] \; < \; \left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha}}(1-\alpha-\alpha\rho) - (1-\alpha)$$

$$< \; \left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha-\alpha\rho}}(1-\alpha-\alpha\rho) - (1-\alpha),$$

which is sufficient to have $\overline{K} > \underline{K}$.

Step 4: To show that the equilibrium in part (i) of Proposition 1 is the only equilibrium for $K < \underline{K}$, suppose that that a measure 0 of firms do not invest. Thus, condition (6) guarantees that any of these firms would have a profitable deviation in investing $k^* = \frac{W\alpha\rho}{n} - 1$. Since $\pi_I(\mu, n) > \pi_{NI}(\mu, n)$ if $\pi_I(n, n) < \pi_{NI}(n, n)$ (by Lemma 2), if a positive measure of firms μ invest, the fact that $K \leq \underline{K}$ guarantees that firms are better off investing when $\mu = n$ is sufficient to establish that investing is always a profitable deviation. This establishes the uniqueness of the equilibrium.

Step 5: To show that the equilibrium in part (ii) is unique, suppose that $K > \overline{K}$ and a zero-measured set of firms invest in R&D. Any of these firms would have a profitable deviation in not investing because of (8). Since $\pi_I(\mu, n) < \pi_{NI}(\mu, n)$ if $\pi_I(0, n) < \pi_{NI}(0, n)$ (by Lemma 2), if a positive measure of firms μ invest, the fact that $K \geq \overline{K}$ guarantees that firms are better off not investing when $\mu = 0$, is sufficient to establish that not investing is always a profitable deviation. This establishes the uniqueness of the equilibrium.

Step 6: In this step, we build an asymmetric equilibrium and we find the equilibrium measure μ of investing firms. Let μ be the measure of firms making a positive investment in R&D. Observe that, since all investing firms face the same problem (2), their investment has to be the same. In particular, the optimal investment problem of an investing firm i (2) has solution $k_i^* = \left(\frac{A(\mu, k_{-i})\alpha\rho}{1-\alpha}\right)^{\frac{1-\alpha}{1-\alpha\rho-\alpha}} - 1$, where $A(\mu, k_{-i}) = \frac{W(1-\alpha)}{\mu(1+k_{-i})^{\frac{\alpha\rho}{1-\alpha}}+(n-\mu)}$ and k_{-i} represents the R&D intensity of the other investing firms. As $dk_i^*/dk_{-i} < 0$, for any μ, we have a unique k^* satisfying the equilibrium condition

$$k^* = \left(\frac{A(\mu, k^*)\alpha\rho}{1-\alpha}\right)^{\frac{1-\alpha}{1-\alpha\rho-\alpha}} - 1,$$

which we denote by $k^*(\mu)$. Let us now denote by $\pi_I(\mu)$ the equilibrium profits of a firm investing in R&D, i.e.,

$$\pi_I(\mu) = \frac{W(1-\alpha)(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}}}{\mu(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu)} - k^*(\mu) - K,$$

and by $\pi_{NI}(\mu)$ the equilibrium profits of a non-investing firm, i.e.,

$$\pi_{NI}(\mu) = \frac{W(1-\alpha)}{\mu(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu)}.$$

In equilibrium, μ should be such that the payoff of the investing firms should be equal to the payoff of the non-investing firms- that is,

$$\pi_I(\mu) = \pi_{NI}(\mu). \tag{10}$$

In order to guarantee that μ is in the range $(0, n)$, let us analyze the extremes of this interval. Suppose that $\mu = 0$, and let us check that $\pi_I(0) > \pi_{NI}(0)$. If $\mu = 0$, we have $k^* = \left(\frac{W\alpha\rho}{n}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} - 1$.

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Because of Assumption 1, we have that $k^* > 0$ if and only if $\frac{W\alpha\rho}{n} > 1$. In this case, $\pi_I(0) > \pi_{NI}(0)$ if $K < \overline{K}$. On the other hand, if a measure $\mu = n$ of firms invest, $\pi_I(n) < \pi_{NI}(n)$ if $K > \underline{K}$. Thus, condition (10) has to be satisfied for at least one $\mu^* \in (0, n)$. To show uniqueness of the equilibrium, follow steps (a) and (b) of Proposition 9 in Baccara (2007).

For the argument we just made on the extremes of the interval $(0, n)$, Steps 1-3 above guarantee that if $K \notin \left(\underline{K}, \overline{K}\right)$, $\pi_I(\mu)$ and $\pi_{NI}(\mu)$ do not cross in the range $(0, n)$ and no μ satisfies (10).

Finally, let us now focus on the case $\frac{W\alpha\rho}{n} < 1$, and let us check if it is possible in this case to sustain asymmetric equilibria in which a measure $\mu \in (0, n)$ of firms invest in equilibrium. Note that, if $\mu = 0$, we have $k^* = \left(\frac{W\alpha\rho}{n}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} - 1 < 0$ (this is guaranteed by Assumption 1, which implies that $k^* > 0$ if and only if $\frac{W\alpha\rho}{n} > 1$). Thus, since $\frac{dk(\mu)}{d\mu} < 0$, the R&D level for investing firms has to be zero for any μ. However, if this is the case, non-investing firms are always better off than investing firms, and there is no asymmetric equilibrium. This guarantees that there are no asymmetric equilibria if $\frac{W\alpha\rho}{n} < 1$ and concludes the characterization. ∎

Proof of Proposition 2. First, note that

$$\underline{K} = \frac{W}{n}\left[1 - \alpha - \alpha\rho - \left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho}{1-\alpha}}(1 - \alpha)\right] + 1$$

is decreasing in n since, after some algebraic manipulation, it is easy to see that

$$\begin{aligned}\frac{\partial\underline{K}}{\partial n} = \ & -\frac{W}{n^2}\left[1 - \alpha - \alpha\rho - \left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho}{1-\alpha}}(1 - \alpha)\right] \\ & -\frac{W^2\alpha^2\rho^2}{n^3}\left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho-1+\alpha}{1-\alpha}} < 0.\end{aligned}$$

Similarly,

$$\overline{K} = \left[\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha-\alpha\rho}}[1 - \alpha - \alpha\rho] - (1 - \alpha)\right]\frac{W}{n} + 1$$

implies

$$\begin{aligned}\frac{\partial\overline{K}}{\partial n} = \ & -\frac{W}{n^2}\left[\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha-\alpha\rho}}[1 - \alpha - \alpha\rho] - (1 - \alpha)\right] \\ & -\frac{W^2}{n^3}[1 - \alpha - \alpha\rho]\frac{\alpha^2\rho^2}{1-\alpha-\alpha\rho}\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho-1+\alpha+\alpha\rho}{1-\alpha-\alpha\rho}} < 0.\end{aligned}$$

Note that if $\frac{W\alpha\rho}{n} = 1$ (recall that the region under study is $\frac{W\alpha\rho}{n} > 1$), then $\overline{K} = \underline{K} = 0$. Moreover, as n goes to zero, both $\overline{K}$ and $\underline{K}$ approach infinity. This allows, for any K, to define $\underline{n}$ and $\overline{n}$ that satisfy (7) and (9) with equality, respectively.

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Now, parts (i) and (ii) are trivial since in these regions $\widehat{\pi}(n)$ is easily shown to be monotonically decreasing and continuous. Let us focus on the region in which $C(n)$ cuts $\widehat{\pi}(n)$ in the region $n \in [\underline{n}, \overline{n}]$. The three step proof below establishes that in this region $\widehat{\pi}(n)$ is constant and continuous. The rest of the proposition follows easily.

First step. Let us now show that $\widehat{\pi}(x)$ is continuous, where $x \in [\underline{n}, \overline{n}]$. As discussed in the proof of Proposition 1, $\widehat{\pi}(x)$ is determined by the unique locus of intersection of two smooth manifolds $\pi_I(\mu, n)$ and $\pi_{NI}(\mu, n)$. Where

$$\pi_I(\mu, n) = \frac{W(1-\alpha)(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}}}{\mu(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu)} - k^*(\mu) - K$$

$$\pi_{NI}(\mu, n) = \frac{W(1-\alpha)}{\mu(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu)}.$$

The continuity of $\pi_{NI}(\mu, n)$ and $\pi_I(\mu, n)$ implies the continuity of the locus of intersection.

Second step:[26] Equilibrium at $\underline{n}$ and $\overline{n}$ requires that $\pi_I = A(1+k)^{\frac{-\alpha\rho}{1-\alpha}} - k - K$ and $\pi_{NI} = A$ are equal. Hence, if $\Pi \equiv \pi_{NI} = \pi_I$, then

$$\Pi = \Pi(1+k)^{\frac{-\alpha\rho}{1-\alpha}} - k - K.$$

Taking the derivative with respect to n yields

$$\frac{\partial\Pi}{\partial n} = \frac{\partial\Pi}{\partial n}(1+k)^{\frac{\alpha\rho}{1--\alpha}} + \Pi\frac{\alpha\rho}{1-\alpha}(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\partial k}{\partial n} - \frac{\partial k}{\partial n}.$$

Noting that $\pi_{NI} = A$ allows k to be written as $k = \left(\frac{\Pi\alpha\rho}{1-\alpha}\right)^{\frac{1-\alpha}{1-\alpha\rho-\alpha}} - 1$, which after substituting into the above expression for $\frac{\partial\Pi}{\partial n}$ and applying the Envelope Theorem, yields

$$\frac{\partial\Pi}{\partial n} = \frac{\partial\Pi}{\partial n}\left(\frac{\Pi\alpha\rho}{1-\alpha}\right)^{\frac{\alpha\rho}{1-\alpha\rho-\alpha}}.$$

Toward a contradiction, suppose $\frac{\partial\Pi}{\partial n} \neq 0$ for some n. Then it must be that $\frac{\Pi\alpha\rho}{1-\alpha} = 1$ for these n, the continuity of the $\pi(n)$ function implies that $\frac{\partial\Pi}{\partial n}$ is defined at these points and hence $\frac{\partial\Pi}{\partial n} = 0$, contradicting the starting assumption. Hence, by contradiction, $\frac{\partial\Pi}{\partial n} = 0$. Note that, since $k = \left(\frac{\Pi\alpha\rho}{1-\alpha}\right)^{\frac{1-\alpha}{1-\alpha\rho-\alpha}} - 1$, $\frac{\partial\Pi}{\partial n} = 0$ implies $\frac{\partial k}{\partial n} = 0$.

Third step. Verify that $\lim_{x\to\underline{n}}\widehat{\pi}(x) = \widehat{\pi}(\underline{n})$ and that $\lim_{x\to\overline{n}}\widehat{\pi}(x) = \widehat{\pi}(\overline{n})$. This is simple to see by evaluating $\pi_I(0, \overline{n})$ and $\pi_{NI}(0, \overline{n})$, and $\pi_I(\underline{n}, \underline{n})$ and $\pi_{NI}(\underline{n}, \underline{n})$, using the appropriate definition of K from the proof of Proposition 1. ∎

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Proof of Proposition 3. Point (i) is straightforward from Proposition 2, as if $c < \underline{c}$, no firm invests in equilibrium and the equilibrium profits are $\frac{W(1-\alpha)}{n}$, which is decreasing in n. Point (ii) follows from the fact that, if $c > \overline{c}$, from Proposition 2, all firms invest and $k^* = \frac{W\alpha\rho}{n} - 1$, which is decreasing in n. Finally, let us then focus on (iii). By Proposition 2, if $c \in (\underline{c}, \overline{c})$, there exists an asymmetric equilibrium where a measure $\widehat{\mu}$ of firms invests in R&D. Note that, since $k = \left(\frac{A\alpha\rho}{1-\alpha}\right)^{\frac{1-\alpha}{1-\alpha\rho-\alpha}} - 1$, from the proof of Proposition 2 $\frac{\partial \pi_{NI}}{\partial n} = \frac{\partial A}{\partial n} = 0$ implies $\frac{\partial k}{\partial n} = 0$. This establishes that R&D intensities stays constant.

Lastly, since a drop in c leads to an increase in n, we need to show that $\frac{\partial \mu}{\partial n} < 0$. As was established in the proof of Proposition 2:

$$\frac{\partial \pi_{NI}}{\partial n} = \frac{\partial A\left(n, \mu\left(n\right), k\left(n\right)\right)}{\partial n} = 0,$$

where

$$\frac{\partial A\left(n, \mu\left(n\right), k\left(n\right)\right)}{\partial n} = \frac{\partial A}{\partial n} + \frac{\partial A}{\partial k}\frac{\partial k}{\partial n} + \frac{\partial A}{\partial \mu}\frac{\partial \mu}{\partial n} = 0.$$

Now, from Lemma 1, $\frac{\partial A}{\partial \mu} < 0$, and from above, $\frac{\partial k}{\partial n} = 0$. Hence

$$\text{sign}\left(\frac{\partial A}{\partial n}\right) = \text{sign}\left(\frac{\partial \mu}{\partial n}\right).$$

Since, trivially, $\frac{\partial A}{\partial n} < 0$ it follows that $\frac{\partial \mu}{\partial n} < 0$. $\blacksquare$

Proof of Proposition 5. Step 1: We begin by showing that, for *fixed n^* and μ*, $\frac{\partial k}{\partial \gamma} < 0$ and $\frac{\partial A}{\partial \gamma} > 0$. To see this, note that, from the first-order condition of the optimal investment problem, we have

$$\frac{W\alpha\rho(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}}{n^* + \mu\left[(1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]} - \gamma = 0.$$

Applying the Implicit Function Theorem, we get

$$\frac{\partial k}{\partial \gamma} = \frac{\left\{n^* + \mu\left[(1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]\right\}^2}{W\alpha\rho(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\left\{\frac{\alpha\rho-1+\alpha}{(1-\alpha)(1+k)}\left[n - \mu + \mu(1+k)^{\frac{\alpha\rho}{1-\alpha}}\right] - \frac{\mu\alpha\rho}{1-\alpha}(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\right\}} < 0. \quad (11)$$

Also, note that, since $A = \frac{W(1-\alpha)}{n^* + \mu\left[(1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]}$, we have

$$\frac{\partial A}{\partial \gamma} = \frac{-W\mu\alpha\rho(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\partial k}{\partial \gamma}}{\left\{n^* + \mu\left[(1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]\right\}^2} > 0.$$

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Now, if all firms invest, the equilibrium investment is $k = \frac{W\alpha\rho}{n^*\gamma} - 1$, and $A = \frac{W(1-\alpha)}{n^*(\frac{W\alpha\rho}{n\gamma})^{\frac{\alpha\rho}{1-\alpha}}}$. Note

that $\frac{W\alpha\rho}{n^*\gamma} > 1$ since $\frac{W\alpha\rho}{n^*} > 1$. Thus, following Step 1 of Proposition 1, a firm is better off investing than not investing if

$$\frac{W}{n}[1 - \alpha - \alpha\rho - \left(\frac{W\alpha\rho}{\gamma n}\right)^{\frac{-\alpha\rho}{1-\alpha}}(1-\alpha)] + \gamma \geq K. \tag{12}$$

Let $\underline{n}''$ be the n which satisfies condition (12) with equality, so that if $n^* < \underline{n}''$ in the post-subsidy environment all firms invest in equilibrium. Similarly, if no firm invests, we have $A = \frac{W(1-\alpha)}{n^*}$, and the optimal investment for an investing firm is

$$k^* = \left(\frac{W\alpha\rho}{n^*\gamma}\right)^{\frac{1-\alpha}{1-\alpha\rho-\alpha}} - 1.$$

Thus, a firm is better off not investing than investing if

$$K \geq \left(\frac{W\alpha\rho}{\gamma n}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}}\left[\frac{(1-\alpha)\gamma}{\alpha\rho} - \gamma\right] - \frac{W(1-\alpha)}{n} + \gamma. \tag{13}$$

Let $\overline{n}'' = \min\{\frac{W\alpha\rho}{\gamma}, \widetilde{n}\}$, where $\widetilde{n}$ satisfies (13) with equality, so that if $n^* > \overline{n}''$, in the post-subsidy industry no firm invest in equilibrium. It is easy to check that $1 - \alpha - \alpha\rho > 0$ implies that $\overline{n}'' > \underline{n}''$. Similarly, it is easy to check that $\overline{n}'' > \overline{n}$ and $\underline{n}'' > \underline{n}$ by observing that $\frac{\partial \overline{K}}{\partial \gamma} > 0$ and $\frac{\partial \underline{K}}{\partial \gamma} > 0$ and using the fact that as K increases, $\overline{n}$ and $\underline{n}$ decrease. Application of the Envelope Theorem along the same lines of the proof of Proposition 2 leads easily to show that $\frac{\partial \Pi}{\partial n} = 0$ for all $n \in [\underline{n}'', \overline{n}'']$.

Step 2: Let us consider each region in turn.

(1) $n^* < \underline{n}$ (case c_1 in Figure 5): In this case, both the pre-subsidy and the post-subsidy equilibria are symmetric ones in which all firms invest. From the first order condition of the optimal investment problem, it is easy to show that $k^* = \frac{W\alpha\rho}{n^*\gamma} - 1$. Thus, the post-subsidy equilibrium profit is

$$\widehat{\pi}'(n^*) = \frac{W(1-\alpha-\alpha\rho)}{n^*} + \gamma - K < \frac{W(1-\alpha-\alpha\rho)}{n^*} + 1 - K = \widehat{\pi}(n^*).$$

Hence, the post-subsidy equilibrium profits at n^* are lower than the pre-subsidy ones, and exit is induced. Since the effects of $\gamma < 1$ and exit push in the same direction, R&D intensity per firm must rise.

(2) $n^* \in [\underline{n}, \underline{n}'']$. The pre-subsidy equilibrium is an asymmetric one (with measure of investing firms μ^*), while in the new one all firms invest. For a fixed pre-subsidy equilibrium n^*, we have $\widehat{\pi}''(n^*) < \widehat{\pi}''(\underline{n}) < \widehat{\pi}(\underline{n}) = \widehat{\pi}(n^*)$, where the second inequality comes from case (1), and the last

equality from the characterization of function $\widehat{\pi}(n)$. Hence, the post-subsidy equilibrium profits at n^* are lower than the pre-subsidy ones, and exit is induced.

(3) $n^* \in [\underline{n}'', \overline{n}]$ (Case c_2 in Figure 3). In this case, the equilibrium is asymmetric both pre- and post-subsidy. For any $\gamma \in (0,1]$, the equilibrium profit is $\widehat{\pi}(n^*) \equiv \pi_I(\mu^*) = \pi_{NI}(\mu^*) = A$, and

$$\pi_I(\mu^*) = A(1+k^*)^{\frac{\alpha\rho}{1-\alpha}} - \gamma k^* - K.$$

Thus, we have

$$\begin{aligned}\frac{\partial \widehat{\pi}(n^*)}{\partial \gamma} &= \frac{\partial \widehat{\pi}(n^*)}{\partial \gamma}(1+k^*)^{\frac{\alpha\rho}{1-\alpha}} + \frac{A\alpha\rho}{1-\alpha}(1+k^*)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\partial k^*}{\partial \gamma} \\ &\quad -k - \gamma\frac{\partial k^*}{\partial \gamma}.\end{aligned}$$

By the Envelope Theorem, we have

$$\frac{\partial \widehat{\pi}(n^*)}{\partial \gamma} = \frac{\partial \widehat{\pi}(n^*)}{\partial \gamma}(1+k^*)^{\frac{\alpha\rho}{1-\alpha}} - k^*,$$

which implies

$$\frac{\partial \widehat{\pi}(n^*)}{\partial \gamma} = \frac{k^*}{\left[(1+k^*)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]} > 0.$$

In turn, this guarantees that the equilibrium profit decreases as the subsidy increases (γ decreases).

(4) Suppose that $n^* \in [\overline{n}, \overline{n}'']$. In this region, we have that the pre-subsidy equilibrium is one in which no firm invests, while the post-subsidy equilibrium at n^* is an asymmetric one in which a measure of firms $\widehat{\mu}$ invests. Thus, for a fixed n^*, we have $\pi_{NI}(0) > \pi_{NI}(\widehat{\mu}) > \pi''_{NI}(\widehat{\mu})$, and, again, the post-subsidy equilibrium profits are lower than the pre-subsidy ones. Thus, exit is induced. To quantify the impact on equilibrium R&D intensity, note that pre-subsidy no investment occurred, while some measure μ engage in investment post-subsidy.

(5) $n^* > \overline{n}''$, the subsidy has no effect on the equilibrium profits or investment as no firms invest.

The rest of the proposition proceeds as per the proof of Proposition 4. ∎

Derivation of expenditure function in Section 5: Note that, from the utility function, we have $m = u - \frac{W}{\alpha}\ln[\mu x^\alpha + (n-\mu)y^\alpha]$, $x = \frac{\alpha W(1+k)^{\frac{\rho}{1-\alpha}}}{\left(\mu(1+k)^{\frac{\alpha\rho}{1-\alpha}}+n-\mu\right)}$, $y = \frac{\alpha W}{\left(\mu(1+k)^{\frac{\alpha\rho}{1-\alpha}}+n-\mu\right)}$, while prices are $(p_m, p_x, p_y) \equiv \left(1, \frac{(1-k)^{-\rho}}{\alpha}, \frac{1}{\alpha}\right)$. Thus, after some manipulation, the expenditure function can be written as

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$$e\left(p_x, p_y, u\right) = u - \frac{W\left(1 - \alpha\right)}{\alpha} \ln\left[n + \mu\left(\left(1 + k\right)^{\frac{\alpha\rho}{1-\alpha}} - 1\right)\right] - W \ln \alpha W + W.$$

∎

Proof of Proposition 6. We have:

$$\Delta CS\left(\mu, k, n\right) = \frac{W\left(1 - \alpha\right)}{\alpha} \ln\left[\mu^1\left(1 + k^1\right)^{\frac{\alpha\rho}{1-\alpha}} + \left(n^1 - \mu^1\right)\right]$$
$$- \frac{W\left(1 - \alpha\right)}{\alpha} \ln\left[\mu^0\left(1 + k^0\right)^{\frac{\alpha\rho}{1-\alpha}} + \left(n^0 - \mu^0\right)\right].$$

Clearly, when $\left\{p_x^0, p_y^0\right\} \equiv \left\{p_x^1, p_y^1\right\}$, $\Delta CS = 0$. We start by showing that whether the market is originally in a symmetric or asymmetric equilibrium a decrease in c results in CS being (weakly) decreasing.

(i) First, let us show that in the regions where equilibria are symmetric

$$\frac{\partial CS\left(\mu, k, n\right)}{\partial c} < 0. \tag{14}$$

First, we consider the region with all firms engage in R&D. Recall that, in this case $k = \frac{W\alpha\rho}{n} - 1$, and we have

$$
\begin{aligned}
\frac{\partial CS\left(\mu, k, n\right)}{\partial c} &= \frac{\partial}{\partial n}\left(\frac{W\left(1 - \alpha\right)}{\alpha} \ln\left[n\left(1 + k\right)^{\frac{\alpha\rho}{1-\alpha}}\right]\right)\frac{\partial n}{\partial c} \\
&= \frac{\partial n}{\partial c}\frac{W\left(1 - \alpha\right)}{\alpha}\frac{\left[\left(1 + k\right)^{\frac{\alpha\rho}{1-\alpha}} + n\frac{\alpha\rho}{1-\alpha}\left(1 + k\right)^{\frac{\alpha+\alpha\rho-1}{1-\alpha}}\frac{\partial k}{\partial n}\right]}{\left[n\left(1 + k\right)^{\frac{\alpha\rho}{1-\alpha}}\right]} \\
&= \frac{\partial n}{\partial c}\frac{W\left(1 - \alpha - \alpha\rho\right)}{\alpha n} < 0.
\end{aligned}
$$

where $\frac{\partial n}{\partial c} < 0$ is implied by Proposition 3. Second, we consider the region where no firm engages in R&D. Here we have

$$
\begin{aligned}
\frac{\partial CS\left(\mu, k, n\right)}{\partial c} &= \frac{\partial}{\partial n}\left(\frac{W\left(1 - \alpha\right)}{\alpha} \ln n\right) \\
&= \frac{\partial n}{\partial c}\frac{W\left(1 - \alpha\right)}{\alpha n} < 0.
\end{aligned}
$$

Let us now consider an asymmetric equilibrium. We have

$$
\begin{aligned}
\frac{\partial CS\left(\mu,k,n\right)}{\partial c} &= \frac{\partial n}{\partial c}\frac{\partial}{\partial n}\left(\frac{W\left(1-\alpha\right)}{\alpha}\ln\left[n+\mu\left[\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right]\right]\right)\\
&= \frac{\partial n}{\partial c}\frac{W\left(1-\alpha\right)}{\alpha\left[\mu\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}+\left(n-\mu\right)\right]}\left(1+\frac{\partial\mu}{\partial n}\left(\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right)\right.\\
&\quad\left.+\frac{\mu\alpha\rho}{1-\alpha}\left(1+k\right)^{\frac{\alpha+\alpha\rho-1}{1-\alpha}}\frac{\partial k}{\partial n}\right)\\
&= \frac{\partial n}{\partial c}\frac{W\left(1-\alpha\right)}{\alpha\left[\mu\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}+\left(n-\mu\right)\right]}\left(1+\frac{\partial\mu}{\partial n}\left(\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right)\right)
\end{aligned}
$$

where the last equality comes from $\frac{\partial k}{\partial n}=0$ as shown in Proposition 3. Thus, for $\frac{\partial CS(\mu,k,n)}{\partial c}=0$ it needs to be shown that $\frac{\partial CS(\mu,k,n)}{\partial n}=0$, or

$$
\frac{\partial\mu}{\partial n}\left(\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right)+1=0. \tag{15}
$$

Given $\frac{\partial\mu}{\partial n}<0$, (15) is equivalent to

$$
\left|\left[\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right]\frac{\partial\mu}{\partial n}\right|=1. \tag{16}
$$

However, note that the fact that the R&D intensity is constant in an asymmetric equilibrium region (as proved in point (iii) of Proposition 3), easily implies that condition (16) is satisfied.

(ii) In the rest of the proof, we will denote by $CS(n)$ and $CS'(n)$ the consumer surplus of the pre- and post- subsidy economy, respectively, when the industry size is n. Let us first consider a change *in the fixed R&D cost* K. Note that in a region where the pre-subsidy industry is in a symmetric equilibrium, the same analysis as in part (i) applies, and we have

$$
\frac{\partial CS(n)}{\partial K}=\frac{\partial}{\partial n}\left(\frac{W\left(1-\alpha\right)}{\alpha}\ln\left[n\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}\right]\right)\frac{\partial n}{\partial K}<0.
$$

Consider now the case in which the pre-subsidy industry is in a asymmetric equilibrium with industry size n^*. In this case, a subsidy to K changes the equilibrium k, μ and eventually n. However, since $\frac{\partial CS'(n)}{\partial n}=0$, we have

$$
\frac{\partial CS\left(n\right)}{\partial K}=\left.\frac{\partial CS\left(n\right)}{\partial K}\right|_{n=n^*}.
$$

In other words, since the consumers' surplus is constant on the flat segment of a given $\widehat{\pi}(n)$, we just need to evaluate how the consumers' surplus changes for a given n^* moving from $\widehat{\pi}(n^*)$ to $\widehat{\pi}'(n^*)$ (see Figure 3). Thus, we have

$$\left. \frac{\partial CS(n)}{\partial K} \right|_{n=n^*} = \frac{W(1-\alpha)}{\alpha \left[\mu(1+k)^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu) \right]} \left(\frac{\partial \mu}{\partial K} \left((1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1 \right) \right.$$
$$\left. + \frac{\mu\alpha\rho}{(1-\alpha)} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} \frac{\partial k}{\partial K} \right)$$
$$= \frac{W(1-\alpha)\frac{\partial \mu}{\partial K}}{\alpha \left[\mu(1+k)^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu) \right]} \left[\left((1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1 \right) \right.$$
$$\left. + \frac{\mu\alpha\rho}{(1-\alpha)} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} \frac{\partial k}{\partial \mu} \right].$$

Let us denote

$$D \equiv (1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1 + \frac{\mu\alpha\rho}{(1-\alpha)} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} \frac{\partial k}{\partial \mu}$$

and note that (5) in the proof of Lemma 1 implies that $D > 0$. However, from Proposition 4 we know that $\frac{\partial \mu}{\partial K} < 0$, Thus, we have

$$\left. \frac{\partial CS(n)}{\partial K} \right|_{n=n^*} = \frac{W(1-\alpha)\frac{\partial \mu}{\partial K}D}{\alpha \left[\mu(1+k)^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu) \right]} < 0.$$

Let us consider now a subsidy changing the variable R&D cost, γ. First, let us consider a region in which, in the pre-subsidy industry, all firms invest. In the symmetric case in which all firms invest we begin by noting that firms profits are given by

$$\widehat{\pi}(\gamma, n) = \frac{W(1-\alpha-\alpha\rho)}{n} + \gamma - K.$$

Holding the profit fixed at the original level (π^*) and allowing n to adjust as γ adjusts (that is, exploiting $\frac{\partial \pi(\gamma, n(\gamma))}{\partial \gamma} = 0$) yields

$$\left. \frac{\partial n}{\partial \gamma} \right|_{\pi=\pi^*} = \frac{n^2}{W(1-\alpha-\alpha\rho)}.$$

Now, we proceed by establishing that consumer surplus, holding profits fixed, is decreasing in γ. Since, $C'(n) > 0$ this effect must have the same sign as the actual effect and be a lower bound on the magnitude. Hence, we evaluate

$$\text{sign} \left(\left. \frac{\partial CS(n)}{\partial \gamma} \right|_{\pi=\pi^*} \right) = \text{sign} \left(\left. \frac{\partial n(1+k)^{\frac{\alpha\rho}{1-\alpha}}}{\partial \gamma} \right|_{\pi=\pi^*} \right).$$

We have

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$$\text{sign}\left[\frac{\partial n\left(1+k\right)^{\frac{\alpha\rho}{1-\alpha}}}{\partial\gamma}\bigg|_{\pi=\pi^*}\right] = \text{sign}\left[\left(\frac{1-\alpha-\alpha\rho}{1-\alpha}\right)\frac{\partial n}{\partial\gamma}\bigg|_{\pi=\pi^*} - \frac{n\alpha\rho}{\left(1-\alpha\right)\gamma}\right]$$

$$= \text{sign}\left[\frac{n}{\left(1-\alpha\right)W\gamma}\left(n\gamma - W\alpha\rho\right)\right] = \left(-\right) \text{ if } \gamma \leq 1$$

due to a symmetric investing equilibrium requiring that $\frac{W\alpha\rho}{n} > 1$ (see Proposition 1).

Let us now consider a pre-subsidy economy in an asymmetric equilibrium (where n^* is the industry size). Let us denote by $CS'(n)$ the post-subsidy consumer surplus at the industry size n. Note that, by Proposition 2, we have $\widehat{\pi}(\underline{n}) = \widehat{\pi}(\mu^*)$. By the step above and part (i) of this proposition, for the symmetric case, we have $\frac{\partial CS}{\partial\gamma}\big|_{\pi=\pi^*} < 0$, which allows to establish that the $CS'(\widetilde{n}) > CS(n^*)$ for $\widetilde{n}$ such that $\widehat{\pi}'(\widetilde{n}) = \pi^*$. However, by part (i) of this Proposition, we know that $CS'(n) > CS'(\widetilde{n})$ for all $n > \widetilde{n}$. In particular, $CS'(n^{**}) > CS'(\widetilde{n}) > CS(n^*)$, which concludes the proof.

∎

Panel 1: Summary Statistics

Year	No. Of Plants	Industry Revenue	No. of R&D Active Plants	Proportion doing R&D	Avg R&D Exp.	Total R&D Exp.
81	1279	234,795,584	339	26.5%	4,072	1,380,501
82	1350	289,466,529	373	27.6%	5,353	1,996,663
83	1469	366,308,102	328	22.3%	6,691	2,194,586
84	1710	525,547,715	342	20.0%	8,437	2,885,322
86	2808	803,053,684	517	18.4%	9,507	4,914,974
87	3023	971,443,116	635	21.0%	11,262	7,151,391
88	3127	1,183,736,845	722	23.1%	12,832	9,264,517
89	3121	1,751,324,399	815	26.1%	15,436	12,580,019
91	3184	1,736,267,113	925	29.1%	21,249	19,655,308
92	3437	2,156,047,175	999	29.1%	22,573	22,550,042
93	3671	2,666,828,799	1089	29.7%	25,026	27,253,825

Panel 2: R&D Expenditure Percentiles (Actual Expenditures)

Year	.75 percentile	.8 percentile	.85 percentile	.9 percentile	.95 percentile	Max
81	324	1,200	2,472	5,260	14,729	536,419
82	200	1,106	2,158	4,546	14,989	1,262,137
83	0	534	1,676	4,792	16,382	1,280,965
84	0	2	1,247	4,526	15,699	1,848,640
86	0	0	719	4,434	17,552	2,961,303
87	0	100	1,509	6,295	21,467	4,336,087
88	0	608	3,300	10,581	30,818	3,374,052
89	120	1,727	4,591	11,197	31,855	7,190,542
91	1,500	5,129	12,440	24,614	60,719	9,924,862
92	1,772	6,203	13,914	29,171	68,304	10,176,789
93	2,004	6,347	13,981	29,282	65,997	9,299,129

Panel 3: R&D Expenditure Percentiles (% of total R&D exp. incurred by plants in equal or lower percentiles)

Year	.75 percentile	.8 percentile	.85 percentile	.9 percentile	.95 percentile	Max
81	0.1%	1.1%	3.2%	7.5%	18.4%	100%
82	0.1%	0.6%	2.1%	5.3%	12.6%	100%
83	0	0.1%	0.9%	3.1%	9.7%	100%
84	0	0.0%	0.4%	2.0%	6.9%	100%
86	0	0	0.1%	1.2%	6.4%	100%
87	0	0.0%	0.3%	1.9%	7.4%	100%
88	0	0.1%	0.8%	3.2%	10.5%	100%
89	0.0%	0.3%	1.2%	3.7%	10.1%	100%
91	0.1%	0.8%	2.8%	7.0%	16.3%	100%
92	0.1%	1.0%	3.2%	7.6%	17.5%	100%
93	0.2%	0.9%	2.9%	6.9%	15.9%	100%

Notes: All expenditures are nominal thousands of TW$. Data unavaliable in 1985 and 1990. In Panels 2 and 3 all 70th percentile entries = 0

TABLE 1 {Label in tex file}

Panel 1: Summary Statistics

Year	No. Of Plants	Industry Revenue	No. of R&D Active Plants	Proportion doing R&D	Avg R&D Exp.	Total R&D Exp.
93	203	6,682,204	41	20.2%	1,906	78,153
94	229	9,646,433	42	18.3%	2,432	102,155
95	238	17,661,540	44	18.5%	3,691	162,402
96	261	17,044,308	40	15.3%	4,824	192,961

Panel 2: R&D Expenditure Percentiles (Actual Expenditures)

Year	.75 percentile	.8 percentile	.85 percentile	.9 percentile	.95 percentile	Max
93	0	1.8	48.9	188.6	888.2	148,801
94	0	0	28	123.2	310.8	177,022
95	0	0	15	124.3	542	360,338
96	0	0	1	150	907	559,258

Notes: All expenditures are nominal Korean Won (in millions)
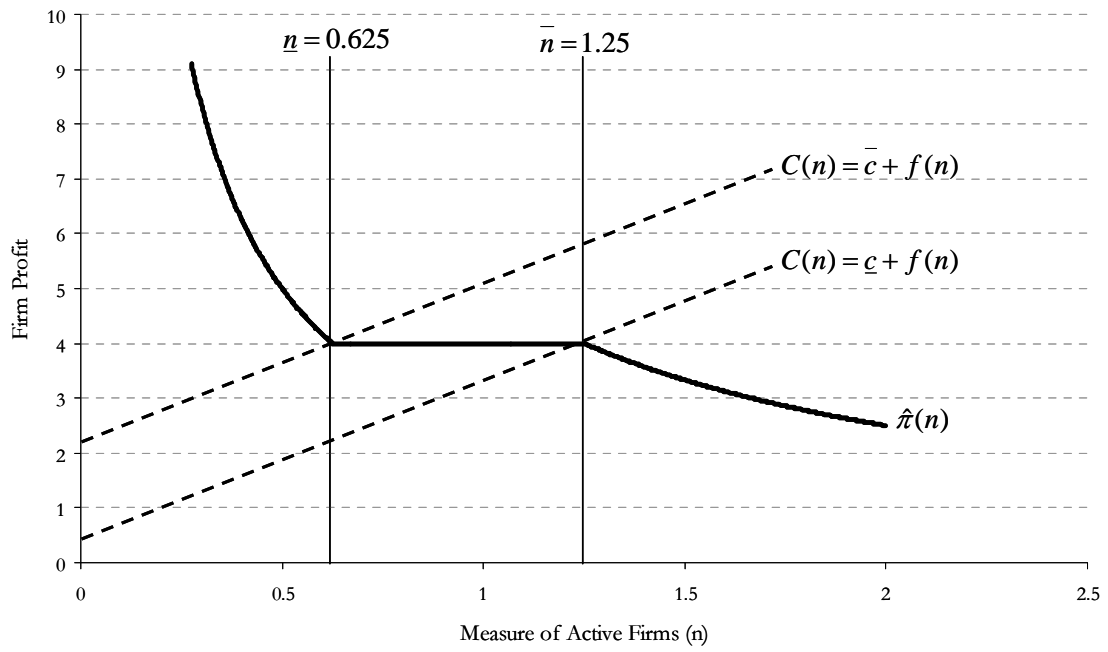
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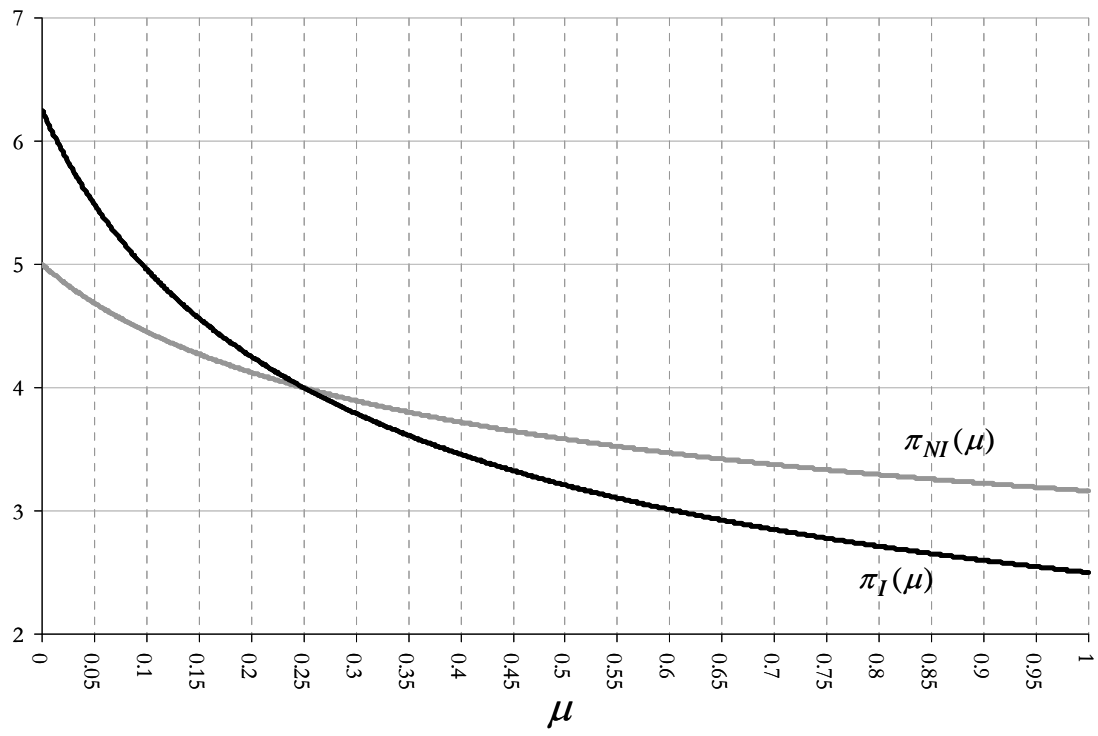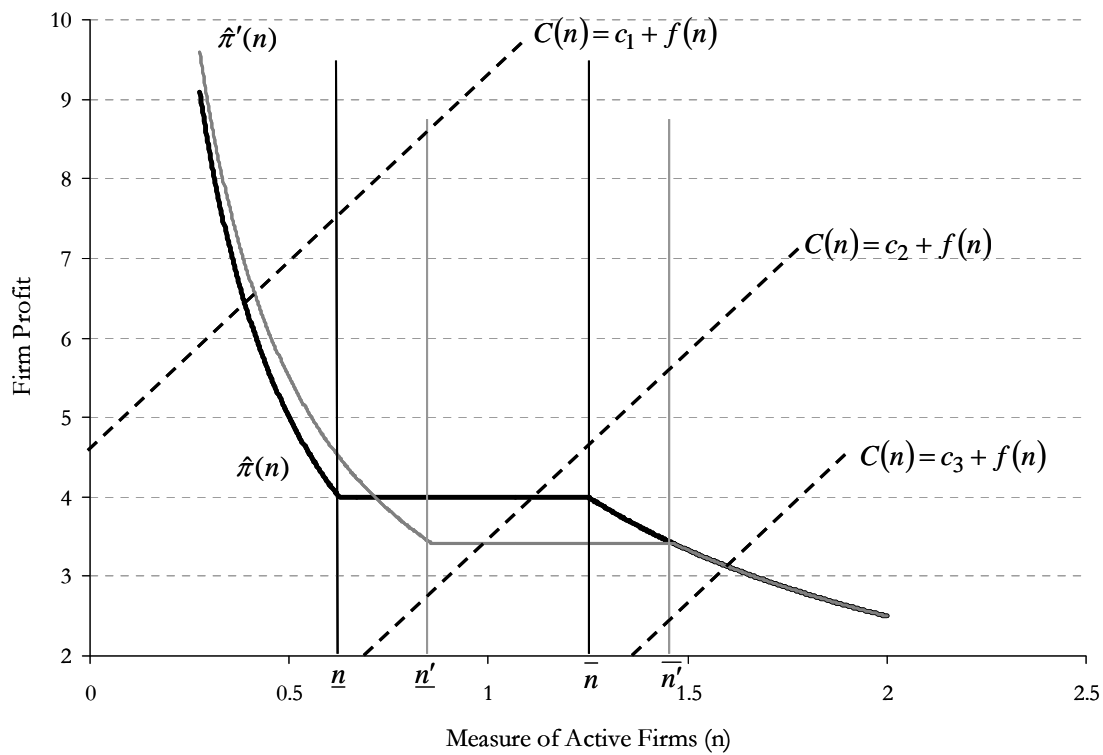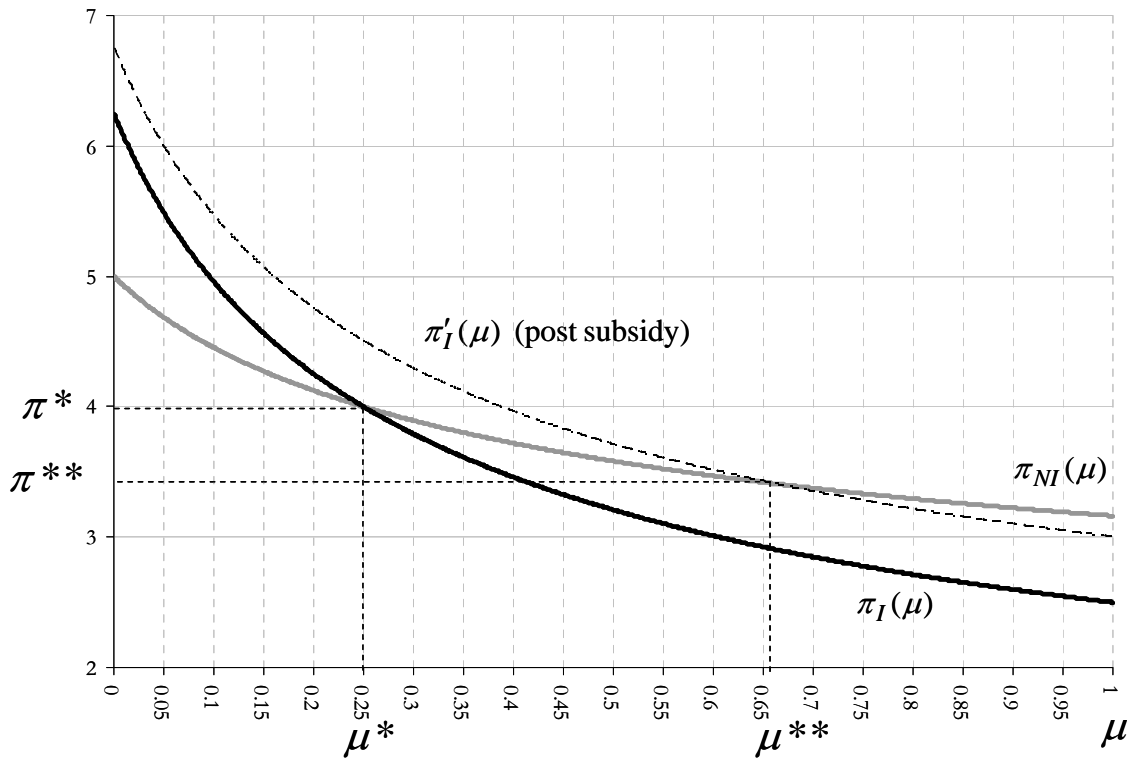
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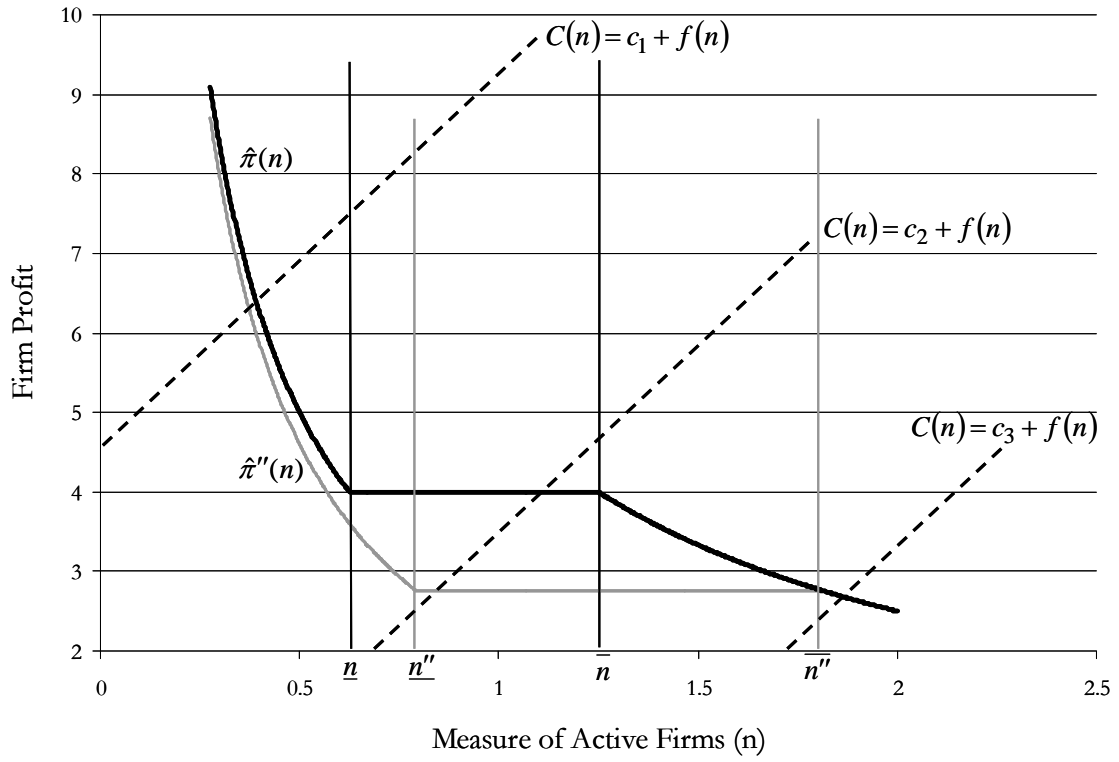
FIG3 {Label in tex file}

FIG4 {Label in tex file}

FIG5 {Label in tex file}