Lecture Notes on Auction Empirics with comments on other topics in Assymetric Information*

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1 Preliminaries

The next three lectures will be on recent emprical work on auctions. There are several reasons for teaching this vibrant area of research:

- it introduces the ideas behind indentification in structural models in a formal yet digestible way
- assymetric information is one of the most fertile areas for empirical work: auctions present an easily understandable environment for examining the impact of assym. info issue (ie we can see the rules of the game, understand the strategy set etc)
- my suspicion is that the tools developed in the auction literature could be adapted to other environments (e.g. flick through Laffont and Tirole for some examples of theory models in this spirit - whether they have direct empricial application is another question)
- auctions can be thought of as a kind of monopoly model (the auctioneer has monopoly power, but less than perfect information and limited capacity, for a similar view see Bulow and Klemperer)
- something like 10% of GDP is transacted through auction markets and their design and application has been, and continues to be, a topic of ongoing interest to both policy makers and business.

*A large part of these notes are based around the excellent survey of work on the identification of auction models written by Athey and Haile in 2005. This survey will appear as a chapter in the Handbook of Econometrics. Any errors contained in these notes are mine. Please let me know if you find any.

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Basically auctions are more central to IO than most economists think.

The pattern of these lectures will go something like

1. what is the point of empirical work on auctions? + themes
2. review of the theory
3. basic emprical results in first price auctions
4. basic empirical results in ascending auctions
5. extensions: auction heterogeniety & bidder heterogeniety

This section will have the assigments for the emprical section. What I will ask you to do is write a reaction paper to each of the following three papers:

1. Hendricks and Porter (this is one of the early seminal papers on auction emprics) (1988)
2. Haile and Tamer (2003) or Campo, Perrigne and Vong (2000)
3. Hortascu (2004) or Cantillon and Pesendorfer (2004)

   A reaction paper should be 1.5 - 2 pages long, should spend half a page outlining the paper and then comment on what about the paper you found striking (i.e. what made you think or what did you find interesting). It is better to be interesting than to comprehensively summarise the paper.

The reaction papers are due to me via email half an hour before the fourth, fifth and sixth classes.

Last Thing: read the Athey Haile survey paper.

2 Auction Empirics: Preliminary Comments

2.1 What is the point of empirical work on auctions?

There are several reasons for empirical work on auctions

- Validating basic assumptions: the theory makes a big deal of the role of assymetric information. To be confident that our models mean anything we should sepnd some time making sure that assymetric information does, indeed, matter. This is how I think of much of the contributions made by Hendricks and Porter

- Testing theory: Theory makes some pretty specific predictions about how model primitives map to outcomes. These seem worth testing. Experimental work has been successful here. Read the Handbook of Experimental Economics Chapter for an introduction to this fruitful area of experimental work.

- Evaluating policy: the optimality of design decisions usually depends on the properties on the underlying primitives we can divide these into two areas:

  - Uncovering the specific distribution of private information: in a FPSB IPV single unit auction the reserve price depends on the specific distribution of private information
  - Uncovering the properties of the structure of private information: in a single unit auction, the attractiveness of the FPSB auction depends on whether we are in the IPV or common value environment or some other private info structure.

The recent work has been on the last dot point mostly. I will talk mostly about the work that seeks to evaluate policy by uncovering the underlying distribution of private information. Testing private values vs common values tends to build on this anyway, often using auxilary data.

2.2 What are we really talking about?

The problem is: given bid data and observable characteristics of the auction and bidders, what can we say about the private infomation possesed by the bidders when they make bids?

Let's start by defining an auction:

The standard simple auction model has

- N bidders

- one indivisable good for sale

- each bidder has some private information represented by a parameter $\theta \in \mathbb{R}$. This sets either values or costs depending on whether we are in a procurement setting or not.

- the auctioneer has a utility function which is given by

$$V = v - p \qquad \text{or} \qquad V = p - v$$

  again depending on whether we are in a procurement setting or not (procurement is first)

- the bidders have a utility function which looks like

$$U_n = p - \theta \qquad \text{or} \qquad U_n = \theta - p$$

It should be obvious that having bidders as buyers or sellers is just a sign change. Since each are equally relevant empirically, I will use whichever is most appropriate for the application.

- we need a joint distribution for the bidders' private information so let $\{\theta_1 ... \theta_N\}$ be drawn from the joint CDF $F \in \mathbf{F}\cdot$
- the auction also needs some rules for allocating the winner and payments and saying what permitted bids are. Let $\mathbf{G}$ denote the set of all distributions over the space of permitted bids.

You should have covered the equilibrium theory of auctions in Micro last year. If you are hazy on that, look through Krishna's excellent text book which I recommend buying as a reference. I will cover it very fast today.

The equilibrium theory gives us a way to map from the private information to bids (which may or may not be the same as the prices (transfers) arising from the auction). Lets call this mapping from the theory $\gamma \in \Gamma$ where $\gamma : \mathbf{F} \to \mathbf{G}$.

2.2.1 Identification

The key idea of these lectures is identification. In most contexts what this means is that given an observed distribution of bids we can say identify a distribution of private information that is "most likely" or has the "best fit" or slightly more formally minimises the loss function of your choice.

I find it useful in structural modelling to draw the distinction between model identification and identification in data

Lets be formal about model identification:

Definition 1 (Identification) *A model* $(\mathbf{F}, \Gamma)$ *is identified if for every* $\left(F, \widetilde{F}\right) \in \mathbf{F}^2$ *and* $(\gamma, \widetilde{\gamma}) \in \mathbf{\Gamma}^2$ *,* $\gamma(F) = \widetilde{\gamma}\left(\widetilde{F}\right)$ *implies* $(F, \gamma) = \left(\widetilde{F}, \widetilde{\gamma}\right)$

This gives us some hope that with real data we can invert the mapping provided by the bid function to get the the private information and thus estimate F. Often we will take a stand on γ which will make the identifation of F much more tractable.

Lastly, bear in mind the difference between this view of identification, which is, in a sense, assymptotic, and the practical issue of identifaction in small samples which is the problem you always face when confronted with data (identification in data).

It is always possible that a model is identified in theory but places demands that the data at hand cannot meet. Always think about this when considering your own research and that of others. This often comes down to judgement but there is some science that can help: things you can do to develop some intuition about your situation

- run monte carlo simulations
- plot the data to examine the avaliable variation in exogenous variables
- look at what other researchers were able to do with similar technology and data

3 Review of Theory

Notation

- random variables in upper case, realisations in lower case
- vectors in bold
- latent variable CDF is $F_Y(\cdot)$
- observed variable CDF is $G_Y(\cdot)$
- where order statistics are used let:
  - $Y^{k:n}$ be the k^{th} order statistic from the sample $\{Y_{1,...,}Y_n\}$ with $Y_1 < Y_n$
  - $F_Y^{k:n}(\cdot)$ is the CDF of this order statistics

Basics

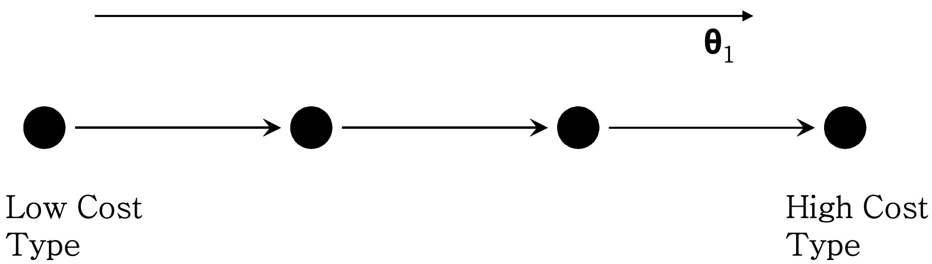
- we consider the sale of a single indivisable good to one of n bidders where $n \geq 2$
- bidders are risk neutral
- $\mathcal{N}$ is the set of bidders (where bidders are ex ante symmetric this is not so as important as $n = |\mathcal{N}|$ will be a sufficient statistic for our purposes)
- $\mathcal{N}_{-i}$ are the opponents of bidder i

Private Information
bidders' private information (or type) is a scalar random variable X_i, $\mathbf{X} = \{X_1, ..., X_n\}$

Aside on the scalar nature of signals

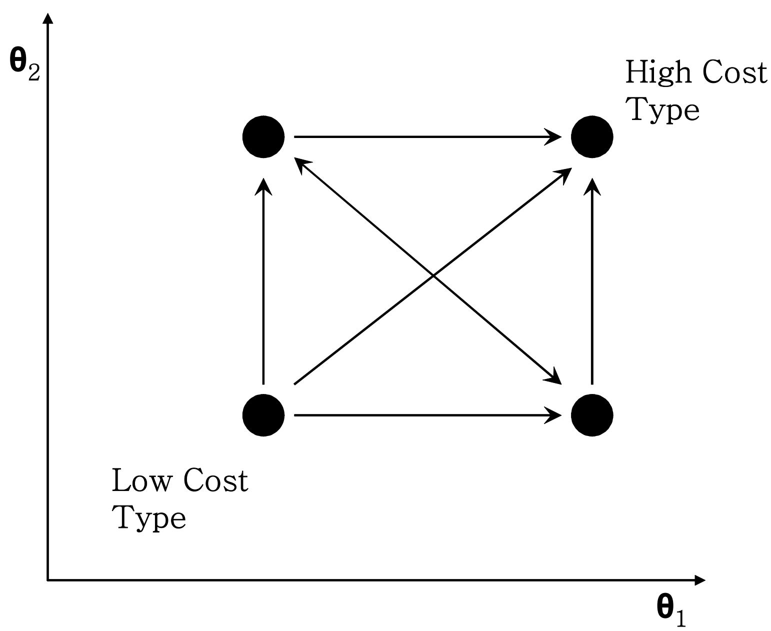
The fact X_i is a scalar is important as it leads to a natural complete ordering of the types from 'best' to 'worst'.

Consider a procurement setting:With four types and scalar info we have



so there is only really one way the IC constraints can bind. (cf Che 1993)

If suppliers have a cost function of the form $\theta_2 + c(q, \theta_1)$ the above diagram becomes



The arrows indicate the ways the downward IC constraints can bind. Note that now we only have a partial ordering over types.

The IC constraints can become a mess and under some mechanisms in some setting we can even have upward binding constraints.

That said, this is a relevant empirical structure for some procurement settings. Asker and Cantillon (2004, 2005) give a theoretical framework for handling it. See also Bajari, Houghton and Tadelis (2004)

Back to Signals

- we have a scalar signal X_i with realisation x_i
- we assume signals are informative in that

$$\frac{\partial E\left[U_i\left|X_i=x_i,X_{-i}=x_{-i}\right.\right]}{\partial x_i}>0 \qquad \forall x_{-i}$$

  so no matter what the signal of the other guys are, my signal always has an impact on the expectation of my utility. Among other things this rules out any one player having a perfect signal in, say, a mineral rights model.

- We often use U_i and X_i interchangably as one is just a monotonic transformation of the other. (note that in some procurement settings this will not make sense, but everything I talk about here works this way)
- Lastly, $\mathcal{N}$, $F_Y(\cdot)$ are common knowledge

Auction Terminology

The auction literature has been around for a long time and a terminology has grown up around it that you should know. So lets run through it.

Definition 2 (Private Values) *Bidders have private values if*

$$E\left[U_i\left|X_i=x_i,X_{-i}=x_{-i}\right.\right]=E\left[U_i\left|X_i=x_i\right.\right] \qquad \forall x_j \text{ and } U_i$$

Definition 3 (Common Values) *Bidders have common values if*

$$E\left[U_i\left|X_i=x_i,X_{-i}=x_{-i}\right.\right]$$

is strictly increasing in x_j for all i,j and x_j

- This division is not exhaustive, but is the space in which most people work. Note in particular that it does not include the work on auctions with externalities by Phillippe Jehiel, Benny Moldovanu and Ennio Stacchetti
- Common values can have the winners curse. The precise meaning of this differs depending on the literature you are reading. In most economics it means the possibility that people have some ex post regret from winning the auction. However, it can also refer to the behavoural phenomenon of people bidding too high relative to the equilibrium and making a loss in expectation.

- A further breakdown is between the independence of signals and affiliation of signals
- Independance means that

$$f_{X_i,X_j} = f_{X_i} f_{X_j}$$

where f_Z is the marginal distribution of Z

- Affiliation is defined as follows

Definition 4 (Affiliation) *A set of random variables* $\mathbf{Y} = \{Y_1, ..., Y_n\}$ *is said to affiliated if for all* $\mathbf{y}$ *and* $\widehat{\mathbf{y}}$ *(which are realisations of* $\mathbf{Y}$*)*

$$f_{\mathbf{Y}}(\mathbf{y} \vee \widehat{\mathbf{y}}) f_{\mathbf{Y}}(\mathbf{y} \wedge \widehat{\mathbf{y}}) \geq f_{\mathbf{Y}}(\mathbf{y}) f_{\mathbf{Y}}(\widehat{\mathbf{y}})$$

where $\vee$ *denotes the component-wise maximum and* $\wedge$ *denotes the component-wise minimum. (See Milgrom and Weber 1982 for more info)*

Affiliation means, very loosely, that the higher is my signal, the the more likely it is that yours is high.

Example:

Let

$$\mathbf{Y} = \left\{ \begin{array}{cc} 1 & 1 \\ 1 & 2 \\ 2 & 1 \\ 2 & 2 \end{array} \right\}$$

$$f_{\mathbf{Y}} = \left\{ \begin{array}{c} \frac{1}{4} + \varepsilon \\ \frac{1}{4} - \varepsilon \\ \frac{1}{4} - \varepsilon \\ \frac{1}{4} + \varepsilon \end{array} \right\}$$

$\mathbf{Y}$ is affiliated if $\varepsilon \geq 0$. You may notice that affiliation is equivalent to imposing a lattice-like structure on the structure of private information.

- So auctions that we often talk about are:
  - IPV: PV with U_i being independant
  - Symmetric IPV: $U_i \sim iid$
  - Affiliated PV (APV)
  - Pure Common Values: $U_i = U_o$
  - Mineral Rights: pure common rights with signals iid conditional on U_o

3.0.2 Applications

Think about how these models apply to the following applications:

1. OCS Oil Drilling Rights Auctions

2. Timber Auctions

3. Treasury Bill Auctions

4. Highway construction contract Auctions

5. Auctions of bus routes

3.1 Equilibrium Theory

Here I will run through what we need for the empirical models we want to explore.

- we divide auctions into the following formats depending on the rules governing who wins etc:

  - * FPSB
    * SPSB
    * Ascending
    * Clock (or Dutch)
    * Other (various multi-unit or combinatorial variants)

3.1.1 FPSB Auctions

- We use Perfect Bayesian Nash equilibrium in pure strategies

- denote the equilibrium bid as B_i, with realisation b_i. Let $\mathbf{B} = \{B_1, ..., B_n\}$

- assume affiliation for this section and also take no stand on private vs common values

- we assume some regularity (see Athey and Haile for details). Notably that values lie on some compact subset of the real line

- we would like some existence and uniqueness results to proceed: Here is what exists

- Existence:

  - $\exists$ an equilibrium with strictly increasing strategies except in some CV auctions with assymetric bidders

- Uniqueness:
  - $\exists$ a unique equilibrium in strictly increasing strategies in PV auctions as long as we have independence or symmetry (or both)
- In CV formats we tend to assume what we need. Note that we don't really need uniqueness, we just need everyone to be playing the same strictly increasing equilibirum.

Equilibrium a bidder with signal $X_i = x_i$ solves

$$\max_{\widetilde{b}} \left(E\left[U_i \middle| X_i = x_i, \max_{j \in \mathcal{N}_{-i}} B_j \leq \widetilde{b} \right] - b \right) \Pr\left(\max_{j \in \mathcal{N}_{-i}} B_j \leq \widetilde{b} \,|X_i = x_i \right) \tag{1}$$

that is, you set your bid to maximise the expected profit if you win (conditional on your signal) times the probability of winning (again, conditional on your signal)

Let

$$\widetilde{v}\left(x_i, m_i; \mathcal{N}\right) = E\left[U_i \middle| X_i = x_i, \max_{j \in \mathcal{N}_{-i}} B_j = m_i \right]$$

and

$$v\left(x_i, y_i; \mathcal{N}\right) = E\left[U_i \middle| X_i = x_i, \max_{j \in \mathcal{N}_{-i}} B_j = \beta_i\left(y_i; \mathcal{N}\right) \right]$$

this expression will turn out to be important as $v\left(x_i, x_i; \mathcal{N}\right)$ is the expected value of winning when i is pivotal.

Let

$$G_{M_i|B_i}\left(m_i \mid b_i; \mathcal{N}\right) = \Pr\left(\max_{j \neq i} B_j \leq m_i \mid B_i = b_i, \mathcal{N} \right)$$

this is the CDF of the maximum bid of the opposing bidders given i's bid and $\mathcal{N}$

Let $g_{M_i|B_i}\left(m_i \mid b_i; \mathcal{N}\right)$ be the corresponding conditional density

Given this notation we can rewrite (1) as

$$\max_{\widetilde{b}} \int_{-\infty}^{\widetilde{b}} \left(\widetilde{v}\left(x_i, m_i; \mathcal{N}\right) - \widetilde{b} \right) g_{M_i|B_i}\left(m_i \mid \beta_i\left(x_i; \mathcal{N}\right); \mathcal{N}\right) dm_i \tag{2}$$

Some technical stuff is required here to prove that this is differentiable a.e. See Athey and Haile, and Krishna's text for technical comments on these points.

We can differentiate (2) w.r.t. $\widetilde{b}$ to get

$$v\left(x_i, x_i; \mathcal{N}\right) = b_i + \frac{G_{M_i|B_i}\left(b_i \mid b_i; \mathcal{N}\right)}{g_{M_i|B_i}\left(b_i \mid b_i; \mathcal{N}\right)} \tag{3}$$

- This is the key equation from theory use for estimation purposes. Note the following
  - It is pretty nice - linear sturcture is simple (this is from risk neutrality)

- LHS is the latent variable we are interested in
- RHS has stuff that this observed (bids) or functions related to observed stuff, this gives hope for estimation.

3.1.2 Ascending Auctions

- Clock or button auctions are the framework used in most theory
- Unattractive for empirical work of very few ascending auctions work like this
- Even if this framework is OK we have to deal with the fact that we never see the value at which the highest bidder would drop out.
- However, we would see all but the first order statistic, which is a lot of information really.
- The key thing from theory, regardless of apporach used is that bidders never bid more than they are willing to pay, and never let anyone win with a price they are willing to beat.

4 Identification and Estimation of FPSB PV Auctions

- Early work by Harry Paarsch and others in the early 1990s explored ways of estimating using bid functions and imposing parametric forms on the distribution of private information.
- Other work by Laffont, Ossard and Vong (in various combinations), and Pat Bajari tried other approaches using different theoretical 'handles'
- The literature seems to have converged on the following basic line of attach due to Guerre, Perrigne and Vong (2000).

Basic idea:

in equilibrium each bidder is acting optimally against the distribution of behaviour of other bidders

How does this help?

- it means that the distribution of opponents bids in (3) can be inferred from the data without parametric restriction.
- ie: "given what others are doing you are doing the best you can *and* given what you are doing they are doing the best they can" - we are taking both parts of this statement very seriously

Lets be formal recall:

$$u_i = b_i + \frac{G_{M_i|B_i}(b_i \mid b_i; \mathcal{N})}{g_{M_i|B_i}(b_i \mid b_i; \mathcal{N})}$$

- first; note that this is the inverse of the bid function
- second; note that the b_i is observed
- third; note that $\frac{G_{M_i|B_i}(b_i|b_i;\mathcal{N})}{g_{M_i|B_i}(b_i|b_i;\mathcal{N})}$ is a ratio of properties of the joint distribution of opponents bids.

The invertability of the bid function is the key thing for indentification (i.e. we rely on the assumption that the bidders use a strictly increasing bid function)

Guerre, Perrigne and Vong (2000), Li, Perrigne and Vong (2002) and Campo, Perrigne and Vong (2003) work all this out for various scenarios.

Summarising: in the APV model, if symmetric just need all the bids, if not, then need the identities of the bidders

4.1 Estimation

How do you operationalise this?

What I want to do is take you through the standard non-parametric approach and then follow up with some comments.

Map of Approach

Say the data is from T auctions. All auctions are the same, although there is some variation in the number of bidders. The estimation proceeds in the following steps:

1. (a) Estimate the $\frac{G_{M_i|B_i}(b_i|b_i;\mathcal{N})}{g_{M_i|B_i}(b_i|b_i;\mathcal{N})}$ from bid data

   (b) Use this to compute $u_i = b_i + \frac{G_{M_i|B_i}(b_i|b_i;\mathcal{N})}{g_{M_i|B_i}(b_i|b_i;\mathcal{N})}$

   (c) Use the psuedo-sample of u_i to estimate F_U

There is an econometric issue that we need to confront: How do you estimate a density or CDF from data?

4.1.1 Aside: Kernel Estimation

- There are several ways to estimate a density from data. Kernels are the most commonly used method in this application. A good reference on this and other apporaches is Pagan and Ullah (1999)

- A kernel estimator is basically an adaption of the idea of a histogram to the case of data that has a support that is an interval of the real line (or for higher dimensional kernels, $\mathbb{R}^n$, where n should be fairly small or else you hit curse of dimensionality problems)
- Descrete data: say you have a random variable x such that the support of x is the set $\{1, 2, 3\}$. To get the density of x you would count the number of times it falls into each bin and then divide these counts by the number of observations. This is consistent etc.
- Data with a Continuous Support
  - This is where kernels come in
  - The idea of a kernel is that we can learn about the value of the density function at a point v, $f(v)$, by looking at how common it is to see realisations of the random variable near this point.

Details

- Imagine we had data on the realisations of a random variable, drawn from a unknown distribution $F(\cdot)$ and want to know the value of the density $f(\cdot)$ at a point x. The data is $\{x_1, ..., x_n\}$
- The closest thing to constructing a histogram we might do is to pick an interval around the point $\widetilde{x}$, and count how many observations fall in this interval.
- Our estimator would look like

$$\widehat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n}\mathbb{I}\left(x - \frac{h}{2} \leq x_i \leq x + \frac{h}{2}\right)$$

where $\mathbb{I}(\cdot)$ is an indicator function and h is a parameter setting the bandwidth. We can rewrite this as

$$\begin{aligned}\widehat{f}(x) &= \frac{1}{nh}\sum_{i=1}^{n}\mathbb{I}\left(-\frac{1}{2} \leq \frac{x_i - x}{h} \leq \frac{1}{2}\right) \\ &= \frac{1}{nh}\sum_{i=1}^{n}K(\psi) \quad \text{where} \quad \psi = \frac{x_i - x}{h}\end{aligned}$$

This is our first kernel estimator. The function $K(\cdot)$ is the kernel, which in this case is a indicator function which equals when when ψ lies in the interval $\left[\frac{-1}{2}, \frac{1}{2}\right]$.

- This kernel is not attractive as it steps, that is, it is not smooth. It has the advantage that it integrates to one, however. Smoothness is generated by choosing a kernel such that

$$\int_{\mathbb{R}} K(\psi)\, d\psi = 1$$

Some examples of commonly used kernels are:

1. standard normal: $K(\psi) = (2\pi)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\psi)^2\right]$

2. epanechnikov: $K(\psi) = \frac{3}{4}\left(1-\psi^2\right)$ if $|\psi| < 1$, zero otherwise (this minimises Intergrated Mean Squared Error)

Many others exist.

A few things about small sample properties:

Kernel estimates are typically biased. The bias is a function of the bandwidth, the kernel and the density to be estimated. Bias can generally be reduced by choosing a smaller bandwidth at the cost of increasing variance in the estimates. Proceedures for generating bias-reducing kernels exist but require that we allow the kernel to take negative values.

How to select bandwidth:

There are two ways to proceed: by plotting the density implied from the density and seeing if it looks "good". Using the data to select the optimal bandwidth by imposing some criteria (like integrated square error[1] or its expectation). Both are easy to impliment, for details see Pagan and Ullah.

Asymptotic Properties

Under regularity conditions and some assumptions on the DGP kernel estimates are CAN. See P&U for these assumptions

The key points are that

1. i.i.d draws are useful but not required (you can handle time series type issues)

2. convergence is different from a parametric estimator which means you have to be a bit careful when using them with parametric estimators in a multi-step proceedure.

3. The assymptotic variance generates a 95% confidence interval that looks like

$$f \pm 1.96\,(nh)^{-\frac{1}{2}}\left[f(x)\int K^2(\psi)\, d\psi\right]^{\frac{1}{2}}$$

4. bootstraps are usually used for small-sample confidence intervals.

[1] ISE is $\int\left(\widehat{f}(x) - f(x)\right)^2 dx$

4.1.2 Application to the FPSB PV Problem

lets start with the simplest case: FPSB with a symmetric IPV specification

1. (a) $F_{\mathbf{U}} = \prod_{i=1}^{n} F_{U_i}$ This makes step (c) easier

   (b) due to independence:

$$\begin{aligned} G_{M_i|B_i}(m_i \mid b_i; \mathcal{N}) &= G_{M_I}(m_i \mid n) \\ &= \Pr\left(\max_{j \neq i} B_j \leq m_i \mid n\right) \end{aligned}$$

   (c) this changes the estimated equation to

$$u = b + \frac{G_B(b \mid n)}{(n-1)\, g_B(b \mid n)}$$

   where notation has been abused a little bit

   i. $G_B(b \mid n)$: marginal distribution of equilibrium bids in n bidder auctions

   ii. $g_B(b \mid n)$: the associatied density

   (d) This formulation makes estimation straightforward using kernels

$$\widehat{G}_B(b \mid n) = \frac{1}{T_n} \sum_{t=1}^{T} \sum_{i=1}^{n} \chi\{b_{it} \leq b, n_t = n\}$$

$$\widehat{g}_B(b \mid n) = \frac{1}{nT_n h_g} \sum_{t=1}^{T} \sum_{i=1}^{n} K\left(\frac{b - b_{it}}{h_g}\right) \chi\{n_t = n\}$$

   so this gives us step (a)

   (e) then

$$\widehat{u} = b + \frac{\widehat{G}_B(b \mid n)}{(n-1)\, \widehat{g}_B(b \mid n)}$$

   gives us step (b)

   (f) then

$$\widehat{f}(u) = \frac{1}{T_n h_f} \sum_{t=1}^{T} \frac{1}{n_t} \sum_{i=1}^{n} K\left(\frac{u - \widehat{u}_{it}}{h_f}\right)$$

- the assymptotic distribution of these estimators are "largely unresolved".
- the easiest way to side-step the issue is use the bootstrap (although always check that you are ok to use it: depending on the dgp you may need to make some adjustments)

4.1.3 Algorithm for Estimation

1. take each observed b_o
2. estimate $\widehat{G}_B(b_o \mid n)$ and $\widehat{g}_B(b_o \mid n)$ using the entire data set
3. infer $\widehat{u}(b_o)$
4. once the above is done for each bid, estimate $\widehat{f}(u)$ and draw a picture of it
5. then do whatever policy stuff you want

4.1.4 Other issues

- – for the theory to work we need a compact support of u_i. Issues arise when the estimator is near the boundary of the support - consistency is no longer assured. Li, Perrigne and Vong (2002, p 180 on) sort this out
  - while nonparametric estimation is very attractive ex ante it may be that the data set you are facing works better with the extra structure imposed by a parametric specification. That is more structure may give you more power. In considering whether to go parametric think about what you want to use the estimates for, where identification is coming from and whether any auxillary data can but used to justify the parametric assumption.
  - there is still a lot of structure being imposed on the data here, particularly in stages (b) and (c). Take some time to think about how much work the functional form assumptions are doing in these stages.
  - lastly, and most importantly, note the big assumptions on auction heterogeniety, bidder heterogeniety etc. More on this later...

4.1.5 Dealing with Affiliation

Now lets confront the more general model

1. $$u_i = b_i + \frac{G_{M_i|B_i}(b_i \mid b_i; \mathcal{N})}{g_{M_i|B_i}(b_i \mid b_i; \mathcal{N})}$$
2. we use a trick (employing bayes rule) to transform the $\frac{G_{M_i|B_i}(b_i|b_i;\mathcal{N})}{g_{M_i|B_i}(b_i|b_i;\mathcal{N})}$ part into something that is easier to estimate.

3. let

$$G_{M,B}(m,b;n) = G_{M|B}(m \mid b;n)\, g_B(b;n)$$
$$g_{M,B}(m,b;n) = g_{M|B}(m \mid b;n)\, g_B(b;n)$$

where g_B is the marginal density of a bidders equilibrium bid. Note that $G_{M,B}(m,b;n)$ is not a CDF.

4. note that

$$\frac{G_{M,B}(m,b;n)}{g_{M,B}(m,b;n)} = \frac{G_{M_i|B_i}(b_i \mid b_i;n)}{g_{M_i|B_i}(b_i \mid b_i;n)}$$

5. so the kernel estimates in step (a) are

$$\widehat{G}_{M,B}(m,b;n) = \frac{1}{nT_n h_g} \sum_{t=1}^{T} \sum_{i=1}^{n} K\left(\frac{b-b_{it}}{h_g}\right) \chi\{m_{it} \le b, n_t = n\}$$

6.

$$\widehat{g}_{M,B}(m,b;n) = \frac{1}{nT_n h_g^2} \sum_{t=1}^{T} \sum_{i=1}^{n} K\left(\frac{b-b_{it}}{h_g}, \frac{b-m_{it}}{h_g}\right) \chi\{n_t = n\}$$

where m_{it} is the maximum of bidder i's opponents bids

7. step (c) is a bit more complicated as well in that we have to estimate a joint density. So

$$\widehat{f}(u) = \frac{1}{T_n h_f^n} \sum_{t=1}^{T} K\left(\frac{u-\widehat{u}_{1t}}{h_f}, ..., \frac{u-\widehat{u}_{nt}}{h_f}\right) \chi\{n_t = n\}$$

Comments:

- - really this is very similar to the IPV case but for the trick to set up the densities at the beginning
  - the demands on data here are considerably tougher. Since estimating bi-variate kernels in step (a). Step (c) is even more demanding given that you are dealing with a potentially higher dimensional kernel. The problem is that the curse of dimensionality make higher dimensional kernels very data internsive.
  - all this suggests some structure on private information, imposed ex ante, will greatly increase power (if they are justifiable).

4.1.6 Dealing with Bidder Assymetry

- The assymetry that we deal with here is differences in the distributions of private infomation (as opposed to covariates observed by other bidders, which we deal with later)
- with symmetry the markdown term was

$$\frac{G_{M_i|B_i}(b_i \mid b_i; n)}{g_{M_i|B_i}(b_i \mid b_i; n)}$$

  without symmetry it is

$$\frac{G_{M_i|B_i}(b_i \mid b_i; \mathcal{N})}{g_{M_i|B_i}(b_i \mid b_i; \mathcal{N})}$$

- the natural apporach, building on what we have done before is to use a kernel estimator such that

$$\widehat{G}_{M,B}(m, b; \mathcal{N}_t) = \frac{1}{T_{\mathcal{N}} h_g} \sum_{s=1}^{T} K\left(\frac{b - b_{is}}{h_g}\right) \chi\left\{m_{it} \leq b, \mathcal{N}_t = \mathcal{N}_s\right\}$$

$$\widehat{g}_{M,B}(m, b; \mathcal{N}_t) = \frac{1}{T_{\mathcal{N}} h_g^2} \sum_{s=1}^{T} K\left(\frac{b - b_{is}}{h_g}, \frac{b - m_{is}}{h_g}\right) \chi\left\{\mathcal{N}_t = \mathcal{N}_s\right\}$$

- this allows each bidder to have their own distribution from which they draw their private information.
- If you think about this apporach for just a second you will realise that it requires a bucket load of data. You need to see the same set of bidders interacting again and again...
- Often we can group bidders together into say class A and class B. Now we just need to make sure that the auctions we use to estimate each density have the same number of class A and B bidders in them. The rest of the adjustments should, by now, be straightforward.
- A good example of this apporach is the Campo, Perrigneand Vong (2003) paper - lets have a look at it

involves two or three firms. A number of arguments for joint bidding have been given in the literature. For instance, joint bidding can weaken financial constraints, reduce costs by pooling cartel members' information and capital through the joint venture and spread risks among firms. See e.g. DeBrok and Smith (1983), Millsaps and Ott (1985), Gilley *et al.* (1985) and Hendricks and Porter (1992). As noted by many economists, however, joint bidding may have introduced some *ex ante* asymmetry among bidders.

Because joint bidding is negligible in the 1950s–1960s, our study focuses on auctions held between December 1972 and 1979.[12] Because of data requirements explained subsequently, we consider auctions with two bidders who can be either joint or solo. This gives a total of 227 auctions from which 55 auctions have two solo bids, 60 auctions have two joint bids and 112 auctions have one solo bid and one joint bid. Among the latter, 63 auctions are won by the joint bidder. Using a normal approximation, the ratio 63/112 is greater than 1/2 at the 10% significance level in a one-sided test, where 1/2 would be the expected ratio if the two participants have equal chance of winning.[13] Thus joint bidding has increased the probability of winning suggesting some *ex ante* asymmetry among participants.

For each wildcat auction, we know the date, the acreage of the tract, the number of bidders, their bids in constant 1972 dollars and whether the bid is a solo or a joint bid. Table I gives some summary statistics in $ per acre for the 454 bids considered in our empirical study as well as on solo and joint bids separately, whether the opponent's bid is of the same type or of a different type.

A first feature revealed by the means displayed in Table I is that joint bids tend to be higher on average than solo bids, as a number of empirical studies have found. Moreover, joint bidders tend to bid higher when they face a joint bidder than when they face a solo bidder. Likewise, though their bids are lower than those of joint bidders, solo bidders tend to bid on average higher when they face a joint bidder than when they face another solo bidder. This suggests that the bidding strategy of each type of bidder depends on the type of their opponent. This could arise from bidders taking into account some possible asymmetry in their bidding strategies. For instance, a test of the equality of means for solo bids versus joint bids in the 112 auctions with one bidder of each type gives a t-statistic equal to 1.66, which (weakly) rejects their equality. It is also interesting to note that the within variability of solo versus solo bids is much smaller than the within variability

Table I. Summary statistics on bids

Variable	# Obs	Mean	STD	Minimum	Maximum	Within STD
All bids	454	687.30	1,431.31	19.51	20,751.32	1,258.91
Joint bids	232	837.32	1,717.54	21.46	20,751.32	—
Solo bids	222	532.53	1,033.20	19.51	11,019.08	—
Joint vs joint	120	875.13	2,056.12	33.94	20,751.32	2,011.99
Joint vs solo	112	796.83	1,266.32	21.46	6,377.94	—
Solo vs joint	112	603.28	1,226.61	19.51	11,019.08	—
Solo vs solo	110	456.45	788.19	20.80	7,009.10	747.43

[12] We exclude auctions after 1979 since the rules of the auction mechanism have changed somewhat after this date. We also exclude the unique sale held in 1970 and the first sale in 1972 because the water depth of the tracts sold at these sales was much greater than usual.

[13] Hereafter, all tests are conducted at the 10% significance level.

private values than solo bidders with a relatively more important variability for the former. As pointed out in Section 3.2, these differences can be explained by unobserved tract heterogeneity and differences between joint and solo bidders. This issue is further investigated below.

As Figure 2 does not provide information on the affiliation between private values within the same auction whether they are both joint or solo, it is useful to test for their independence. We use the non-parametric test proposed by Blum *et al.* (1961) (BKR hereafter), which is consistent and distribution free. For two variables X and Y, the test statistic is equal to $(1/2)\pi^4 B$, with $B = N^{-4}\sum_{\ell=1}^{N}(N_1(\ell)N_4(\ell) - N_2(\ell)N_3(\ell))^2$, with N the number of observations and $N_1(\ell), N_2(\ell), N_3(\ell), N_4(\ell)$ the numbers of points lying respectively in the regions $\{(x, y)|x \leq X_\ell, y \leq Y_\ell\}$, $\{(x, y)|x > X_\ell, y \leq Y_\ell\}$, $\{(x, y)|x \leq X_\ell, y > Y_\ell\}$ and $\{(x, y)|x > X_\ell, y > Y_\ell\}$. To impose symmetry among bidders of the same type, we duplicate the observations so that $N = 2 \times L$. We find a test statistic equal to 6.57 using observed bids and to 4.69 using trimmed private values for joint bidders. For solo bidders, we obtained a test statistic equal to 9.52 using observed bids and equal to 6.92 using trimmed private values. The null hypothesis of independence is clearly rejected in all cases.

Case $(n_1, n_0) = (1,1)$

The potentially asymmetric case is estimated using the 112 auctions with both types. Because there is only one bidder of each type, (4) and (5) simplify as B_1^* and B_0^* are void. In particular, their denominators reduce to the conditional densities $g_{b_0|b_1}(b_1|b_1)$ and $g_{b_1|b_0}(b_0|b_0)$. Hence, (4) and (5) reduce to

$$v_1 = \xi_{10}(b_1) = b_1 + G_{b_0|b_1}(b_1|b_1)/g_{b_0|b_1}(b_1|b_1) \tag{18}$$

$$v_0 = \xi_{01}(b_0) = b_0 + G_{b_1|b_0}(b_0|b_0)/g_{b_1|b_0}(b_0|b_0) \tag{19}$$

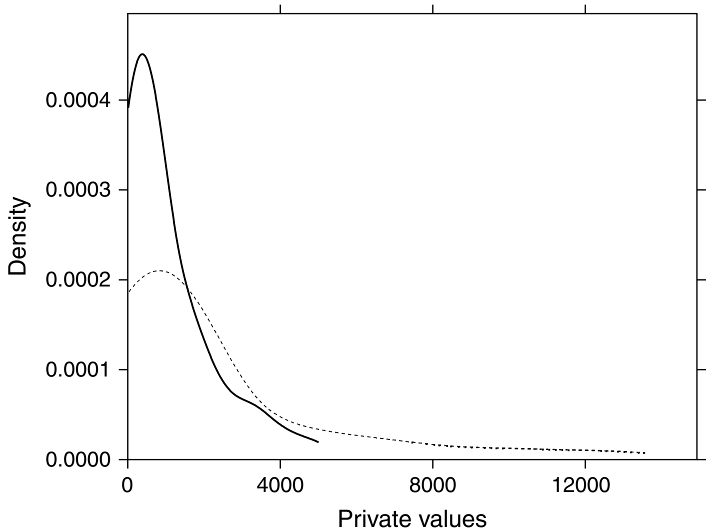


Figure 2. Marginal densities of private values

$2195.14 per acre with a standard deviation equal to $3024.10 and a range of [$33.94;$13 605.86]. For the (0,2) case, these numbers are $1027.90, $1170.15 and [$26.89; $5031.39], respectively from the 98 trimmed private values.

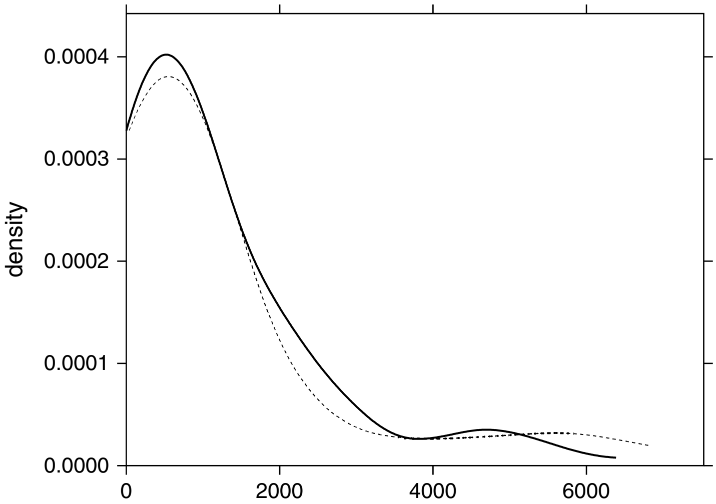


Figure 4. Marginal densities of private values

asymmetry though weak between joint and solo bidders as the empirical cumulative distribution functions slightly differ with a single crossing.[22]

Such an asymmetry leads the solo bidders to shade less their private values than joint bidders, as found in Figure 3, so as to increase their probability of winning the auctions. See also Maskin and Riley (2000a) and Pesendorfer (2000). However, the shading effect does not counterbalance fully the asymmetry in terms of valuation distributions, as indicated by the bid averages for joint versus solo and solo versus joint in Table I and the empirical probability of winning. As is well known, the aggressiveness of the weak bidder relative to the strong bidder may introduce some inefficiency in the auction in the sense that the winner of the auction has the lowest valuation. It turns out that this does not happen in our data set, which can be explained by the relatively weak asymmetry and the important variability of private values within each auction.

It is interesting to compare these results to the first two cases where bidders are of the same type. Figure 5 displays the inverse bidding strategies for a joint bidder when facing a joint bidder ($\hat{\xi}_{11}(\cdot)$) and when facing a solo bidder ($\hat{\xi}_{10}(\cdot)$), the former being to the right of the latter. Given a same tract valuation, a joint bidder will bid more aggressively when facing a joint bidder than when facing a solo bidder. For, the joint bidder faces less 'competition' when facing a solo bidder who is more likely to draw a lower private value. Figure 6 displays the inverse bidding strategies for a solo bidder when facing a solo bidder ($\hat{\xi}_{00}(\cdot)$) and when facing a joint bidder ($\hat{\xi}_{01}(\cdot)$), the former being to the left of the latter. Thus, a solo bidder will bid slightly more aggressively when facing a joint bidder than when facing a solo bidder to compensate for his lower private value. These results confirms the descriptive statistics of Table I. Both figures indicate that bidders have integrated the type of their opponents in their bidding strategies.

[22] The graph is available upon request from the authors. A Kolmogorov–Smirnov test does not reject the equality of the c.d.f.s on either private values or bids. Note, however, that the Kolmogorov–Smirnov test is based on the independence of the two samples. This is not the case as joint and solo private values (or bids) are affiliated, which decreases the power of the test.

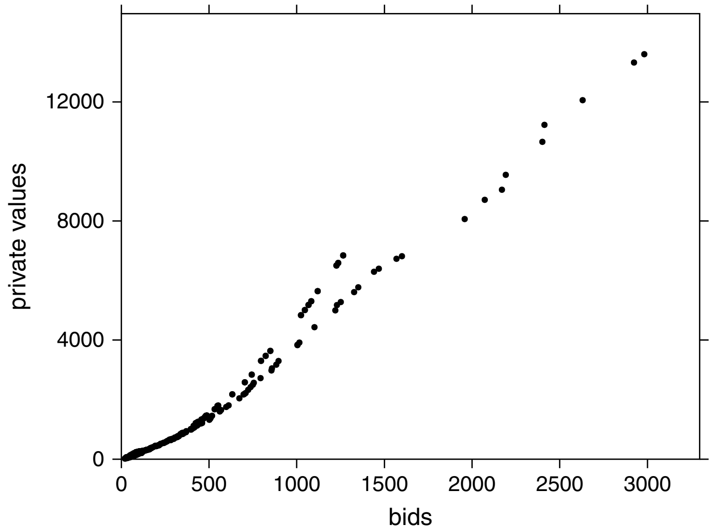


Figure 5. Inverse bidding strategies of joint bidders

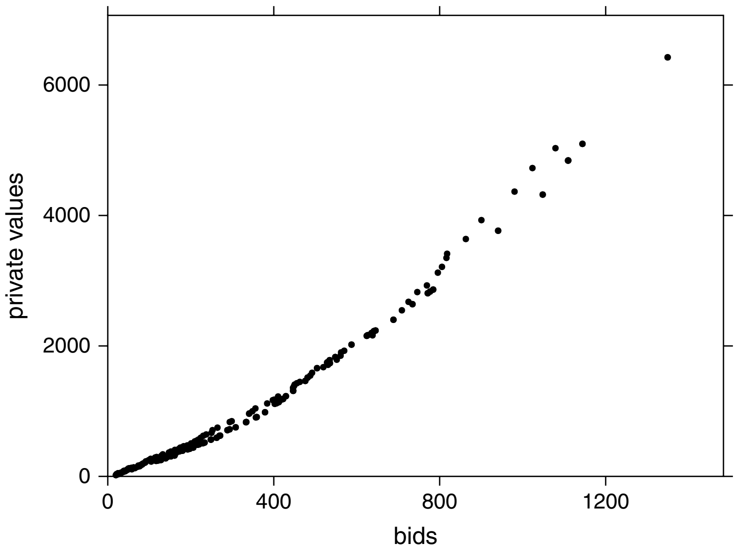


Figure 6. Inverse bidding strategies of solo bidders

Lastly, as indicated in Section 3.2, the comparisons of $\hat{f}_1^{(2,0)}(\cdot)$ and $\hat{f}_1^{(1,1)}(\cdot)$ as well as of $\hat{f}_0^{(0,2)}(\cdot)$ and $\hat{f}_0^{(1,1)}(\cdot)$ provide some information about unobserved tract heterogeneity. A Kolmogorov–Smirnov test gives a test statistic equal to 0.1917 and 0.0965 for the former and latter comparisons, respectively. This represents a clear rejection of $\hat{f}_1^{(2,0)}(\cdot) = \hat{f}_1^{(1,1)}(\cdot)$. Under the assumptions of Section 3.2, this means that there is some unobserved tract heterogeneity as tracts attracting two joint bidders differ significantly from tracts attracting one joint bidder and one solo

4.1.7 Incomplete data:

- - Often you just observe the transaction data from FPSB auctions and little else.
  - Some results exist on indentification suggesting that a feasible estimator may exist. Indeed a simple approach may be to exploit the properties of order statistics. Athey and Haile has a little discussion of this.
  - To my knowledge no-one has really implemented an estimator with incomplete data to get at a substantive issue.

5 Ascending Auctions

- Ascending auctions are often modelled as button auctions
- these are pretty poor descriptors of what the data generating process really looks like since the information transmisson in a button auction is too good.
- Haile and Tamer investigate a bounds approach to which is elegant and likely to have application in other assymetric information problems where the data generating process is not perfectly modelled.
- They start with the following two assumptions

1. (a) i. [A1] Bidders do not bid more than they want to pay
   ii. [A2] Bidders do not let someone win at a proce they are willing to beat

- - note: this allows jump bidding, bids $\neq$ valuations, bidders that merely watch the action without entering a bid etc
  - the idea here is to use these restrictions to provide partial identification of valuation. That is, provide bounds.

Formally:

1. Assume we see the bids made by all the bidders in the data set.
2. [A1] is equivalent to $b_i \leq u_i \quad \forall i$
3. it follows that in an n-bidder auction $b^{(i;n)} \leq u^{(i;n)}$ [this is easy but not immediate]
4. it follows that

$$G_B^{(i;n)}(u) \geq F_U^{(i;n)}(u) \;\; \forall i, u, n \tag{4}$$

5. now we need to do a little statistics:

6. the distribution of an order statistic from an iid sample of size n from an arbitary distribution $F(\cdot)$ has distribution

$$F^{(i:n)}(s) = \frac{n!}{(n-i)!\,(i-1)!} \int_0^{F(s)} t^{i-1}(1-t)^{n-i}\,dt$$

7. since the RHS is strictly increasing in $F(\cdot) \in [0,1]$, $F^{(i:n)}(s)$ uniquely determines a value for $F(s)\ \forall s$

8. this is a useful result since it allows us to define the following function, ϕ, implicitly. ϕ is just the mapping between $F^{(i:n)}(s)$ and $F(s)$

$$H = \frac{n!}{(n-i)!\,(i-1)!} \int_0^{\phi} t^{i-1}(1-t)^{n-i}\,dt \qquad H \in [0,1]$$

so that

$$F_U(u) = \phi\left(F_U^{i:n}(u); i, n\right) \tag{5}$$

9. since $\phi : [0,1] \to [0,1]$ is strickly increasing (4) and (5) give

$$\phi\left(G_B^{i:n}(u); i, n\right) \geq F_U(u)$$

so, given an estimate of $G_B^{i:n}(u)$ we can get an upper bound on $F_U(u)$. Lets set the estimation of $G_B^{i:n}(u)$ aside for one moment.

10. We have several versions of $G_B^{i:n}(u)$, so we have a bunch of upper bounds to choose from. What we do is choose the least upper bound (the most informative bound)

$$F_U^{+}(u) = \min_{i,n} \phi\left(G_B^{i:n}(u); i, n\right)$$

11. The lower bound is similar, although we have slightly less data to work with...

12. [A2] implies that all losing bidders have valuations less than $b^{n:n}+\Delta$, where Δ is the minimum bid increment

13. this implies

$$u^{n-1:n} < b^{n:n} + \Delta$$

14. this gives

$$G_\Delta^{n:n}(u) \leq F_U^{n-1:n}(u) \qquad \forall n, u$$

where $G_\Delta^{n:n}(u)$ is the distribution of $B^{n:n} + \Delta$

15. now things proceed as above, but for the fact we only have $|\{\underline{n}, ..., \overline{n}\}|$ lower bounds. Also we should look for the greatest lower bound.

Comments

- - This seems a pretty flexible approach: do not really need all the bids, just the transaction price.
  - Also, note the importance of knowing the number of bidders. Without this it is a bit hard to exploit the order statistics.
  - It might be possible to proceed if you had a distribution over the possible number of bidders.

5.0.8 Estimation

- for this to work we need estimates of $G_B^{i:n}(u)$ and $G_\Delta^{n:n}(u)$
- as before we use non-parametric estimates which, given these are CDFs, are actaully pretty trivial
- the estimators are

$$
\begin{aligned}
G_B^{i:n}(b) &= \frac{1}{T_n}\sum_{t=1}^{T} \varkappa\left\{n_t = n, b^{i:n_t} \leq b\right\} \\
G_\Delta^{n:n}(b) &= \frac{1}{T_n}\sum_{t=1}^{T} \varkappa\left\{n_t = n, b^{n_t:n_t} + \Delta_t \leq b\right\}
\end{aligned}
$$

- after these estimates have been taken, we plug it into the formulae above to get the bounds (there is alittle bit of computation to be done here)
- the assymptotic distribution of the final estimates is a little weird due to the min and max operators but Haile and Tamer show that the bootstrap is able to be applied here.
- there is a problem in that in finite samples the upper and lower bounds may overlap. H&T discuss this and propose a solution that bascially involves taking wieghted averages rather than the min or max.

5.0.9 So What?

- The key issue is whether these bounds allow you to say anything useful about the world.
- The authors show that careful inspection of the basic auction model allows to say things about reserve prices (bound the optimal reserve price)

- An adaptation of Manski and Tamer allows them to say things about how valuations are affected by covariates
- The extent to which you can make useful inference about policy variables will depend on your apllication and model
- Lets have a look at the H&T results

TABLE 2
Gaps Between First- and Second-Highest Bids

Quantiles	High Bid	Gap	Minimum Increment	Gap ÷ Increment
10%	9,151	30	4.1	1.2
25%	22,041	92	10.1	6.9
50%	55,623	309	23.4	14.8
75%	127,475	858	52.1	20.0
90%	292,846	2,048	110.5	76.4

dollars) on the median tract.[23] Forest Service officials report that jump bidding is common. Table 2 provides some support, showing a gap between the highest and second-highest bid of several hundred dollars (roughly 10–20 times the minimum increment) in the majority of auctions. Since the cost of jump bidding—the risk that one wins with the jump bid and pays too much—is highest at the end of the auction, jump bidding is likely to be more significant early in the auctions. However, these gaps themselves are generally quite small relative to the total bid, suggesting that we may be able to obtain tight bounds.

B. *Reserve Price Policy*

The Forest Service's mandated objective in setting a reserve price is to ensure that timber is sold at a "fair market value," defined as the value to an "average operator, rather than that of the most or least efficient" (U.S. Forest Service 1992). Many observers have argued that Forest Service reserve prices fall short of this criterion and are essentially nonbinding floors (see, e.g., Mead, Schniepp, and Watson 1981, 1984; Haile 1996; Campo et al. 2000). Bidders, for example, claim that the reserve prices never prevent them from bidding on a tract (Baldwin et al. 1997). As discussed above, for our purposes it is sufficient to assume only that the actual reserve prices are below the profit-maximizing reserve prices.

There is an ongoing controversy over so-called below-cost sales—sales generating revenues insufficient to cover even the costs to the Forest Service of administering the contract (see, e.g., U.S. General Accounting Office 1984, 1990, 1991; U.S. Forest Service 1995). Obviously, this is possible only with reserve prices below profit-maximizing levels. However, reserve prices are not set with the goal of profit maximization nor

[23] Forest Service rules actually require only that total bids rise as the auction proceeds, although local officials often specified discrete increments. In the time period we consider, the 5 cent increment was a common practice in this region. Sometimes increments of 1 cent per MBF were used, and many sales used no minimum increment. We use the 5 cent increment since this results in a more conservative bound, although variations of this magnitude have very little effect on the results: 5 cents represents about 0.05 percent of the average bid in our sample.

TABLE 3
Summary Statistics

	Mean	Standard Deviation	Minimum	Maximum
Number of bidders	5.7	3.0	2	12
Year	1985.2	2.6	1982	1990
Species concentration	.68	.23	.24	1.0
Manufacturing costs	190.3	43.0	56.7	286.5
Selling value	415.4	61.4	202.2	746.8
Harvesting cost	120.2	34.1	51.1	283.1
Six-month inventory*	1,364.4	376.5	286.4	2,084.3
Zone 2 dummy	.88		0	1

* In millions of board feet.

are quite tight. The shape of the true distribution suggested by these bounds resembles a lognormal distribution, which has been used in several prior studies.

To construct estimates of bounds on the optimal reserve price, an estimate of v_0, the cost of allowing the harvest of the tract, is needed. We consider a range of possible values based on Forest Service estimates (U.S. Forest Service 1995; U.S. General Accounting Office 1999).[27] Table 4 shows the results of simulations used to evaluate the trade-offs between net revenues and the probability that a tract goes unsold with alternative reserve prices. Values of v_0 between \$20 and \$120 are considered and the implied bounds on the optimal reserve prices calculated. For each value of v_0, we consider three possible reserve prices: $\hat{p}_L$, $\hat{p}_U$, and the average of the two. The table reports simulated gains in profit per MBF relative to actual profits, using each value of v_0 as the measure of costs. This is done both assuming $F(\cdot|\mathbf{X}) = \hat{F}_L(\cdot|\mathbf{X})$ and assuming $F(\cdot|\mathbf{X}) = \hat{F}_U(\cdot|\mathbf{X})$, providing estimated bounds on the profit gains (losses) from using each reserve price considered. Note that lemma 4 enables us to use equilibrium bids in a second-price sealed-bid auction to obtain revenue predictions.

As foreshadowed by our simulations, despite the tightness of the bounds on $F(\cdot)$ in figure 8, the bounds on the optimal reserve price for each v_0 are fairly wide. Because the bounds on $F(\cdot)$ are tight, however, our estimates of the expected revenues obtained with reserve prices

[27] For sales in region 6 in 1993, the Forest Service estimated that costs of the timber sales program were between \$85 and \$113 per MBF (U.S. General Accounting Office 1999). On the basis of sales in 1990–92, nationwide cost-based reserve prices between \$18 and \$47 per MBF were suggested as appropriate (U.S. Forest Service 1995), depending on which timber sales program costs are to be covered by auction revenues. Both calculations include some costs that are sunk at the time of the auction and, therefore, should be excluded from v_0. However, other costs, such as forgone return on investment and adverse environmental impacts, are excluded. Obtaining more precise estimates of v_0, ideally as a function of tract characteristics, would be an important step toward a more definitive analysis of reserve price policies.

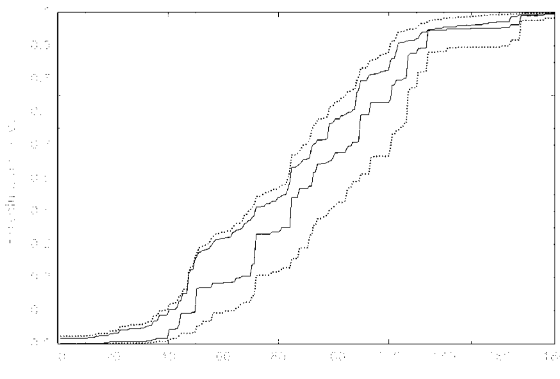


FIG. 10.—U.S. Forest Service timber auctions. Solid curves are estimated bounds, and dotted curves are bootstrap confidence bands.

between $\hat{p}_L$ and $\hat{p}_U$ differ little, with v_0 held fixed. The calculated bounds on the optimal reserve prices provide strong support for the assumption that the actual reserve price (around \$54) is well below the optimum. Even with $v_0 = 0$, the estimated lower bound on p^* is still slightly larger than the average actual reserve price. These results also suggest that, at least on average tracts in our sample, reserve prices could be raised considerably without causing many tracts to go unsold. Even if $F(\cdot) = \hat{F}_U(\cdot)$, a reserve price nearly twice the actual average would be required to drive the probability that a tract will go unsold past 15 percent—a key threshold given a Forest Service policy of ensuring that at least 85 percent of all offered timber volume is actually sold (U.S. Forest Service 1992).

The potential gains in profit from raising reserve prices obviously depend on v_0. With $v_0 = \$20$, for example, we estimate that gains would be less than 10 percent (and not necessarily positive) even when $F(\cdot) = \hat{F}_L(\cdot)$.[28] With $v_0 = \$80$, however, the potential gains are much larger. In that case, the Forest Service might achieve net gains of \$10 per MBF or more, which would represent more than an 80 percent increase in profits. With opportunity costs above the average gross revenues of \$92.08 per MBF, sales typically lead to a net loss. Hence, for costs of \$100 or \$120, substantial gains (reductions in losses) from im-

[28] Note that, in general, revenues need not be higher with a given reserve price between p_L and p_U given one particular CDF between $F_L(\cdot)$ and $F_U(\cdot)$. However, if $\Delta = 0$ or if Myerson's regularity condition is assumed, then lemma 4 implies that we can rule out the optimality of reserve prices that yield a (statistically significant) reduction in expected revenues when $F(\cdot) = F_L(\cdot)$ is assumed. This follows from the fact that a rightward shift in $F(\cdot)$ raises expected revenues at any reserve price. In our simulations, reductions in expected revenues appear for a few reserve prices, but only when $F(\cdot) = F_U(\cdot)$ is assumed.

TABLE 4
SIMULATED OUTCOMES WITH ALTERNATIVE RESERVE PRICES

	Reserve Price					
	p_L		$(p_L+p_U)/2$		p_U	
	Distribution of Valuations					
	F_L	F_U	F_L	F_U	F_L	F_U
Reserve price when v_0=\$20	62.40		86.02		109.65	
Change in profit	6.96	−2.78	6.67	−2.74	1.74	−18.57
Pr(no bids)	.00	.02	.07	.12	.19	.41
Reserve price when v_0=\$40	74.93		92.29		109.65	
Change in profit	7.64	−.61	7.61	−1.14	6.30	−10.04
Pr(no bids)	.03	.05	.11	.18	.19	.41
Reserve price when v_0=\$60	85.67		103.39		121.11	
Change in profit	9.23	1.92	12.04	3.14	7.21	−6.05
Pr(no bids)	.07	.12	.15	.28	.35	.58
Reserve price when v_0=\$80	98.20		112.34		126.48	
Change in profit	13.65	7.63	15.03	6.82	10.44	.96
Pr(no bids)	.13	.24	.28	.46	.46	.72
Reserve price when v_0=\$100	111.09		122.54		134.00	
Change in profit	20.09	15.94	21.65	16.87	17.00	14.30
Pr(no bids)	.28	.45	.45	.60	.67	.80
Reserve price when v_0=\$120	144.74		156.01		167.29	
Change in profit	32.06	31.31	33.72	31.64	31.56	28.87
Pr(no bids)	.84	.86	.84	.89	.88	.97

NOTE.—Profit and reserve price figures are given in 1983 dollars per MBF. See text for additional details.

posing higher reserve prices would be obtained by selling only tracts receiving unusually high bids. While revenue maximization is not the objective of the Forest Service timber sales program, these estimates suggest the magnitudes of revenues and costs that must be weighed against other objectives in determining optimal policy.

To evaluate the effects of auction observables on bidder valuations, we estimate the simple semiparametric model

$$v_{it} = \mathbf{X}_t\boldsymbol{\beta} + \epsilon_{it}$$

assuming $\text{med}[\epsilon_{it} \mid X_{it}] = 0$. Table 5 presents estimated bounds on the parameter vector $\boldsymbol{\beta}$. Following Manski and Tamer (2002), we construct confidence intervals using the bootstrap. Since zero lies outside the 95 percent confidence interval for each coefficient, we can reject the hypothesis that any of these conditioning variables has no effect on valuations. The implied signs are as expected: larger inventories, higher harvesting costs, or higher manufacturing costs reduce valuations. Greater species concentration and higher selling value of end products

6 Extensions to the basic framework

There are many extensions to this basic framework. I want to deal with the two that seem to me to be crucial to pretty much any empirical investigation you might wish to conduct. Mainly I discuss the FPSB auction.

These are:

- Auction Heterogeniety (both observed and unobserved)
- Bidder Heterogeniety (both observed and unobserved)

6.1 Observed Auction Heterogeniety - FPSB Auctions

- Basically the news here is good: All the identification results we have used before go through.
- Here I show how to handle observed heterogeniety in empirical implementation
- Let $\mathbf{Z}$ be a vector of auction covariates
  - Now the variables we have been playing with become:
  - $\beta_i(\cdot;\mathcal{N},\mathbf{Z})$, $F_{\mathbf{U}}(\cdot|\mathbf{Z})$, $G_{M_i|B_i}(b \mid b;\mathcal{N},\mathbf{Z})$, $g_{M_i|B_i}(b \mid b;\mathcal{N},\mathbf{Z})$
  - and the bidding function we use for estimation changes accordingly
- One approach to the previous estimation is to use standard kernel smoothing over covariates
  - This can also be used in the ascending auction application (see Haile and Tamer)
  - However as is often the case with kernels this apporaqch is vulnerable to curse of dimensionality problems
- An alternative suggested by Haile, Hong and Shum (2003) [and applied in Krasnokutskaya (2004), Bajari and Tadelis (2004), and Shneyerov (2005)] is as follows:
- We exploit the fact that additive seperability is preserved by equilibrium bidding
- Let

$$u_i = \Gamma(z_t) + a_{it}$$

where a_{it} is the bidders private information. (note multiplicative separability has also been explored)

- Let a normalisation exist such that

$$\Gamma(z_0) = 0$$

- so now (this is shown in HHS):

$$\begin{aligned} \beta_i(u_i; \mathcal{N}, \mathbf{z}) &= \Gamma(\mathbf{z}) + \beta_i(u_i; \mathcal{N}, \mathbf{z}_0) \\ &= \beta(\Gamma(\mathbf{z}) + a_{it}; \mathcal{N}) \end{aligned} \tag{6}$$

- Now we can write the inverse bid function as

$$a_{it} + \Gamma(\mathbf{z}_t) = \beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}) + \frac{G_{M_i|B_i}(\beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}) \mid \beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}); \mathcal{N})}{g_{M_i|B_i}(\beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}) \mid \beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}); \mathcal{N})} \tag{7}$$

- Now we have to note a few things which are due to the additive separability of the last few equations
  - the events $\{\beta_i(\Gamma(\mathbf{z}) + A_i; \mathcal{N}) = \beta_i(\Gamma(\mathbf{z}) + a_i; \mathcal{N})\}$ and $\{\beta_i(\Gamma(\mathbf{z}_0) + A_i; \mathcal{N}) = \beta_i(\Gamma(\mathbf{z}_0) + a_i; \mathcal{N})\}$ are equivalent for any $\mathbf{z}$.
  - the events $\left\{\beta_j(\Gamma(\mathbf{z}) + A_j; \mathcal{N}) = \beta_i(\Gamma(\mathbf{z}) + a_i; \mathcal{N})\right\}$ and $\left\{\beta_j(\Gamma(\mathbf{z}_0) + A_j; \mathcal{N}) = \beta_i(\Gamma(\mathbf{z}_0) + a_i; \mathcal{N})\right\}$ are also equivalent for any $\mathbf{z}$ and $j \neq i$.
  - it follows that the expression

$$\frac{G_{M_i|B_i}(\beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}) \mid \beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}); \mathcal{N})}{g_{M_i|B_i}(\beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}) \mid \beta_i(\Gamma(\mathbf{z}_t) + a_{it}; \mathcal{N}); \mathcal{N})}$$

    is invariant to $\mathbf{z}_t$.

- The upshot is that (6) implies (7) holds for all $\mathbf{z}_t$ whenever it is for $\mathbf{z}_t = \mathbf{z}_0$
- The whole point of this is that auction heterogeniety can now be controlled for by what amounts to a hedonic regression
  - Let

$$b_{it} = \alpha(\mathcal{N}) + \Gamma(\mathbf{z}_t) + \varepsilon_{it}$$

  - This is estimated using standard regression techniques.
  - From this little regression we get a homogenized bid

$$b_{it}^h = b_{it} - \widehat{\Gamma}(\mathbf{z}_t)$$

  - Then we run through the by now usual approach.

- The only technical point to note is that equilibrium bidding implies that the distribution of the sampling error, ε_{it}, should vary with $\mathcal{N}$. Hence the final stage, where the distribution of the private information is estimated, should be done separately for each $\mathcal{N}$

6.2 Unobserved Auction Heterogeniety

- The problems here are that things which are observed by all the bidders but not by the econometrician that affects valuations, and these things vary accross auctions.

- There are at least three issues here:

1. (a) whether this auction heterogeniety is empirically distinguishable from other assumptions about the private information (i.e. is and IPV auction iwth unobserved heterogeniety distinguishable from an APV auction?)
   (b) is the distribution of the private information identified (assuming you know the dgp)?
   (c) is the identifiation adequate? That is, can we use the inference to answer useful questions if we don't see the unobserved stuff. This will depend on the project you have in mind.

- This is an area where more applied econometric work would be useful.

6.2.1 Dealing with it in a FPSB Auction

- currently the state-of-the-art for dealing with this unobserved auction heterogeniety is Krasnokutskaya (2004)

- She does something similar to the homogenisation trick we have seen in dealing with observed heterogeniety

- I leave you to go through it if you are interested. However, the key thing to note is that identification of $F_{\mathbf{U}}(\cdot|\mathbf{Z})$ is now possible only up to a locational normalisation. So you can work out the shape of the distribution but not where it sits.

- So in dealing with unobserved heterogeniety we put a constraint on the applicability of the methods. I think there is scope for more research here, although any breakthrogh is likely to come from a different apporach.

6.2.2 Ascending Auctions

- Because the map from bidding to valuations is much easier here the central issue is how to deal with the fact that most approaches deal with independence, but unobserved heterogeniety implies a violation of independance.

- An adapation of the FPSB tricks may be useful here ??

6.3 Bidder Heterogeniety

- As far as I can tell people have very little idea of how to handle bidder heterogeniety that is observed by bidders but not the econometrician. As a result I just talk about what people have done w.r.t. observed heterogeniety.

- FPSB: When it comes to observed bidder heterogeniety, it seems likely that the tricks we use for dealing with auction heterogenity could be adapted here. Of course, repeated observations of the same bidder would make identification more compelling. The only thing is to think deeply about how you preserve the assymetry between the bidders in the equilibrium equation.

- Ascending Auctions: Athey and Haile have some identification results that suggest that bidder specific covariates would be usful for the identification of ascedning auctions where independance breaks down. With a good application, this would be an interesting avenue to pursue. No applied work has been done here, at least to my knowledge.

```
% this is code generating data for bidding in a FPSB SIPV auction and then estimating
% it a la GPV(2000)

% there is a mistake in this code in the construction of the bids I think

% set the number of bidders

nb = 3;
% Below is the code that generates the bids.
% % generate the data:
% % values:
% vsimsize = 20000;
% V = randn(vsimsize/2,1);
% V = V + 4;
% x1 = find(V<1);
% V(x1) = [];
% x2 = find(V>7);
% V(x2) = [];
% V = [V;V+3];
% V = sortrows(V);
% % so max V = 10, min V = 1
% % now we need to work out the value of the CDF of V - its easiest to just do this via s ↙
imulation
% supp = (9./(vsimsize*5)).*[1:(vsimsize*5)] + 1;
% supp = supp';
% [n k] = size(supp);
% CDF = [supp zeros(n,2)];
% for i = 1:n
%     x = V < supp(i);
%     if sum(x) > 0
%         CDF(i,2) = V(sum(x));
%     end
%     CDF(i,3) = sum(x)./vsimsize;
% end
% % Now work out the bids
% % bid = v - integral, 0 to v of (F(y)/F(v))^(n-1) dy
% % note that the way I am going to compute integrals puts a very slight bias into
% % things that i would fix if this were for reasearch
% [n k] = size(CDF);
% integral = zeros(n,1);
% for i = 1:n
%     if CDF(i,2) > 0
%         integral(i) = ((sum(CDF(1:i,3).^(nb-1))/i).*(CDF(i,1)-1))./(CDF(i,3).^(nb-1));
%     end
% end
% CDF = [CDF integral];
% x = find(CDF(:,2)==0);
% CDF(x,:) = [];
% % now can do the bids
% % need to pick out first time each V element turns up in CDF
% x = find(sign([0;diff(CDF(:,2))]));
% CDF = CDF(x,:);
% % now bids
% bid = CDF(:,2) - CDF(:,4);
% CDF = [CDF bid];
% disp('done')
%
```



```
% % now implement the kernel estimates
%
%
% % lets draw a kenernel of the density
% [n k] = size(CDF);
% PDF = [CDF(:,2) zeros(n,1)];
% h = 1;
% for i = 1:n
%     k = (CDF(:,2) - PDF(i,1))./h;
%     x = find(abs(k)<1);
%     k = k(x);
%     k = sum(0.75*(1-k.^2));
%     PDF(i,2) = k/(n*h);
% end
%
% save CDF CDF
% save PDF PDF
% return
load CDF
load PDF
[n k] = size(CDF);

% % % lets draw a kernel of the bids
% % [n k] = size(CDF);
% % bids = [CDF(:,2) zeros(n,1)];
% % h = 1;
% % for i = 1:n
% %     k = (CDF(:,5) - bids(i,1))./h;
% %     x = find(abs(k)<1);
% %     k = k(x);
% %     k = sum(0.75*(1-k.^2));
% %     bids(i,2) = k/(n*h);
% % end
% %
% %
% % plot(PDF(:,1),PDF(:,2),'-y',bids(:,1),bids(:,2),'-b')
% return
% keyboard

% now lets do the GPV proceedure:
% rand('seed',2)
samplesize = 250;
x = ceil(rand(samplesize,1)*n);
sample = sortrows(CDF(x,5));

n = samplesize;
h = .05;
Ghat = zeros(n,1);
ghat = zeros(n,1);
for i = 1:n
x = sample < sample(i);
Ghat(i) = sum(x)./n;
    k = (sample - sample(i))./h;
    x = find(abs(k)<1);
    k = k(x);
    k = sum(0.75*(1-k.^2));
    ghat(i) = k/(n*h);
```

```
end
u = sample + Ghat./(ghat.*(nb-1));
h = 1;
Updf = [u zeros(n,1)];
for i = 1:n

    k = (u - u(i))./h;
    x = find(abs(k)<1);
    k = k(x);
    k = sum(0.75*(1-k.^2));
    Updf(i,2) = k/(n*h);
end
Updf = sortrows(Updf,1);
plot(Updf(:,1),Updf(:,2),'b-',PDF(:,1),PDF(:,2),'y-')

% PDF = Updf;
% save PDF PDF
% I have saved PDF as the estimated kernel (samplesize = 16000 with h=.3)
% from that data, it shoul be the real values, but I dont
% want to use them cos there is a problem in tthe way have computed the bids.
```